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Infrastructure access charges, service differentiation, and strategic competition in the EU railway passenger market.

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1. Introduction

Since the beginning of the XXI century, four legislative railway packages have come into force in the EU. Increasing competition, enhancing the supervision of national authorities responsible for railway activities, and encouraging private and public investment are the main targets. Accordingly, EU Member States have adopted a variety of legislative acts.[1] The current framework for an effective liberalization of the railway passenger sector has been defined in the 4th Railway Package, in particular the amended Directive 2012/34/EU and Regulation 1370/2007. In this paper, we focus on the effect of liberalization on long distance railway passenger services. On the way towards liberalization, other sectors such as air transport, telecommunications and postal services, merely eliminate exclusive rights in order to allow the entry to new operators. This policy is referred to as *competition in the market*. Industrial organization literature also called this model *open access competition* or *on-track competition*, when it refers to rail transport. Although the 4th Railway Package promotes competition in the market, it has also envisaged an alternative model with regard to long distance (both, conventional and high-speed) commercial passenger services. Indeed, EU Member States are initially allowed to limit the number of competitors and to appoint them through a tendering procedure. The competition takes place during the tender, not in the service provision.[2] This model of competition will be referred to as *competition for the market*. Literature on railways also use *franchising* and *competition for the tracks* to describe this situation (see for instance Geroski 2003, and Cherbonnier et. al., 2018). Indeed, competition for the market will be the standard regime within EU Member States for domestic passenger services under public service obligations (PSOs, hereinafter), according to Regulation 1370/2007.[3]

The model presented here provides a useful paradigm to study the liberalization process where duopoly competition is generally the option in the early stages. Our framework involves three types of agents: regulatory authorities, the network infrastructure manager (NIM, hereinafter) and train operation companies (TOCs, hereinafter). First, regulatory authorities determine infrastructure

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[1] For a worldwide survey, see Laurino et al., 2015.

[2] The 4th Railway Package allows Member States to limit the number of competitors and to appoint them through tenders in two different scenarios. In the first scenario, it is possible to grant exclusive rights to provide bundles of services including commercial long distance services and services under PSOs. The benefits derived from the provision of commercial services can fully or partially finance the provision of services under PSOs. In the second scenario, EU Directives allow Member States to restrict the number of competitors for a transitory period (until 25 December 2026) if a tender for the assignment of a second authorization to compete with the incumbent took place before 25 December 2018.

[3] Regulation EU 2016/2338, art. 192: "Public service contracts for public passenger transport services by rail should be awarded on the basis of a competitive tendering procedure, except for those cases set out in this Regulation".

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access charge rules that the NIM may apply in two differentiated market segments: peak and off-peak time slots. The separation between peak and off-peak periods is also assumed in Börjesson et al. (2021). They use this framework to simulate how the profit, welfare, fares, frequencies, modal shares and train size depend on the level of the track charges. Second, taking into account the regulatory authority constraints, the NIM determines infrastructure access charges to operate at each time slot. Finally, in a Hotelling[4] product differentiation model, competition takes place in the service provision, setting fares in peak and off-peak market segments. We assume that TOCs want to capture as high a market share as possible. Accordingly, the NIM grants the whole network capacity, and time slots are assigned in a balanced manner (i.e., each operator gets 50% of the total capacity).

In our setting, service differentiation arises from two aspects. First, there is horizontal differentiation concerning peak and off-peak services. Although peak and off-peak train times vary across routes and railway companies, peak trains usually run from 6:30 to 9:30 and from 16:00 to 19:00 on weekdays, whereas off-peak travel times are normally between around 9:30 to 16:00 and from 19:00 onwards. Peak time is related with business hours and commuting to get to meetings or office jobs, evening hours to get back home, Friday evenings to get to a weekend second residence, etc. In these cases, trains have low substitutability. Off-peak time include those departures and arrivals with high substitutability among them, mostly those identified with leisure trips and consumers who are unwilling to pay a lot. Second, we introduce a parameter measuring consumers' extra valuation when travelling at peak times, which in turn is related with the low substitutability of these services. Thus, from TOCs' point of view, there is an incentive to set higher fares in peak hours due to consumers' extra willingness to pay. Although this extra valuation can be considered either as horizontal or vertical differentiation, the interpretation does not alter our results. There are some papers including a vertical parameter in a Hotelling model to explain why some consumers derive more utility from purchasing a given variety. In Cremer and Thisse (1991), a general framework is defined to point out that both types of differentiation models can yield similar equilibria. Degryse (1996) includes both types of differentiation to explain why bank consumers derive an extra disutility when they come to a physical visit.

Our contribution to the literature is twofold. On the one hand, vertical separation between the NIM and the service provision is studied under different track access charge regulatory regimes. On the other hand, we explicitly introduce market segmentation (peak and off-peak) and study the effect that this feature has on final fares, NIM and TOCs profits, and welfare. Overall, our results strongly depend on the regulatory scheme applied to the NIM by the regulatory authorities. We found that consumers' extra quality valuation of peak services and NIM costs determine infrastructure access charges, which in turn determines the level of fares, and the number of trains in each time slot. In particular, when regulatory authorities allow the NIM to extract surplus from the peak market, peak fares increase whereas train frequencies decrease, and the contrary holds in the off-peak market. Moreover, there is an inverse relationship between infrastructure access charges and consumers' aggregate utility.

The rest of the paper is organized as follows. Section 2 reviews the literature related with the issue under study, whereas Section 3 presents the current situation of the EU passenger railway market. Section 4 reviews network access principles. Section 5 presents and solves the model. In Section 6, we analyze and discuss the results. Section 7 concludes. Appendix 1 includes Table A summarizing notation whereas Table B presents acronyms used thorough the paper. Proofs of propositions are relegated to Appendix 2.

2. Related literature

The recent development of the European railway sector has attracted attention from practitioners and researchers. The ongoing liberalization process and the opportunity to compete with state-owned operators raise a number of interesting questions concerning the evolution of fares, frequencies and the overall social surplus (in particular, the extent to which liberalization benefits or damages consumers and TOCs).

Transport markets benefit from liberalization in different ways. Although in some cases there is complementarity among transport services (see for instance Clark et. al., 2014), in other cases, competition takes place within the same transport mode. High-speed long distance trains may compete directly with air transport in distances below 800-900 kms or 4 hours. Besides, there is no effective competition with long-distance coach transport, which is a marginal mode in Europe. Usually, it connects marginal origin-destination relationships without rail transport, especially in Western and Southern Europe. Although some recent experiences of liberalization may represent a change in the coach transport sector, it remains a narrow market (see for instance Grimaldi et al. 2017 for the case of Germany and Italy).

A general vision of the implementation of high-speed railway services around the world can be found in Campos and de Rus (2009). They draw conclusions over various aspects that are important to railway stakeholders and end users. In Fraszczyk et al. (2016), statistical data on various passenger-related parameters of the railway system in a number of selected European countries are compared. They also study operating, maintenance costs, and some economic facts related to liberalization processes by country. Stead et al. (2019) analyze the increasing evidence of competition in the British railway market. Preston et al. (1999) analyze several scenarios of duopolistic on-track competition including product differentiation. They conclude that on-track competition usually cannot benefit passengers more than it harms suppliers, and therefore reduces overall welfare. Álvarez-SanJaime et al. (2015) also study on-track competition between two train operator companies on a high-speed railway line and air transport services including private and/or public operators. Private operators are assumed to maximize profits, whereas a public operator maximizes social welfare subject to a break-even constraint. Simulation results show that the entry of a TOC increases welfare only if it entails very large increases in rail-traffic demand. They also found that such welfare gains are substantially larger if the incumbent TOC is not privatized.

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[4] There is an extensive literature concerning the classical model by Hotelling (1929). Here, we use the standard approach with linear transportation cost.

Industrial organization and game theory provide a useful framework for analyzing duopoly scenarios in a simplified environment where agents, strategies, and payoffs yield to different equilibria (Rasmussen, 2006). There are many examples of game theoretical models applied to transportation problems (Adler et al., 2010, 2014; Cantarelli et al., 2013; Takebayashi, 2015, among others). Most of these papers find stable Nash equilibria outcomes assuming that agents behave rationally by maximizing their payoffs. For instance, Palacín and Ruiz-Rúa (2013) assume Bertrand competition. They provide insights to quantify the minimum requirements for a new operator to stay in the market as well as the equilibrium price and the level of investment required. Alvarez-SanJaime et al. (2016) analyze the vertical structure and endogenous access charges in competition on high-speed railway lines. They find that on-track competition generates larger welfare gains if infrastructure and rail operation activities are vertically integrated, and implement marginal access pricing. Overall, if market entry does not generate enough demand, then it cannot enhance welfare at all. Gutiérrez-Hita and Ruiz-Rúa (2019) present a quantity-setting duopoly model in a railway passenger market. They focus on the differences between the monopoly situation and a duopoly market where operators compete by offering a homogeneous service. They conclude that duopoly competition may enhance welfare as long as operational variable costs and the access charges to enter the network are low enough. Broman and Eliasson (2019) also study a duopoly market for passenger rail services and thereby assume that price competition occurs between trains with close departure times. They show that welfare increases when moving from a profit-maximizing monopoly to competition when the regulatory board may prevent TOCs from purchasing the respective competitor's access rights and restoring monopoly. If operators offer closer departures to each other (i.e., a departure of a given operator is always followed by a departure of the competitor), welfare is maximized. According to this result, it seems that the number of time slots and their distribution between competitors is of great importance. In a horizontal differentiation model, van der Weijde et al. (2014) explore simultaneous and sequential competition among transport operators. They found that departure times can be strategic instruments and services are scheduled closer together than optimal. In van Reeven (2006), operators' pricing, departure timing and frequency decisions are analyzed in a horizontal product differentiation model. It is found that operators choose intervals between their own departures that minimize consumers' waiting costs, and set prices that do not depend on the level of competition in terms of frequency.

Related to the distribution of time slots, track access charges are of great importance. In Sánchez-Borrás et al. (2010) and Ljungberg (2013) the effect of track access charges in fares is studied. The first one assumes that increases in charges are directly passed on to the consumers through higher fares, whereas the second one assumes that it occurs partially. In contrast, Börjesson et al. (2021) found that increases in track access charges have a limited impact on fares but larger impacts on frequencies. They also provide a simulation based on the Stockholm-Gothenburg line by considering welfare optimal track charges under different levels of congestion. In this sense, they use infrastructure access charges as a pricing instrument to allocate train slots efficiently. As we will see, we reach a similar result concerning the allocation of trains between the two different types of time slot.

3. Competition in the EU railway market

Although a significant number of EU Member States had already promoted competition in the railway passenger market even before the 4th Railway Package, in recent years we have seen that due to entry barriers, duopolies have proliferated. With respect to operator ownership, European railway liberalization facilitates the entry of private operators in the market while the former legal monopoly operator remains in public hands. Indeed, this is the most observed configuration across EU Member States. Moreover, no case of privatization of the former legal monopoly passenger operator within the EU has been reported so far except for the case of the UK (see for instance Preston, 1996 and Welsby & Nichols, 1999). Market entry experiences across European countries are classified into four groups; (i) fringe competition (or line-niche competition); (ii) non line-intensive competition (service extension); (iii) line-intensive competition; and (iv) large-scale competition in the whole network against the former legal monopolist. Table 1 reports TOCs' newcomers by country in the European railway market by November 2020. With regard to the criteria for choosing the TOCs, we follow Perennes (2017), where she defines the COAPS terminology (competing open access passenger services) that classified the different open access services across Europe.[5]

Line-niche occurs when a private TOC offers some services on a line without competition. In this case, although the country is open to competition, effective competition does not take place. Line-intensive competition holds when there is effective competition in a railway corridor (within the entire line) whereas non line-intensive or service extension only takes place on part of the line or when frequencies are significantly less than the main TOC. Finally, large-scale competition refers to those cases where frequencies are similar between TOCs in practically the entire nationwide network.

As Table 1 states, line-niche or fringe competition is observed especially in Germany, France (Ivaldi and Pouyet, 2018, Perennes, 2017, and Ramos, 2020), Sweden (Vigren, 2017) and Italy (Beria and Grimaldi, 2016, Grimaldi et. al., 2017), whereas non line-intensive competition on one line is observed in Slovakia, Poland, and Hungary (Ramos, 2020). There are three cases of line-intensive competition: Austria (ÖBB/Westbanh, in Salzburg-Vienna on a conventional line), Italy (NTV/Trenitalia, in the Italian high-speed network), and Sweden (SJ/MTR, on the Gothenburg-Stockholm conventional line; see Vigren, 2017). A triopoly line-intensive configuration can be observed in the Czech Republic market (where CD/Regiojet/Leo Express compete on the conventional Prague-Ostrava line; see Tomeš et al., 2014, 2016). The only successful case of network large-scale entry is the operator Italo-NTV in the Italian high-speed network (Bergantino et al., 2015; Beria and Grimaldi 2016, Beria and Grimaldi 2017, among

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[5] The interested reader can visit https://www.irg-rail.eu/download/5/722/IRG8thMMReport-Dataset-final.xlsx (point 9). This table by IRGRail reports operators and number of trains run by 2018.

Table 1
TOC newcomers in countries already open to competition (November 2020) and expected TOC newcomers since December 2020.

Country	Private operators Italics: non-operating TOCs. Bold: effective competition	Type of competition
Sweden	SNALLTAG (Since 2010)	Line-niche (Low cost)
	BLA TAG (2011-2019)	Line-niche (Luxury)
	GRONA TAG (2016-2018)	Line-niche (Low cost)
	MTRX (Since 2015)	**Line-intensive**
	FLIXTRAIN (Since August 2021)	Non line-intensive
United Kingdom	FIRST HULL (Since 2000)	Line-niche (route)
	GRAND CENTRAL (Since 2007)	Line-niche (route)
Germany	HEX (Since 2005)	Line-niche (Ultra Low Cost / route)
	HKX (2012-2017)-FLIX[1]	Line-niche (Low Cost)
	LOCOMORE (2016-2017)-FLIX[2]	Line-niche (Low Cost)
Italy	*ARENAWAYS (2012)*[3]	Line-niche luxury
	NTV (Since 2012)	**Line-intensive, large-scale**
Austria	**WESTBAHN (Since 2013)**	**Line-intensive**
Czech Rep.	**REGIOJET (Since 2011)**	**Line-intensive**
	LEO EXPRESS (Since 2012)	**Line-intensive**
Slovakia	*REGIOJET (2014-2017)*	*Non line-intensive (service extension)*
	LEO EXPRESS (Since 2014)	Non-line intensive (service extension)
Poland	LEO EXPRESS (Since 2018)	Non-line intensive (service extension)
Netherlands	*LOVERS RAIL (1996-1999)*	Line-niche (ultra-low cost)
Romania	5 regional small operators[4]	Line-niche
Hungary	REGIOJET (Since 2020)	Non-line intensive (service extension)
Spain[7]	**ILSA-TRENITALIA[5]**	**Line-intensive (large-scale)**
	DB[6]	Non line-intensive
	OUIGO SPAIN (SNCF)	**Line-intensive (Low Cost)**
France	**FLIXTRAIN[8]**	**Line-intensive**
	RENFE	Line-niche (service extension)
	THELLO (2011-2020)-THELLO[9]	Line-niche (service extension)
Belgium	No entrants expected	
Finland		
Greece		
Slovenia		

Source: Authors' own elaboration. (1) The operation continues as *FlixTrain* from March 2018. (2) Locomore services were taken over by *Leo Express* after bankruptcy. *FlixTrain* sells tickets. (3) When the Torino-Milano high-speed infrastructure was established, *Trenitalia* began to operate services on this new infrastructure, leaving the old infrastructure to the *Arenaways* operator. Arenaways used conventional luxury coaches, offering a differentiated service. In this sense, *Arenaways* occupied a line-niche. (4) These small private companies operate subject to a public service obligation, through directly awarded contracts. It is not therefore a pure case of competition in the market. (5) The first attempt to enter the network was October 2018 in the line Madrid–Montpellier with cabotage services at Barcelona and Zaragoza, but access right was withdrawn. (6) From 7/17/19, it has rights of access from Porto to A Coruña, with cabotage services at Santiago de Compostela, Pontevedra and Vigo. (7) ADIF, the NIM in Spain has granted three packages in response to market liberalization. Package A is endowed with the largest capacity and allows new TOCs a ten-year operation agreement; package B is medium-sized and also ensures ten years of operation; finally, package C is the smallest one in terms of capacity, under the same operative conditions. (8) *FlixTrain* requests paths in conventional lines for routes: Paris–Brussels North; Paris–Bercy–Lyon–Perrache; Paris–Bercy–Nice (night train); Paris–Bercy–Tolouse; and Paris–Austerliz–Bordeaux, by 2020. Entry is subject to Arafer (French Railway Regulator) approval. Finally, *FlixTrain* has decided to postpone its plans to launch these services indefinitely. (9) Although *Thello* stopped international services Venice-Paris and Milan-Nice by November 2020, the operator has already requested train paths to operate the Paris-Milan route.

others). Finally, Spain and France have reported information about new operators during 2020. In both countries, competition is expected to occur with the state-owned operator, so may be observed on some lines as from 2021.

In accordance with the empirical evidence, the current situation across EU Member States where line-intensive and large-scale competition takes place is a duopoly structure (except the aforementioned case in the Czech Republic). Moreover, in those countries that already have a monopoly in the provision of the service, like France and Spain, duopoly competition is expected on most of the main routes in the first stages of the liberalization process.[6]

4. Network access principles

Following the spirit of the 4th Railway Package (according to Directive 2016/45 EC), let us assume that the NIM grants time slots under equal opportunities, taking into account the requests submitted by TOCs and the network capacity, resulting in a balanced schedule assignment.[7] Among others features of the market, the market pillar (which comprises Directive 2016/2370 EC, Regulation (EU) 2016/2338 and Regulation (EU) 2016/2337) determines the access charge to the rail network infrastructure. It must allow non-discriminatory access for all TOCs, providing efficient economic signals in order to promote an effective use of the network – a higher total traffic volume under an efficient distribution of frequencies in any time slot-, enhance cost efficiency of the NIM, and optimize network investments.[8] Moreover, Member States shall establish a charging framework while respecting management independence. Thus, specific charging rules may be established or such powers may be delegated to the infrastructure manager.[9] In Section 2, Article 31 established that the minimum access charge to the rail network infrastructure shall be set at the cost directly attributable to the operation of the railway service, ensuring a traffic volume that can pay at least the average cost of the NIM. This principle includes the recovery of investment costs.

NIMs may have different objectives such as to maximize profits or enhance consumers' utility by keeping fares stable and moderate, among others (for a wide review of NIM objectives see Svedberg, 2018). According to the above discussion, we study two regulatory schemes:

- **Hard regulation**. Under this scheme, we assume that regulatory authorities oblige the NIM to set infrastructure access charges equal to average costs. Nevertheless, under this price scheme, network infrastructure profits are zero and governments usually need to implement subsidies in order to maintain and improve railway infrastructure. Thus, the NIM depends on the government budget constraint.
- **Skimmed regulation**. Under this system, regulatory authorities set off-peak access charges equal to average costs, leaving peak access charges as a NIM's strategic variable. EU legislation allows track access charges to be higher than the marginal social cost to achieve cost recovery.[10] It resembles Ramsey prices where there are large fixed costs to cover. Admittedly, as investments in infrastructure are huge due to depreciation of the infrastructure and network improvements, government's subsidies may not cover total investments and maintenance. As a result, it may leave the NIM with negative profits. To solve this problem, let us assume that regulators allow the NIM to extract surplus from the peak business market segment, aimed at recovering part of the depreciation and infrastructure investment. Off-peak infrastructure access charge remains fixed at average cost due to social concerns, allowing access to the railway at moderate fares. Under this scheme, the government may avoid subsidies but, as we will see, it strongly affects consumers' aggregate utility.

In the next section, we present and solve the model. Notice that the above regulatory schemes determine the NIM's market behavior and thus, indirectly affect market outcomes.

5. The model

Our game has two stages under perfect and complete information. Once regulatory authorities set rules, at the first stage the NIM fixes infrastructure access charges and grants TOCs' capacity at both peak and off-peak time slots. At the second stage, competition in prices takes place in order to capture demand. The model is solved by backward induction to get subgame perfect Nash equilibria under the two regulatory schemes. Each regulatory scheme is considered a different state of the nature (the reader may assume that it is at stage zero, when the player *nature* moves). Once equilibria are characterized, we will discuss our results and present simulations based

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[6] Recently, the Spanish NIM, ADIF, has granted three packages of time slots to three operators; the state-owned RENFE, Rielsfera (a French brand), and ILSA (a joint-venture brand comprising Italian and Spanish firms). However, time slots have been granted on separate packages, limiting competition.

[7] Although in most of the cases, the NIM grants time slots, there are some cases where an independent authority decides the assignment of slots.

[8] These ideas are mainly covered in the fourth chapter, Section 2.

[9] It is interesting to note that in some EU Member States regulatory authorities may oblige the NIM to allocate time slots in a balanced manner whereas in others the NIM actually acts as a regulator.

[10] For instance, in the UK, track access charges cover the short run marginal cost, demanding large subsidies from the government to cover the fixed infrastructure costs (Nash et al., 2018). In the case of Germany and France, track access charges aim for full cost coverage (except for investments in Germany), resulting in some form of Ramsey pricing, with the result that the track access charges make up 40% of the ticket price (Nash et al., 2016).

on the empirical evidence in Section 6.

5.1. Second stage: competition in fares

We consider a duopoly model in a Hotelling fashion where TOCs provide a differentiated passenger service by offering frequencies in both peak and off-peak hours. Frequencies are granted under the assumption that the state-owned operator, I, and an entrant operator, E, compete for running trains interspersing one after the other both at peak and off-peak hours. This assumption is in line with Broman and Eliasson (2017), and with the ongoing liberalization process that promotes that newcomers compete under equal conditions.[11]

Consumers are uniformly distributed along the unit interval, $x \in [0, 1]$. Peak services (p) are on the left hand side ($x = 0$), whereas off-peak services (v) are on the right hand side ($x = 1$). Accordingly, consumers located close to zero prefer to travel by using peak services (those consumers tend to be commuters) whereas those located close to one have strong preferences for off-peak services (those consumers can be identified as people with a low willingness to pay). In this respect, Lovrić et al. (2016) has studied an activity-based demand framework to evaluate off-peak pricing strategies within a rail context. Moreover, consumers identify peak services with an extra quality aspect according with the parameter $\mu > 0$. Thus, consumers travelling on peak-time services derive utility $U_p = \rho + \mu - x - P_p$, whereas those travelling on off-peak services bear a utility function $U_v = \rho - (1 - x) - P_v$. Parameter ρ denotes the gross surplus that a consumer at point x enjoys from purchasing the service, whereas P_p and P_v are equilibrium fares observed by consumers once price competition between TOCs has taken place. Let us define by $\widehat{x}$ the indifferent consumer between both types of services by setting $U_p = U_v$,

$$\widehat{x} = \frac{1+\mu}{2} + \frac{P_v - P_p}{2}. \tag{1}$$

5.1.1. Demands

Taking [1], we define demand functions for both types of services. Demand for peak services is $D_p(P_p, P_v) = \widehat{x}$, whereas demand for off-peak services is $D_v(P_p,P_v) = 1 - D_p(P_p,P_v)$.[12] Demand for each TOC is $d_{ih}(P_h,P_{-h})$, where $i = \{I, E\}$, and $h, -h = \{p, v\}$, $h \neq -h$. Then,

$$D_h(P_h, P_{-h}) = \sum_{i=I,E} d_{ih}(P_h, P_{-h}) \tag{2}$$

We assume that trains' capacity is given and equal for all TOCs.[13] The number of trains each TOC runs is defined by[14],

$$q_{ih}(P_h, P_{-h}) = \frac{d_{ih}(P_h, P_{-h})}{\theta}, \tag{3}$$

where θ is a factor to convert demand (as trains' capacity is exogenously given, θ decreases as demand increases). In other words, it is the inverse of frequency that the NIM offers in accordance with the expected demand. Then, in those railway corridors when demand is high, the number of trains run will be larger. We introduce the following assumption:

Assumption 1. Demand is partitioned as a multiple of eight (a round trip for each TOC at each market niche). Thus, $\theta = \frac{1}{8n}$, ($n \in \boldsymbol{N}$, times that demand is partitioned into eight parts).

This assumption is useful for introducing different levels of demand: in those corridors where the volume of passengers is high, total demand is partitioned more times.[15] Thus, the number of trains at each type of time slot is,

$$Q_h(P_h, P_{-h}) = \sum_{i=I,E} q_{ih}(P_h, P_{-h}). \tag{4}$$

Finally, the total number of trains running thorough the network is $Q = \sum_{h=p,v} Q_h(P_h, P_{-h})$. We consider a standard origin-destination pair and that TOCs use identical rolling stock. These assumptions mean that costs per train kilometer and costs per train run are the same for all TOCs.

5.1.2. Costs structure of TOCs

Each operator running trains bears the following costs. First, the infrastructure access charges to enter the network. It includes the right to run trains, electric power consumption, stops at stations and other technical operations such as switching rolling stock at

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[11] Regulatory authorities make it compulsory for the state-owned operator to run trains in both types of time slots in order to introduce effective market competition and cover those routes under PSOs.

[12] Notice that customers are likely to have preferences for when to travel within peak and off-peak periods. Hence, a customer with a preferred departure time, t^*, within each type of period will choose (ceteris paribus) between the last train arriving before t^* and the first train arriving after t^*. Our model does not delve into this particular detail of spatial competition within each type of period. It deserves a further research and it is out of the scope of this paper.

[13] On average, trains capacity is 450 seats under single configuration and one floor.

[14] Notice that $q_{ih}(P_h,P_{-h})$ stands for one trip (one-way).

[15] We assume there is at least a round trip train per operator; i.e., the maximum value of θ is 1/8.

terminals, among others. Across EU Member States, different types of infrastructure access charge are observed. Usually, they should work as two-part tariff schemes, where the fixed component that the NIM charges, covers the right to enter the network and run trains. However, this fixed component may represent an entry barrier for newcomers because usually they obtain lower economies of scale than the state-owned operator does. Indeed, competition authorities and some academic studies advise the reduction or almost total disappearance of that fixed component. For instance, in Nash, 2005 it is argued that EU Directives state that the average and marginal charges in the infrastructure access should be close, which rule out the possibility of two-part tariffs. Following this spirit, EU Member States tend to withdraw the fixed component of the network access (in France and Germany, it is almost zero). Hence, let us assume that the fixed component is zero, letting the infrastructure access costs $T_{ih}(q_{ih}) = e_h \cdot q_{ih}(P_h, P_{-h})$. It depends on the number of trains each operator runs, where the parameter $e_h > 0$ is the infrastructure access charge; i.e., the marginal cost to run a train. Second, labor costs (mainly due to train crew salaries) are $L_{ih}(q_{ih}) = \sigma \cdot q_{ih}(P_h, P_{-h})$, where $\sigma > 0$. Finally, operational costs due to ticketing, advertising, passenger attendance, etc. are $C_{ih}(d_{ih}) = \alpha \cdot d_{ih}^2(P_h, P_{-h})$ where $\alpha > 0$.[16] Then, total costs are,

$$\varphi_{ih}(d_{ih}) = T_{ih}(q_{ih}) + L_{ih}(q_{ih}) + C_{ih}(d_{ih}). \tag{5}$$

5.1.3. TOCs profit maximization

Each TOC maximizes profits and competition takes place in prices in order to capture demand at each type of time slot,

$$\pi_i = \sum_{h=p,v} \left[P_h \cdot q_{ih}(P_h, P_{-h}) - \varphi_{ih}(d_{ih}(P_h, P_{-h})) \right]. \tag{6}$$

The service offered at each market niche is homogeneous.[17] It means that $U_{ih} = U_{jh}$, $(i, j = \{I, E\}, i \neq j)$ so that consumers are indifferent as to whether they purchase a ticket from operator I or E. Thus, $P_{ih} = P_{jh}$ and at equilibrium a unique fare P_h is observed. It yields to the standard Bertrand paradox where TOCs get zero profits. Notice that in our model, once the level of demand is known, the number of trains (related with the parameter θ) is decided. We assume that there are no capacity constraints (i.e., the network is not saturated). Indeed, an operator i captures zero demand if $P_{ih} > P_{jh}$ and demand is split into two when $P_{ih} = P_{jh}$. Thus, fares are fixed at the level of average costs, $P_h = \frac{e_h}{\theta} + \frac{\sigma}{\theta} + \alpha \cdot d_{ih}(P_h, P_{-h})$. This condition yields second stage fares,

$$P_h = \frac{\sigma}{\theta} + \frac{\alpha}{4} + \frac{\alpha\left(e_p + e_v + r(h)\right) + 4e_h}{2\theta(2+\alpha)}, \tag{7}$$

where $r(h) = \begin{cases} \theta\mu \text{ if } h = p, \\ -\theta\mu \text{ if } h = v. \end{cases}$

It is interesting to report the gap between fares,

$$P_p - P_v = \frac{\alpha\theta\mu + 2\left(e_p - e_v\right)}{\theta(2+\alpha)} > 0. \tag{8}$$

The above expression reveals that the peak fare is greater than the off-peak fare. It comes from two forces: (i) the gap between infrastructure access charges, $e_p > e_v$, and (ii) the extra valuation of consumers for peak services.[18] This rationale arises from the concept of Ramsey prices (although in our model there is no elasticity of demand, the willingness to pay for peak hours is higher than that for off-peak hours and congestion is also high; then the operator can charge a higher fare at peak hours). Moreover, although EU legislation allows track charges to be higher than the marginal social cost to achieve cost recovery, Ramsey pricing (see for instance Besanko and Cui, 2016) would in many cases significantly reduce social welfare by increasing distortions in the transport market.[19] If $e_p - e_v = 0$, and $\mu = 0$, fares will be identical, no matter the frequencies and the level of α. This means that, on one hand, the NIM has the capacity to influence fares when it sets the infrastructure access charges and, accordingly, it should be controlled by regulatory authorities. On the other hand, the consumers' valuation for peak services as positive (negative) affects peak (off-peak) fares. Moreover, fare difference decreases as frequency decreases, $\partial(P_p - P_v)/\partial\theta = -2(e_p - e_v)/\theta^2(2 + \alpha) < 0$. Given θ, second stage demands are set at,

$$d_{ih}(e_h, e_{-h}|\theta) = \frac{1}{4} - \frac{e_h - e_{-h} - r(h)}{2\theta(2+\alpha)}, \tag{9}$$

5.1.4. Consumers' aggregate disutility

We are interested in the effects that liberalization has on the railway sector. In addition, we evaluate how consumers' quality

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[16] This technical assumption reflects the idea that in public transportation operations, economies of scale can be observed only up to a certain level of demand. A higher number of passengers may lead to the marginal social cost exceeding the average social cost, which will yield diseconomies of scale (see for instance Coulombel and Monchambert, 2019). It also provides interior solutions. This scheme is usually assumed in the literature; see for instance Álvarez-SanJaime et al. (2016)

[17] Admittedly, this could be a weakness of the model because consumers could view different TOCs as heterogeneous services.

[18] The difference $e_p - e_v$ is positive with quadratic costs. Moreover, we have tested the model with linear costs, and the positive difference already holds.

[19] Fares including marginal cost plus mark-ups determined by Ramsey pricing for rail infrastructure are only economically optimal when other transport modes are appropriately charged.

valuation and NIM costs determine the infrastructure access charges. We assume that both the state-owned operator and the private operator are profit maximizers. In our model, TOCs' profits are zero (when the cost structure is symmetric) and then, welfare is equal to aggregate consumer surplus plus the NIM profits (when they are positive). In a Hotelling model, where consumers have an exogenous level of gross utility ρ, the relevant measure to evaluate aggregate consumers' surplus is consumers' aggregate disutility. In this sense, the lower the level of consumers' disutility, the greater the welfare.[20] Consumers' aggregate disutility (*CD*) is,

$$\int_0^{\widehat{x}}(P_p+x)dx+\int_{\widehat{x}}^1(P_v+(1-x))dx. \tag{10}$$

We will study consumers' aggregate disutility under the two regulatory schemes.

5.2. The first stage: setting capacity and infrastructure access charges

As we assume perfect information, the NIM incorporates the information at the second stage of the game to maximize profits at the first stage. The following subsection describes NIM costs. Later on, the equilibrium is characterized.

The NIM bears two type of costs. First, there are operational costs that we assume to be increasing. This assumption comes from the following facts. On one hand, managerial duties to control trains' operations become more complex as long as the number of trains increases. This is notorious in peak hours when commuter, regional and High Speed trains operate. On the other hand, the network (stations, traffic signals…) experience physical wear as the level of use increases. Moreover, as congestion takes place, operational costs during peak time slots are higher than those during off-peak time slots. We define operational costs in peak and off-peak time slots as $N_p(Q_p)=c_p\cdot Q_p^2,$ and $N_v(Q_v)=c_v\cdot Q_v^2$, respectively, where $c_p>c_v>0$. Let us normalize $c_v=1$, without loss of generality. Second, there are costs due to network infrastructure investments and amortization. We assume that $f=F/n$ is a fixed cost per period, where F are total infrastructure costs over a number of amortization periods n. As n is usually large, $f\to\varepsilon$, where ε takes reduced values. Thus, in order to maintain the tractability of the model, let us assume that fixed cost per period f (per year amortization and annual investment cost) is included as part of the operational costs in order to recover infrastructure investments and amortization. It represents only a positive constant step in the cost function by year, which does not alter the model's results.

Concerning the allocation of infrastructure capacity, we follow the spirit of the European Law. Chapter 4, Section 3 states that infrastructure shall be allocated in a fair and non-discriminatory manner and one operator shall not transfer capacity to another.[21]

Once second stage fares are set, the number of trains each TOC will run and equilibrium fares are obtained. Taking [3] the number of trains in each type of slot are $q_{ih}=d_{ih}(e_h,e_{-h}|\theta)/\theta$. It is interesting to note that μ has a different impact on the number of trains run in each market niche. Indeed, $\partial q_{iv}(\alpha,\theta,\mu)/\partial\mu<0$, whereas $\partial q_{ip}(\alpha,\theta,\mu)/\partial\mu>0$. Moreover, the gap between the number of trains at each market niche is,

$$q_{iv}-q_{ip}=\frac{2(e_p-e_v-\theta\mu)}{2\theta^2(2+\alpha)}. \tag{11}$$

The number of trains run in off-peak time slots will be larger than those run in peak time slots as long as $(e_p-e_v-\theta\mu)>0$. As we will see, the level of e_p is always larger than $e_v+\theta\mu$ and thus the above expression is positive.

NIM chooses the level of infrastructure access charges, e_p ,e_v in order to maximize,

$$\pi_N(e_h,e_{-h}|\theta)=\sum_{h=p,v}(e_h\,Q_h-N_h(Q_h)), \tag{12}$$

where Q_h is the function $Q_h(e_h,e_{-h}|\theta)$ which comes from $Q_h(P_h,P_{-h})$ by substitution of fares in [4]. Although the NIM is a profit-maximizing firm, regulatory authorities may impose restrictions, as we have already stated. In what follows, we present solutions for the aforementioned regulatory schemes in Section 3.

5.2.1. Hard regulation

When regulatory authorities fix infrastructure access charges equal to average cost, NIM's profits are zero. Notice that although fixed costs per period are included as part of the average variable costs, there can be financial constraints due to infrastructure requirements and/or improvements. In this case, the government would need to subsidize the NIM. Let us assume that the monetary amount of the subsidy equals the profitability that the NIM can extract under Skimmed regulation (when it is allowed to extract surplus from the peak market, as we will see in the next subsection). Thus, from the social welfare point of view, both measures are equivalent (either a subsidy or, alternatively, allowing the NIM to extract surplus in order to cover extra fixed costs).

Letting $e_p=c_p\cdot Q_p(e_h,e_{-h}|\theta)$ and $e_v=Q_v(e_h,e_{-h}|\theta)$ the level of infrastructure access charges are set. Proposition 1 and Corollary 1 provide equilibrium values under Hard regulation.

Proposition 1. Under Hard regulation, the level of infrastructure access charges is as follows:

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[20] External effects may come from possible interaction with other transport modes. As in our model of price competition à la Hotelling when there is no elasticity of demand, external effects cannot be explicitly included.

[21] Articles 38 and 39.

$$e_p^H = \frac{c_p}{2\theta}\left[1+\gamma\left(c_p,\theta,\mu,\alpha\right)\right],\ e_v^H = \frac{1}{2\theta}\left[1-\gamma\left(c_p,\theta,\mu,\alpha\right)\right].$$

whereas fares and demands at peak and off-peak time slots are, respectively,

$$P_p^H = \frac{\sigma}{\theta}+\frac{\left(2c_p+\alpha\theta^2\right)}{4\theta^2}\left[1+\gamma\left(c_p,\theta,\mu,\alpha\right)\right],\ P_v^H = \frac{\sigma}{\theta}+\frac{\left(2+\alpha\theta^2\right)}{4\theta^2}\left[1-\gamma\left(c_p,\theta,\mu,\alpha\right)\right].$$

$$d_{ip}^H = \frac{1}{4}\left[1+\gamma\left(c_p,\theta,\mu,\alpha\right)\right],\ d_{iv}^H = \frac{1}{4}\left[1-\gamma\left(c_p,\theta,\mu,\alpha\right)\right],\ \text{where}\ \gamma(c,\theta,\mu,\alpha) = \frac{1-c_p+2\theta^2\mu}{1+c_p+\theta^2(2+\alpha)}.\blacksquare$$

Taking fares from Proposition 1, it is interesting to show here the difference in fares between time slots,

$$P_p^H - P_v^H = \frac{\alpha}{2}\gamma\left(c_p,\theta,\mu,\alpha\right) + \frac{c_p-1+\left(c_p+1\right)\gamma\left(c_p,\theta,\mu,\alpha\right)}{2\theta^2} > 0. \tag{13}$$

As we can see from Proposition 1, infrastructure access charges and fares in peak time slots are higher than those in off-peak time slots. These expressions and demands at equilibrium depend on $\gamma(c_p,\theta,\mu,\alpha)$. It is interesting to characterize the function $\gamma(c_p,\theta,\mu,\alpha)$ in order to understand the results of the model. To do so, we introduce here Assumption 2 and Definition 1.

Assumption 2. $c_p \geq 1 + 2\theta^2\mu$.

Assumption 2 deserves an explanation. We assume that c_p is larger than c_v (that we have normalized to one) and the term $2\theta^2\mu$. It means that c_p exceeds the extra value that consumers grant to peak services and the effect of frequencies. As frequencies decrease, there is lower substitutability between peak and off-peak services, increasing the congestion at peak hours. In addition, as μ increases, there may be a higher number of peak trains and thus, costs due to congestion will increase accordingly. As the number of trains must be a discrete number, we present Definition 1 prior to Corollary 1.

Definition 1. Let us define $\gamma(c,\theta,\mu,\alpha)$ as the average load factor function.[22] The average trains' load factor (ALF_h) for each type of slot is,

$$ALF_h = \begin{cases} \dfrac{Q_h}{ROUND[Q_h]} = 1,\ when\ \gamma\left(c_p,\theta,\mu,\alpha\right) = 0, \\ \dfrac{Q_h}{ROUND[Q_h]+1} < 1,\ when\ \gamma\left(c_p,\theta,\mu,\alpha\right)\langle 0. \end{cases}$$

The interpretation of ALF_h is as follows. On the one hand, when trains run at full capacity ($\gamma(c_p,\theta,\mu,\alpha) = 0$, and thus $ALF_h = 1$) the effective number of trains operated is equal to the frequency previously offered by the NIM. On the other hand, when trains run are not at full capacity ($\gamma(c_p,\theta,\mu,\alpha) < 0$, and thus $ALF_h < 1$) there is a redistribution of frequencies between peak and off-peak time slots and the total number of trains can be higher than the amount offered by the NIM in advance. This case often occurs in real dispatch due to daytime adjustments. Usually it does not differ by more than one or two trains from those previously scheduled.

Corollary 1. *The number of trains each TOC runs at each time slot is* $q_{ih}^H = \frac{1}{4\theta}d_{ih}^H$, *whereas the total number of trains is* $Q^H = \sum_{h=p,v} Q_h^H(e_h, e_{-h}|\theta) = \frac{1}{\theta}$. ■

Corollary 1 characterizes the number of trains once demands are obtained. It is interesting to study the cases of fully occupied and not fully occupied trains. Indeed, the former can be used as a paradigm case. According with ALF_h, when $\gamma(c_p,\theta,\mu,\alpha) = 0$, it holds that $Q_h = ROUND[Q_h]$, and thus $ALF_h = 1$. It means that trains are fully occupied and, accordingly, equilibrium values are,

$$P_p^H = \frac{\sigma}{\theta}+\frac{\left(2c_p+\alpha\theta^2\right)}{4\theta^2},\ P_v^H = \frac{\sigma}{\theta}+\frac{\left(2+\alpha\theta^2\right)}{4\theta^2}.\ P_p^H - P_s^H = \frac{c_p-1}{2\theta^2} > 0.$$

$$d_{ih}^H = \frac{1}{4},\ e_p^H = \frac{c_p}{2\theta},\ e_v^H = \frac{1}{2\theta},\ q_{ih}^H = \frac{1}{16\theta}.$$

In order to evaluate consumers' aggregate disutility, we first report the indifferent consumer, $IC = \frac{1}{2} + \gamma(c_p,\theta,\mu,\alpha)$. When $\gamma(c_p,\theta,\mu,\alpha) = 0$ (fully occupied trains) consumers are equally distributed among time slots, whereas when $\gamma(c_p,\theta,\mu,\alpha) < 0$ there is an asymmetric distribution of consumers. Consumers' aggregate disutility is reported in Appendix 2.

5.2.2. Skimmed regulation

In this case, regulatory authorities allow the NIM to extract surplus from peak time slots in order to recover cost due to depreciation and investments. Off-peak infrastructure access charge, e_v, remains equal to average cost whereas peak infrastructure access charge, e_p, is taken as a strategic variable by maximizing [12]. Proposition 2 and Corollary 2 provide equilibrium values under Skimmed regulation.

Proposition 2. *Under Skimmed regulation, the level of access charges is as follows:*

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[22] The operator $ROUND[x]$ is the integer closest in value to x. For instance, $ROUND[3.7] = 4$.

$$e^S_p = \frac{1+2c_p+(2+\alpha)\theta^2}{4\theta}\left[1+\gamma\left(c_p,\theta,\mu,\alpha\right)\right],\text{ and } e^S_v = \frac{3}{4\theta}\left[1-\gamma\left(c_p,\theta,\mu,\alpha\right)\right],$$

whereas fares and demands at peak and off-peak time slots are, respectively,

$$P^S_p = \frac{\sigma}{\theta}+\frac{\left(2+4c_p+(4+3\alpha)\theta^2\right)}{8\theta^2}\left[1+\gamma\left(c_p,\theta,\mu,\alpha\right)\right],\ P^S_v = \frac{\sigma}{\theta}+\frac{\left(2+\alpha\theta^2\right)}{8\theta^2}\left[3-\gamma\left(c_p,\theta,\mu,\alpha\right)\right],$$

$$d^S_{ip} = \frac{1}{8}\left[1+\gamma\left(c_p,\theta,\mu,\alpha\right)\right],\ d^S_{iv} = \frac{1}{8}\left[3-\gamma\left(c_p,\theta,\mu,\alpha\right)\right].\ \blacksquare$$

From Proposition 2, the difference in fares is,

$$P^S_p - P^S_v = \frac{c_p+\theta^2-1}{2\theta^2}+\frac{\left(4\left(1+c_p+\theta^2\right)+5\alpha\theta^2\right)\gamma\left(c_p,\theta,\mu,\alpha\right)}{8\theta^2} > 0. \tag{14}$$

Notice that Skimmed regulation yields positive profits to NIM at the level,

$$\pi^S_N = \frac{2+(2+\alpha)\theta^2+2\theta^2\mu}{16\theta^2}\left[1+\gamma\left(c_p,\theta,\mu,\alpha\right)\right]. \tag{15}$$

As one can see by comparison of Proposition 1 and Proposition 2, this regulatory scheme produces a reorganization of the demand. Accordingly, the number of trains at peak and off-peak times also varies. In particular, there is a business-stealing effect from the off-peak market.

Corollary 2. *The number of trains each TOC runs at each time slot is* $q^S_{ih} = \frac{1}{\theta}d^S_{ih}$, *whereas the total number of trains is* $Q^S = \sum_{h=p,v} Q^S_h(e_h, e_{-h}|\theta) = \frac{1}{\theta}$. ■

Although as in Corollary 1, the number of trains run depends on the parameter c_p,α, and μ, for each value of θ, under Skimmed regulation the business-stealing effect observed in Proposition 2 translates in a higher number of trains in off-peak time slots. We also report here the full capacity case for $\gamma(c_p,\theta,\mu,\alpha) = 0$,

$$P^S_p = \frac{\sigma}{\theta}+\frac{\left(2+4c_p+(4+3\alpha)\theta^2\right)}{8\theta^2},\ P^S_v = \frac{\sigma}{\theta}+\frac{3\left(2+\alpha\theta^2\right)}{8\theta^2},$$

$$d^S_{ip} = \frac{1}{8},\ d^S_{iv} = \frac{3}{8},\ q^S_{ip} = \frac{1}{8\theta},\ q^S_{iv} = \frac{3}{8\theta},\ e^S_p = \frac{1+2c_p+(2+\alpha)\theta^2}{4\theta},\ e^S_v = \frac{3}{4\theta},$$

$$P^S_p - P^S_v = \frac{\left(c_p-1+\theta^2\right)}{8\theta^2},\ \pi^S_N = \frac{2+(2+\alpha)\theta^2+2\theta^2\mu}{16\theta^2}.$$

Finally, in order to evaluate consumers' aggregate disutility, we report the indifferent consumer, $IC = \frac{1}{4}[1+\gamma(c_p,\theta,\mu,\alpha)]$. As $c_p > 1$, we know that $\gamma(c_p,\theta,\mu,\alpha) \leq 0$ and thus, there are more consumers travelling in off-peak time slots than those travelling in peak time slots. Again, this effect is related with the higher peak infrastructure access charge under Skimmed regulation and the business-stealing effect from the off-peak market. In addition, off-peak (peak) fare is now lower (higher) than that under Hard regulation. Consumers' aggregate disutility is reported in Appendix 2.

6. Discussion of results and simulation of different scenarios

In this section, we extend the analysis and discuss the above results. Hard and Skimmed regulations provide different insights concerning infrastructure access charges, fares, number of trains, consumers' aggregate disutility and the NIM profits. TOCs' profits are equal to zero due to the price competition at each market niche.

6.1. Main results

First, we discuss the results under Hard and Skimmed regulations. Regulatory regimes are compared in section 5.2. Qualitative results are the same in the ideal case where trains run fully occupied ($\gamma(c_p,\theta,\mu,\alpha) \cong 0$, which yields $ALF_h = 1$), and trains not running at full capacity (when $\gamma(c_p,\theta,\mu,\alpha) < 0$, which yields $ALF_h < 1$). Throughout this section and without loss of generality, we focus on parameters c_p, μ and θ, whereas we set $\sigma = 0$ and $\alpha = 1$. This is because both parameters do not affect qualitative results. Throughout this section we use superscript index $k = \{H, S\}$.

With respect to propositions 1 and 2, we found that infrastructure access charges are positive when affected by c_p and μ, whereas they are negative when affected by θ. In particular, this effect is higher in the peak market, due to the extra cost of operating trains in this market niche. This effect is translated to fares. Explicitly,

$$\frac{\partial e^k_p}{\partial c_p} > \frac{\partial e^k_v}{\partial c_p} > 0,\text{ and } \frac{\partial P^k_p}{\partial c_p} > \frac{\partial P^k_v}{\partial c_p} > 0.$$

Moreover, TOCs' extract surplus by charging fares that are greater than infrastructure access charges, yielding double marginalization. Although TOCs' profits are zero due to price competition in each market segment, as long as peak cost increases so does the intensity of double marginalization,

$$\frac{\partial\left(P_p^k - e_p^k\right)}{\partial c_p} > \frac{\partial\left(P_v^k - e_v^k\right)}{\partial c_p} > 0.$$

The impact of consumers' extra valuation of peak services depends on the market segment. Indeed, while μ positively affects peak infrastructure access charge as c_p increases, the contrary holds in the off-peak market. Moreover, the intensity of this effect is higher in the peak market and, as in the case of c_p, it is translated to fares and the intensity of double marginalization,

$$\frac{\partial e_p^k}{\partial \mu} > \left|\frac{\partial e_v^k}{\partial \mu}\right| > 0,\ \frac{\partial P_p^k}{\partial \mu} > \left|\frac{\partial P_v^k}{\partial \mu}\right| > 0,\ \text{and}\ \frac{\partial\left(P_p^k - e_p^k\right)}{\partial c_p} > \left|\frac{\partial\left(P_v^k - e_v^k\right)}{\partial c_p}\right| > 0.$$

Concerning parameter θ, we observe a negative relation with respect to infrastructure access charges, fares, and the intensity of double marginalization,

$$\frac{\partial e_v^k}{\partial \theta} < \frac{\partial e_p^k}{\partial \theta} < 0,\ \frac{\partial P_v^k}{\partial \theta} < \frac{\partial P_p^k}{\partial \theta} < 0, \text{and}\ \frac{\partial\left(P_p^k - e_p^k\right)}{\partial \theta} < \frac{\partial\left(P_v^k - e_v^k\right)}{\partial \theta} < 0.$$

Indeed, as frequencies increase, the value of θ decreases and thus, infrastructure access charges increase. This occurs because as the use of the infrastructure increases, the NIM increases e_h^k due to the increasing marginal cost. Notice that, as the peak marginal cost c_p increases, the mark-up also increases in both markets, although the effect is larger in the peak market because it is directly affected by c_p. In other words, the ability to exert market power in the off-peak market is lower.

The effect that c_p, μ, and θ have on peak and off-peak demands, complete the discussion of propositions 1 and 2. An increase in c_p negatively affects demand for peak services and, accordingly, enhances the demand for off-peak services. The intuition is straightforward: the consumers' utility function is inversely affected by the evolution of peak and off-peak fares and thus, fares determine the evolution of the indifferent consumer, which in turn determines demands. However, μ positively affects the utility of those consumers travelling during peak service times and the contrary holds for those travelling off-peak. Finally, as frequencies increase θ decreases) there is a business-stealing effect from the off-peak services due to the lower fare,

$$\frac{\partial d_{ip}^k}{\partial c_p} < 0,\ \frac{\partial d_{iv}^k}{\partial c_p} > 0,\ \frac{\partial d_{ip}^k}{\partial \mu} > 0,\ \frac{\partial d_{iv}^k}{\partial \mu} < 0,\ \text{and}\ \frac{\partial d_{ip}^k}{\partial \theta} > 0,\ \frac{\partial d_{iv}^k}{\partial \theta} < 0.$$

Concerning corollary 1 and 2, the number of trains each TOC runs at each market niche is a consequence of the above results. Partial derivatives with respect to the parameters under study are the following,

$$\frac{\partial q_{ip}^k}{\partial c_p} < 0,\ \frac{\partial q_{iv}^k}{\partial c_p} > 0,\ \frac{\partial q_{ip}^k}{\partial \theta} < 0,\ \frac{\partial q_{iv}^k}{\partial \theta} > 0,\ \text{and}\ \frac{\partial q_{ip}^k}{\partial \mu} > 0,\ \frac{\partial q_{iv}^k}{\partial \mu} < 0.$$

Indeed, as c_p and θ increase, frequencies in the off-peak market increase. Indeed, extra costs to operate trains during peak hours (with congestion) yields to a more aggressive behavior by the NIM in the off-peak market in order to alleviate the higher e_p^k. Then, peak fare increases and off-peak fare decreases, as we have seen above. With respect to demand increases (a lower θ), as costs for operating trains in the off-peak market are lower than in the peak market (with the effects described in propositions 1 and 2), demand expansions increase off-peak market more than the peak market, resulting in a higher number of trains in the off-peak market. Finally, as μ increases, although the NIM found it profitable to charge a higher infrastructure access charge, the number of trains in the peak market decreases whereas in the off-peak market it increases. The overall effect is an increase in the total number of trains being operated.

Concerning consumers' aggregate disutility, the following evolution is observed as the parameters under study change,

$$\frac{\partial CD^k}{\partial c_p} > 0,\ \frac{\partial CD^k}{\partial \mu} > 0,\ \text{and}\ \frac{\partial CD^k}{\partial \theta} < 0.$$

On one hand, consumers' aggregate disutility increases as c_p increases. This result follows from the previous insights in propositions

Table 2
Number of trains per TOC and time-slot.

		$\gamma(c, \theta, \mu, \alpha) < 0$	$\gamma(c, \theta, \mu, \alpha) = 0$
Hard regulation	$q_{ip}^H = \frac{1}{4\theta}d_{ip}^H$	$2n(1 + \gamma(.))$	$2n$
	$q_{iv}^H = \frac{1}{4\theta}d_{iv}^H$	$2n(1 - \gamma(.))$	$2n$
Skimmed regulation	$q_{ip}^S = \frac{1}{8\theta}d_{ip}^S$	$n(1 + \gamma(.))$	n
	$q_{iv}^S = \frac{1}{8\theta}d_{iv}^S$	$n(3 - \gamma(.))$	$3n$

1 and 2: as costs of operating trains in the peak market increase, peak fares also increase and there is a contagion effect in the off-peak market. Indeed, as both services are substitutes, there is an incentive to increase off-peak fare when the peak fare increases. On the other hand, a similar effect occurs when the extra valuation of consumers for peak services increases. As μ increases, peak fare also increases, yielding to the above mentioned effect in the off-peak fare. Finally, when total demand increases the amount of demand's partitions also increase (because we assume that train's capacity is given); then, θ decreases. As θ decreases, the use of infrastructure is higher and thus e_p^k and e_v^k increase (proposition 1 and 3). This yields to higher fares that enlarge consumers' aggregate disutility.

Regarding consumers' disutility differences under both regulatory regimes, it is important to note that, given our parameter constellation the following property holds,

$$CD^S - CD^H > 0.$$

As we will see in the simulation, consumers' disutility is higher under Skimmed regulation than under Hard regulation, because of the market power exerted by the NIM and the corresponding double marginalization applied by the TOCs.

6.2. Simulation

In this subsection, we conduct a simulation of the model based on data from the main passenger railway corridors in central and western EU Member States. In accordance with Table 1, we include those corridors where effective competition takes place. We have summarized in Table 3 real dispatch from the European Rail Timetable (ERT, hereinafter). We use this timetable to elaborate the simulation reported in Table 5, in line with the theoretical values from our model reported in Table 2 and Table 4. Simulations in Table 5 covers real dispatches reported in Table 3 (leaving some degree of freedom at both, the lowest and the highest values).

Table 2 reports the characterization of the equation for number of trains under Assumption 1. We report two cases: when γ(*c*, θ, μ, α) is lower than zero, and equal to zero. Although fully occupied trains are difficult to observe in a real world, it is a stylized case to make a comparison with the usual case where trains are not fully occupied.

Empirical evidence included in Table 3 suggests that within EU Member States, *n* usually takes values from 8 up to 10, depending on the corridor considered. Table 3 includes those corridors where effective competition takes place or it is expected to occur as from 2021, in accordance with Table 1.

In Table 4, we summarize the upper and lower bound of γ(c_p,θ, μ, α) according to our assumptions and the parametrization presented above. Relevant values of *n* are included to capture the empirical evidence reported in Table 3. In addition, lower values of *n* corresponding to corridors with low demand are also considered. We assume that the NIM cost parameter $c_p \in (0, 1.3]$. According to Odolinski and Boysen (2019) the cost elasticity with respect to train density is 0.25 (an increase of 10 per cent on the number of trains is produced and an increase of 2.5 per cent in the maintenance costs). Hence, we argue that the assumption covers the empirical evidence. Finally, μ ∈ (0, 1]. It means that consumers travelling at peak times may have a 100% extra valuation of these services. According to this parametrization, equilibrium values are reported in Table 5.

First, we can observe that when trains are not fully occupied ($ALF_h < 0$) frequencies are larger than those observed when trains run fully occupied ($ALF_h = 0$), irrespective of whether Hard or Skimmed regulations are implemented. This is due to a higher c_p: when this parameter increases, TOCs run a lower number of trains and consumers experience a lower substitutability. However, when c_p approaches unity (and thus it is almost equal to c_v), running trains is relatively cheap and TOCs increase the number of trains. Second, as demand increases (*n* increases), the business-stealing effect from off-peak trains also increases. This effect is large when trains run fully occupied and under Skimmed regulation. The intuition behind this is related with c_p, which is always larger than c_v. Thus, TOCs have an incentive to operate more trains during off-peak times, extracting a higher aggregate surplus from those consumers traveling during off-peak times than from those traveling during peak times.

To conclude this section, we graphically compare Hard and Skimmed regulation when infrastructure access charges, fares, consumer disutility, and the number of trains vary. We consider $c_p \in (1, 1.3]$. As maximum and minimum values of γ(c_p,θ, μ, α) are reached for μ = 1 and $\mu \sim 0$, respectively, we take $\mu = \frac{1}{2}$, as an average value.[23]

Fig. 1 reports the differences in fares and infrastructure access charges. Let us define gaps between fares and infrastructure access charges as $g_P^k = P_p^k - P_v^k$, and $g_e^k = e_p^k - e_v^k$, respectively. One can see that the evolution of the gaps depends on the level of NIM costs. Under Hard regulation, fare gap and infrastructure access charge gap are lower than under Skimmed regulation. The intuition behind this is related with the use of the peak infrastructure access charge as a strategic variable by the NIM under Skimmed regulation. In this case, the ability to exert market power in the peak market increases the infrastructure access charge and thus, TOCs translate these increases to final fares (double marginalization).

Fig. 2 shows the evolution of the consumers' aggregate disutility according to the expressions reported in Appendix 2. One can observe that, as the cost parameter c_p increases, disutility also increases. Moreover, this effect is higher under Skimmed regulation, as one can see by comparison of the vertical axis. In both regulatory regimes, this fact is related to the intensity of the double marginalization: as infrastructure access charges increase, fares also increase. Indeed, under Skimmed regulation, the NIM is able to extract surplus from the peak time slots and thus, infrastructure access charges and fares are higher not only in peak but in the off-peak market. Although there is a business-stealing effect from the off-peak market, the overall effect yields to a higher aggregate consumers' disutility under Skimmed regulation. In other words, individual consumers travelling during peak times are worse off, whereas

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[23] We also know that this relationship is monotonic, so this value is taken without loss of generality.

Table 3
Line-intensive competition, average frequencies.[*] (ERT, June-December 2021).

Country	Line	One-way trains (frequency)		State-owned operator		Private operator 1		Private operator 2	
		Workdays	Weekend	Workdays	Weekend	Workdays	Weekend	Workdays	Weekend
Italy	Rome-Milan[a]	34	33	22[b]	28	12[c]	5	-	-
Sweden	Stockholm-Gothenburg[d]	34	25	23[e]	15	8[f]	7	3[g]	3
Austria	Vienna-Salzburg[h]	75	52	57[i]	45	18[j]	7	-	-
Czech Republic	Prague-Ostrava	37	37	19[k]	18	11[l]	11	7[m]	8
Spain	Madrid-Barcelona[n]	38[p]	27	33	22	5	5	16[q]	-

Source: authors' own compilation from the European Rail Timetable (ERT), 2021.

[*] Average frequencies in round numbers (integer numbers)

[a] Restricted frequencies due to Covid-19. Regional services by Trenitalia excluded

[b] Trenitalia

[c] Italo

[d] Services from August 22nd 2021. Frequencies by alternative regional trains (Västerås and Örebro Lines) excluded

[e] SJ

[f] MTRX

[g] FlixTrain;

[h] Subject to alteration until April 11th by Covid-19 restrictions

[i] ÖBB

[j] Westbahn

[k] ČD

[l] Regiojet

[m] Leo Express

[n] Frequencies of the alternative regional Renfe trains by Iberian gauge excluded

[p] 85 services might be withdrawn (on average 13.6) during working days, and 17 services (on average 8.5 during weekends) due to Covid-19 restrictions

[q] The new operator ILSA will operate 16 trains from the winter schedule of 2022.

Table 4
Upper and lower values of $\boldsymbol{\gamma(c_p, \theta, \mu, \alpha)}$ for $\boldsymbol{n} = \{2, 4, 6, 8, 10\}$.

	θ = 1/16	θ = 1/32	θ = 1/48	θ = 1/64	θ = 1/80
Upper bound values, $\overline{\gamma}_\theta$ $\overline{c}_p = 1.3,\ \overline{\mu} = 1.$	-0.0010	-0.0032	-0.0039	-0.0047	-0.0048
Upper bound mean $\overline{\boldsymbol{\gamma}}(\overline{\boldsymbol{c}}_p, \boldsymbol{\theta}, \overline{\boldsymbol{\mu}}, 1) = -0.0035 \cong 0$					
Lower bound values, $\underline{\gamma}_\theta$ $\underline{c}_p = 1.001,\ \underline{\mu} = 0.001.$	-0.1297	-0.1302	-0.1302	-0.1303	-0.1304
Lower bound mean $\underline{\boldsymbol{\gamma}}\left(\underline{\boldsymbol{c}}_p, \boldsymbol{\theta}, \underline{\boldsymbol{\mu}}, 1\right) = -0.1298$					

individual consumers travelling off-peak are better off; however, at an aggregate level, consumers' disutility increases because both fares increase (peak and off-peak). This result is interesting from a policy-making perspective: there is a trade-off between NIM's profits and the consumers' aggregate disutility. As regulatory authorities allow extracting surplus from the peak market and thus allowing NIM profits to increase, then consumers' aggregate disutility increases. This fact draws attention to the possibility of setting a price cap on the infrastructure access charges in order to limit consumers' aggregate disutility.

Finally, in Fig. 3 we present the number of trains run under both regulatory regimes. The evolution of the distribution of trains between both types of time slots comes from the discussion on infrastructure access charges and fares. It reveals that off-peak trains strongly increase under Skimmed regulation as c_p increases, which yields to a higher gap between the peak and off-peak infrastructure access charge and thus, a large fare gap between both markets. Although the business-stealing effect from the off-peak market also occurs under Hard regulation as c_p increases, it is more intensive under the Skimmed regime.

From a policy-making perspective, allowing the NIM to extract surplus from the peak market may alleviate the amount of government subsidies. Moreover, it means that train passengers contribute to the maintenance of the network in line with the use of the service and the extra valuation for peak trains, whereas population who do not use the service may reduce their contribution to the sustainability of the railway transportation mode.

6.3. *Some extensions of the model*

In this subsection, we discuss two extensions of the basic model. In particular, we investigate the extent to which our results are affected by the number of TOCs and possible asymmetries in costs.

As we have reported in Table 1, duopoly competition is the most common structure in the railway passenger sector across Europe. However, there are some cases where more than two TOCs compete (for instance in Czech Republic, line-intensive competition).

Table 5

Simulation: number of trains for $\boldsymbol{\theta} = 1/8\boldsymbol{n}$. ($\boldsymbol{n} = \{2,4,6,8,10\}$).

Regulation	Slot	n	$\underline{\gamma}_\theta \le \gamma(c_p,\theta,\mu,\alpha) < 0$				$\overline{\gamma}_\theta(c_p,\theta,\mu,\alpha) \cong 0$			
			q_{is}	Q_S	ALF_h	Q_T	q_{is}	Q_S	ALF_h	Q_T
Hard	Peak	2	4	8	0,8	18	4	8	1	16
	Off-peak		5	10	0,8		4	8	1	
Skimmed	Peak		2	4	0,8	18	2	4	1	16
	Off-peak		7	14	0,9		6	12	1	
Hard	Peak	4	7	14	0,9	34	8	16	1	32
	Off-peak		10	20	0,9		8	16	1	
Skimmed	Peak		4	8	0,9	34	4	8	1	32
	Off-peak		13	26	0,9		12	24	1	
Hard	Peak	6	11	22	0,9	50	12	24	1	48
	Off-peak		14	28	0,9		12	24	1	
Skimmed	Peak		6	12	0,9	50	4	8	1	48
	Off-peak		19	38	0,9		18	36	1	
Hard	Peak	8	14	28	0,9	66	16	32	1	64
	Off-peak		19	38	0,9		16	32	1	
Skimmed	Peak		7	14	0,9	66	8	16	1	64
	Off-peak		26	52	0,9		24	48	1	
Hard	Peak	10	18	36	0.97	82	20	40	1	80
	Off-peak		23	46	0.97		20	40	1	
Skimmed	Peak		9	18	0.94	82	10	20	1	80
	Off-peak		32	64	0.98		30	60	1	

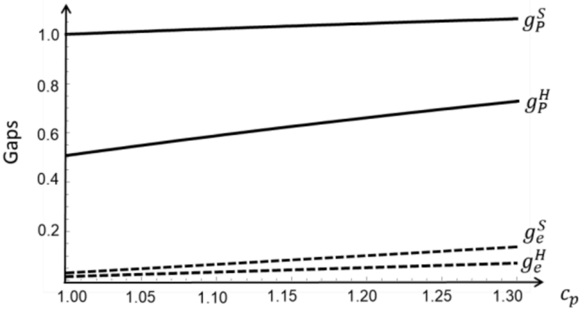

Fig. 1. Gaps between peak and off peak time slots in infrastructure access charge and fares.

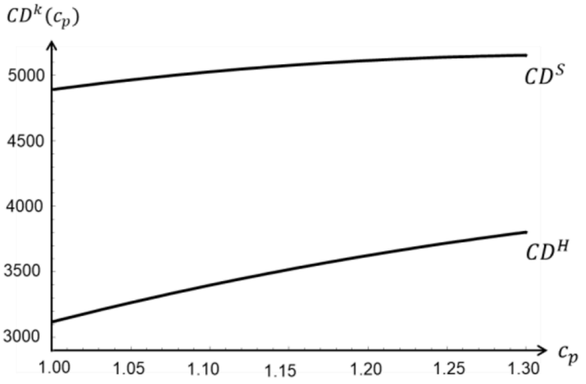

Fig. 2. Consumers' disutility.

Moreover, it is expected that the number of TOCs may increase in the near future as liberalization evolves. Let us assume that there is more than two operators ($N > 2$). At the second stage, price competition sets fares equal to average cost, regardless of the number of TOCs. Thus, second stage equilibrium is not affected. Concerning the first stage, when the NIM sets infrastructure access charges (whether it follows Hard or Skimmed regulation), their level is not affected. This is because infrastructure access charges only depend on the total number of trains (Q_T), which in turn, depends on θ. Indeed, once the number of time slots is decided, the assignation of time slots among TOCs does not alter the level of the infrastructure access charges.

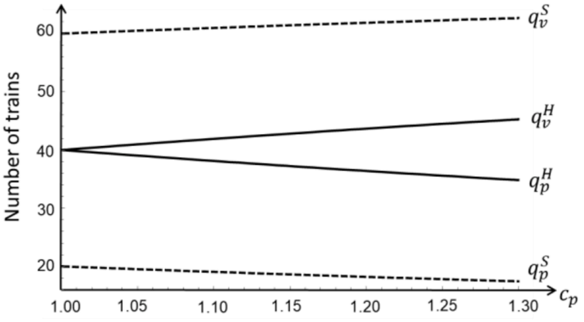

Fig. 3. Number of trains by time slot under Hard and Skimmed regulation.

Let us now introduce costs asymmetries. First, infrastructure access charges $T_{ih}(q_{ih})$ are equal across TOCs because they are set by the NIM. Second, labor costs $L_{ih}(q_{ih})$ can be different for a number of reasons (for instance, state-owned firms usually enjoy greater labor benefits than new TOCs; moreover, wages of train crews are usually higher in state-owned TOCs). Then, we assume that $\sigma_I > \sigma_E$. Finally, operational costs, $C_{ih}(d_{ih})$, due to ticketing, advertising, etc. are assumed to be equal. Then,

$$L_{Ih}(q_{Ih}) > L_{Eh}(q_{Eh}).$$

Taking our equilibrium characterization under Hard and Skimmed regulation, we can argue the following results. First, infrastructure access charges, level of demand, and the number of trains operating during peak and off-peak time slots, do not depend on σ (see Proposition 1 and 2). Second, a higher level of σ (in particular $\sigma_I > \sigma_E$) implies fare differences. Explicitly, fares under Hard and Skimmed regulation will be set at,

$$\widehat{P}_h^k = \max\left\{P_{Ih}^k, P_{Eh}^k\right\}.$$

As there is no incentive to cut prices for the TOC with the lower costs, final prices will be $\widehat{P}_{Ih}^k$. It yields to a positive mark-up for the entrant TOC whereas the incumbent remains at zero profits. This situation provides an incentive for the state-owned TOC to increase efficiency aimed at reducing costs in order to achieve positive profits. Thus, in the long run, competitiveness should yield lower fares and zero TOCs' profits. Notice that peak fares remain greater than off-peak fares according to Proposition 1 and 2: $P_p^k - P_v^k > 0$.

7. Conclusions and policy implications

In this paper, we have presented a railway competition model where two TOCs offer a differentiated service (peak and off-peak). Consumers may horizontally choose between peak and off-peak services, where peak services are endowed with an extra quality value in a vertical sense. Moreover, the NIM in charge grants slots to TOCs under two different regulatory schemes.

First, it is found that, the level of NIM costs, the consumers' extra quality valuation for peak services, and frequencies determine infrastructure access charges. Second, infrastructure access charges affect fares and determine the number of trains each TOC runs at each type of time slot. Moreover, regardless of the regulatory scheme, the number of trains run at off-peak hours is equal or higher than those run under peak hours, although this effect is more intensive under Skimmed regulation. The intuition behind this, is that as a result of the extra value for peak services, and the higher maintenance costs at peak times, the infrastructure access charge at peak hours is high. As a result, the higher fares during peak hours cancel the effect of the extra valuation μ, which yields to scheduling a low number of trains at peak times, increasing the number of trains at off-peak times; i.e., there is a higher frequency at off-peak times.

Overall, we can argue that the regulatory regime chosen by authorities depends on the final objective. If the objective is to minimize consumers' aggregate disutility, Hard regulation should be chosen. It benefits consumers that use both types of service because it leads to lower fares being set. In contrast, a higher number of trains are run in off-peak services when Skimmed regulation is applied. As off-peak trains increase, substitutability increases. However, fares are higher than those set under Hard regulation. Hence, Skimmed regulation can be viewed as a transport policy aimed at managing demand, alleviating traffic in peak time slots. However, it does not promote moderate fares in the off-peak market. Moreover, from an efficiency point of view, allowing the NIM to recover infrastructure costs, depreciation and to promote future investments in infrastructure, Skimmed regulation might be advised.

The contribution of the paper to the literature on competition in the railway sector is twofold. On one hand, we used an industrial organization approach to analyze the effects that the liberalization process has on passenger railway services. In particular, in the distribution of frequencies, fares, TOCs and NIM profits, and consumer surplus (measured in our model by consumers' disutility). On the other hand, the model can be used to provide insights about further transport policy developments affecting the railway sector, as a consequence of the current liberalization process. In this sense, our framework is suitable for studying other features of the railway competition arena. Future research may include the analysis of multimodal transportation networks, intermodal competition between the air and rail transport modes, and competition between traditional services and low-cost services, among others. Differences in quality services offered by TOCs can also be of interest as they may affect consumers' choice (for instance, high-speed services versus standard services, complimentary facilities on board…). Moreover, the study of a semi-public operator is also interesting (i.e. the state-owned TOC can be semi-privatized) because as long as the operator becomes private, the distribution of welfare may be modified. Another field of research is related with the threats of a duopoly structure. Indeed, operators may be parties to collusive agreements (either tacit or explicit), splitting the market and increasing fares.

Author Statement

The authors declare that the model included in the manuscript is original. Simulations are based in real data. There is no conflict of interests. All funder institutions are included.

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Appendix 1

Table A
Summary of notation

Notation	Meaning
$\boldsymbol{x}$	Indifferent consumer
ρ	Consumer’s gross surplus
P_h	Fares by service
U_h	Utility that a consumer at point x enjoys from purchasing the service
D_h	Demand by service
q_{ih}	Number of trains by service and operator
Q_h	Total number of trains by service
$\boldsymbol{\theta}$	Inverse of times that total demand is partitioned
e_h	Infrastructure access charge by service
$\boldsymbol{\sigma}$	Labor costs parameter
$\boldsymbol{\alpha}$	Operational costs parameter
$\boldsymbol{\pi_i}$	TOCs’ profit function
$\boldsymbol{\varphi_h}$	Total costs function by service
$\boldsymbol{T_{ih}(q_{ih})}$	Infrastructure access costs by service
$\boldsymbol{L_{ih}(q_{ih})}$	Labor costs by service
$\boldsymbol{C_{ih}(d_{ih})}$	Operational costs by service
$\boldsymbol{\mu}$	Extra valuation of consumers for peak services
c_h	Operational costs of NIM
$\boldsymbol{F}$	Total infrastructure costs over n periods
$\boldsymbol{f}$	Fixed costs per period
γ	Average load factor function

Table B
Summary of acronyms

Acronym	Meaning
NIM	Network infrastructure manager
TOCs	Train operator companies
PSO	Public service obligations
CD	Consumers’ aggregate disutility
$\mathrm{ALF_h}$	Average trains’ load factor by service
COAPS	Competing Open Access Passenger Services
RENFE	Public operator, *Red Nacional de los Ferrocarriles Españoles.*
MTRX	Private operator *Mass Transit Railway*.
ÖBB	Public operator, *Österreichische Bundesbahnen.*
ČD	Public operator, *České dráhy.*
ILSA	Private operator, *Intermodal de levante, S.A.*
ERT	European Rail Timetable.

Appendix 2

Proof of Proposition 1. Under Hard regulation, infrastructure access charges are set at the level of average cost. Thus, $e_p^H = c_p \cdot Q_p(e_p, e_v|\theta)$ and $e_v^H = Q_v(e_p, e_v|\theta)$. Solving this system of equations, we get: $e_p^H = c_p\left[\frac{(2+\theta^2(2+\alpha+2\mu)}{2\theta(1+c_p+(2+\alpha)\theta^2)}\right] = \left[\frac{(c_p+c_{p+}c_p2-c_p2+c_p\theta^2(2+\alpha)+2c_p\theta^2\mu)}{2\theta(1+c_p+(2+\alpha)\theta^2)}\right] =$

$\frac{c_p}{2\theta}+\frac{c_p(1-c_p+2\theta^2\mu)}{2\theta(1+c_p+(2+\alpha)\theta^2)}$. We define $\gamma(c_p,\theta,\mu,\alpha)\ =\frac{1-c_p+2\theta^2\mu}{1+c_p+(2+\alpha)\theta^2}$, which yields $e_p^H\ =\frac{c_p}{2\theta}[1\ +\gamma(c_p,\theta,\mu,\alpha)]$ In the same way, off-peak network access is $e_v^H\ =\frac{(2c_p+\theta^2(2+\alpha+2\mu)}{2\theta(1+c_p+(2+\alpha)\theta^2)}=\left[\frac{(c_p+c_{p+}\theta^2(2+\alpha)+2\theta^2\mu)}{2\theta(1+c_p+(2+\alpha)\theta^2)}\right]=\frac{1}{2\theta}+\frac{(1-c_p+2\theta^2\mu)}{2\theta(1+c_p+(2+\alpha)\theta^2)}=\frac{1}{2\theta}[1\ +\gamma(c_p,\theta,\mu,\alpha)]$ Now, to find fares, we take expressions [7]. By substitution of e_p^H and e_v^H fares are set at,

$$P_p^H=\frac{\sigma}{\theta}+\frac{(2c_p+\alpha\theta^2)}{4\theta^2}\left[1+\gamma\left(c_p,\theta,\mu,\alpha\right)\right],\ \text{and}\ P_v^H=\frac{\sigma}{\theta}+\frac{(2+\alpha\theta^2)}{4\theta^2}\left[1-\gamma\left(c_p,\theta,\mu,\alpha\right)\right].$$

Finally, from expressions in [9] and by substitution of e_p^H and e_v^H demands are set at,

$$d_{ip}^H=\frac{1}{4}\left[1+\gamma\left(c_p,\theta,\mu,\alpha\right)\right],\ \text{and}\ d_{iv}^H=\frac{1}{4}\left[1-\gamma\left(c_p,\theta,\mu,\alpha\right)\right].$$

This completes the proof. ■

Proof of Corollary 1. It is straightforward by taking $q_{ip}=\ d_{ip}(e_p,e_v|\theta)/\theta$, and $q_{iv}=\ d_{iv}(e_p,e_v|\theta)/\theta$, and the expressions from Proposition 1. By substitution, $q_{ip}^H\ =\frac{1}{4\theta}d_{ip}^H$, and $q_{iv}^H\ =\frac{1}{4\theta}d_{iv}^H$, whereas adding by time slots and TOCs total number of trains is at $\frac{1}{\theta}$. This completes the proof. ■

Consumers' disutility under Hard regulation

$$CD^H=\frac{1-3\gamma()+c(1+\gamma())(1+2\gamma())+2\gamma()^2\left(1+(2+\alpha)\theta^2\right)+\theta(\theta+\alpha\theta+4\sigma)}{4\theta^2}.$$

Proof of Proposition 2. Under Skimmed Regulation, peak infrastructure access charge is a strategic variable. Before find the first order condition, it is necessary to insert in [6] the level of e_v equal to average cost, $e_v\ =\frac{\theta(2+\alpha-2\mu)+2e_p}{2(1+\theta(2+\alpha))}$. Making the partial derivative $\partial\pi_N(e_p,e_v|\theta)/\partial e_p=0$, the optimal value of e_p is set at $e_p^S\ =\frac{(1+2c_{p+}\theta^2(2+\alpha))(2+\theta^2(2+\alpha+2\mu))}{4\theta(1+c_{p+}\theta^2(2+\alpha))}$. Rewriting this expression and by substitution of the function $\gamma(c_p,\theta,\mu,\alpha)$ it yields $e_p^S\ =\frac{2c_p+1+\theta^2(2+\alpha)}{4\theta}(1\ +\ \gamma(c_p,\theta,\mu,\alpha))$, and $e_v^S\ =\frac{3}{4\theta}(1\ -\ \gamma(c_p,\theta,\mu,\alpha))$. Now, to find fares we take expressions [7]. By substitution of e_p^S and e_v^S fares are set at,

$$P_p^S=\frac{\sigma}{\theta}+\frac{(2+4c_p+(4+3\alpha)\theta^2)}{8\theta^2}\left[1+\gamma\left(c_p,\theta,\mu,\alpha\right)\right],\ P_v^S=\frac{\sigma}{\theta}+\frac{(2+\alpha\theta^2)}{8\theta^2}\left[3-\gamma\left(c_p,\theta,\mu,\alpha\right)\right],$$

Finally, from expressions in [9] and by substitution of e_p^S and e_v^S demands are set at, $d_{ip}^S\ =\frac{1}{8}[1\ +\ \gamma(c_p,\theta,\mu,\alpha)]$, $d_{iv}^S\ =\frac{1}{8}[3\ -\ \gamma(c_p,\theta,\mu,\alpha)]$ This completes the proof. ■

Proof of Corollary 2. It is straightforward by taking $q_{ip}=\ d_{ip}(e_p,e_v|\theta)/\theta$, and $q_{iv}=\ d_{iv}(e_p,e_v|\theta)/\theta$, and the expressions from Proposition 3. By substitution, $q_{ip}^S\ =\frac{1}{\theta}d_{ip}^S$, and $q_{iv}^S\ =\frac{1}{\theta}d_{iv}^S$, whereas adding by time slots and TOCs total number of trains is at $\frac{1}{\theta}$. This completes the proof. ■

Consumers' disutility under Skimmed regulation

$$CD^S=\frac{10+2c(1+\gamma())^2\left(1+\theta^2\right)-2\gamma()\left(2+\theta^2\right)+\gamma()^2\left(2+\theta^2+2\alpha\theta^2\right)+\theta\left(5\theta+6\alpha\theta+16\sigma\right)}{16\theta^2}.$$

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