

Measuring dynamic inefficiency through machine learning techniques

Juan Aparicio^{a,b}, Miriam Esteve^{a,*}, Magdalena Kapelko^c

^a Center of Operations Research (CIO), Miguel Hernandez University of Elche (UMH), 03202 Elche (Alicante), Spain

^b valgrAI - Valencian Graduate School and Research Network of Artificial Intelligence, 46022 Valencia, Spain

^c Department of Logistics, Wrocław University of Economics and Business, Komandorska 118/120, 53-345, Wrocław, Poland

ARTICLE INFO

Keywords:

Data envelopment analysis
Free disposal hull
Dynamic inefficiency
Classification and regression trees
Dairy manufacturing industry

ABSTRACT

This paper contributes by developing new models for assessing dynamic inefficiency that incorporate machine learning techniques. In particular, the new approaches apply decision trees models for the estimation of dynamic production technologies that account for investment adjustment costs. Methodologically, the new models build on the recently developed techniques of Efficiency Analysis Trees (EAT) and Convexified Efficiency Analysis Trees (CEAT) and extend them even further to a dynamic framework comprising dynamic EAT and CEAT models. The study compares dynamic inefficiency scores estimated assuming the new models against the traditional dynamic free disposal hull (FDH) and dynamic data envelopment analysis (DEA). Our empirical application focuses on dairy manufacturing firms in the main dairy processing countries in the European Union for the years 2014 and 2018. The results show that inefficiency related to the dynamic CEAT or EAT is higher than their corresponding values calculated through the dynamic DEA or FDH. The discriminating power of dynamic DEA (dynamic FDH) drastically improves when switching to dynamic CEAT (dynamic EAT). Finally, the differences between countries are observed regarding the development of dynamic inefficiency in the period associated with milk quota abolition.

1. Introduction

Production (in)efficiency assessment using data envelopment techniques, including Free Disposal Hull (FDH) (Deprins et al., 1984) and Data Envelopment Analysis (DEA) (Banker et al., 1984; Charnes et al., 1978), continues to attract considerable attention in the literature. This analysis allows to answer the question on how well the unit transforms its inputs into outputs relative to the benchmark of best practice companies, and as a result to measure firms' performance. Within this literature, the recent line of research concerns the assessment of dynamic (in)efficiency. In differentiation to a static approach, the idea of dynamic measures of (in)efficiency is to account for interdependence between time periods. In particular, the measurement of dynamic (in) efficiency recognizes the effects of transition elements between subsequent observations (e.g., the degree to which input consumption in a specific period affects output levels over consecutive periods) and/or assume that firms cannot adjust instantaneously (Fallah-Fini et al., 2014; Mariz et al., 2018). These conceptualizations allowed the development of two types of dynamic models. The first category of models, initially

proposed by Färe and Grosskopf (1996), is known as a dynamic network DEA approach in which outputs and/or investments in some stages of production are used as inputs to other stages, assuming multiple periods of time (see e.g., Nemoto and Goto, 2003; Tone and Tsutsui, 2010, 2014; Fukuyama and Weber, 2017). The second group of models is rooted in the theory of adjustment costs (Eisner and Strotz, 1963; Treadway, 1970; Epstein, 1981) which is based on the idea that incorporating new processes and installations (i.e., changing firms' quasi-fixed factors) can bring about adjustment costs such as learning costs. This line of research was developed more fully in the efficiency context by Silva and Stefanou (2003; 2007)¹. The present study contributes to this second line of dynamic efficiency analysis research.

The literature devoted to dynamic inefficiency analysis grounded on the idea of adjustment costs is restricted to descriptive analysis with the sample of data at hand without attempts to estimate the actual production function from which the data was generated. In particular, the models feature an overfitting problem, in that they underestimate firms' inefficiency by generating the frontier that is always located, by definition, below the theoretical frontier. This problem is inherited from the

* Corresponding author at: Center of Operations Research (CIO), Miguel Hernandez University of Elche (UMH), 03202 Elche (Alicante), Spain.

E-mail addresses: j.aparicio@umh.es (J. Aparicio), miriam.estevec@umh.es (M. Esteve), magdalena.kapelko@ue.wroc.pl (M. Kapelko).

¹ Recent research on the measurement of dynamic efficiency also proposes a model that is both explicitly costly and endogenously determined (Tsionas et al., 2020). It is operationalized using a modified version of a Bayesian Exponentially Tilted Empirical Likelihood.

use of standard DEA and FDH approaches, which suffer from overfitting, (see, for example, Valero-Carreras et al., 2021). Some contributions have already tried to estimate technical efficiency at a theoretical frontier level in the nonparametric setting (see, for instance, bootstrapping with Simar and Wilson, 1998; Simar and Wilson, 2000a; and Simar and Wilson, 2000b). Furthermore, some studies have incorporated Machine Learning (ML) techniques into the assessment of the efficiency with the aim of the estimation of the actual production function (see, for example, Kuosmanen and Johnson, 2010, 2017; Parmeter and Racine, 2013; Daouia et al., 2016; Esteve et al., 2020; Valero-Carreras et al., 2021; Guerrero et al., 2022; Olesen and Ruggiero, 2018, 2022; Esteve et al., 2023; Thaker et al., 2022; Tsolas et al., 2020). However, none of these attempts have dealt with the dynamic model.

With regard to the adaptation of machine learning techniques into the estimation of production frontiers, very recently, the literature has witnessed the introduction of a new method called Efficiency Analysis Trees (EAT) developed by Esteve et al. (2020) that links frontier analysis with machine learning techniques and attempts to estimate the true theoretical frontier that generated the observations under study. EAT applies decision tree models; in particular it adapts Classification and Regression Trees (CART) (Breiman et al., 1984) to estimate production frontiers. The EAT algorithm generates a step function, satisfying free disposability and making it similar to the FDH approach. However, through cross-validation and pruning (Breiman et al., 1984), it is free of the overfitting problem and outperforms FDH in terms of bias and mean squared error, as shown by Esteve et al. (2020) through a simulation study. In Esteve et al. (2020), the EAT approach was implemented as the output-oriented radial model. Nevertheless, it was further extended in the study by Aparicio et al. (2021) to deal with the directional distance function (Chambers et al., 1998) and the assumption of convexity, giving rise to the Convexified Efficiency Analysis Trees (CEAT). While EAT competes against FDH, CEAT presents similarities with standard DEA. The studies of EAT and CEAT, however, so far have not considered the dynamics of firms' production decisions, since they were conceived in the standard static production framework.

The considerations made so far have led us to identify certain gap in the literature: usual dynamic inefficiency models are based on standard FDH and DEA techniques that suffer from overfitting problems when estimating the level of technical inefficiency is the concern. This problem should be overcome to get accurate estimations of technical inefficiency under the dynamic approach. Otherwise, managerial decisions based on these estimates made by policy makers and managers could be systematically mistaken. Moreover, recently developed techniques for estimating production frontiers based on Machine Learning (ML), such as EAT, do not account for dynamics of firm's production decisions. Hence, it seems necessary to extend the ML-based models to deal with the dynamic approach. In this paper, we aim to fill in this gap in the literature and propose to augment dynamic inefficiency measurement through frontier machine learning techniques: EAT and CEAT. The new techniques developed are denominated dynamic EAT and dynamic CEAT and have three main advantages: (1) they allow to account for the dynamic nature of firms' productivity decisions associated with adjustment costs, (2) they avoid the overfitting problem, and (3) they can also be considered as an extension for the application fields of EAT and CEAT as recently proposed machine learning techniques. These advantages are demonstrated through an empirical application that compares dynamic EAT and dynamic CEAT with traditional dynamic FDH and dynamic DEA. The empirical illustration focuses on a recent dataset of dairy firms that are representative of the most significant dairy processors in the European Union (EU): Spain, Poland, France, Italy, and Germany. We analyze two years of data (2014 and 2018) for the purpose of taking a glance at a year before dairy quota removal, which regulated milk production in the EU until 2015, and a year with a quota-free market in the dairy sector.

It is worth noting that, from a computational point of view, the new approach is linked to a very intensive computational procedure. This

feature contrasts sharply with the simplicity of the standard FDH technique, which is based on Mixed Integer Linear Programming, or the standard DEA technique, which is grounded on Linear Programming. Nevertheless, this weakness of the new approach is counterbalanced by the results that it may provide, as we will show by an empirical illustration. For example, we will demonstrate that the discriminating power among efficient DMUs of dynamic FDH and dynamic DEA drastically improves when switching to the application of the new models, which are based on machine learning.

The rest of the paper is organized as follows. Section 2 reviews the relevant literature. Section 3 introduces FDH and DEA, dynamic FDH and dynamic DEA as well as static EAT and CEAT. Section 4 develops new dynamic approaches based on linking dynamic DEA with CEAT and dynamic FDH with EAT. Section 5 presents an empirical application of the methods developed to recent data on the dairy manufacturing sector in the EU. Section 6 offers concluding comments and suggests lines of future research.

2. Literature review

This section reviews the most relevant literature on the measurement of dynamic efficiency, as well as linking the assessment of efficiency with machine learning techniques, all in the non-parametric setting.

The dynamic DEA research grounded on the idea of cost adjustment was initiated by the studies of Silva and Stefanou (2003; 2007). Since then, this line of research has flourished. The primal and dual dynamic directional distance function models were developed by Silva et al. (2015), while extensions of these models were proposed by Kapelko et al. (2014). Dynamic input-specific inefficiency measurement was proposed in Kapelko et al. (2017), while models contemplating full input-output-investment space were proposed by Aparicio and Kapelko (2019) and Ang and Lansink (2018). Extensions also include the measurement of dynamic environmental inefficiency through the dynamic by-production model (Dakpo and Lansink, 2019) and inclusion of corporate social responsibility in the dynamic framework (Engida et al., 2020) with consideration of input indivisibilities and non-convexities through the FDH approach (Kapelko et al., 2021). Furthermore, a novel measurement of opportunity costs in providing an environmental good was proposed within the dynamic framework (Ang et al., 2018). Subsequently, other approaches have been published: Luenberger-based approaches to measure dynamic productivity change (Lansink et al., 2015; Kapelko, 2019), input-specific dynamic-productivity change (Kapelko et al., 2017) and dynamic biased technical change (Baležentis and Oude Lansink, 2020) as well as many empirical applications (e.g., Setiawan, 2019; Kapelko, 2017; Baležentis, 2016; Martínez-Victoria et al., 2019)². None of the aforementioned dynamic efficiency studies were developed with the purpose of the estimation of the actual production function that is behind the generation of the observations.

Additionally, the increasing interest in the literature to link efficiency analysis with machine learning techniques should be highlighted. Most of the previous studies that connected efficiency analysis and machine learning techniques did not aim to relate these two concepts in the estimation of the frontier itself, but they used machine learning techniques in conjunction with conventional measures of efficiency, usually in a two-step procedure (that is, first, a standard efficiency method is applied and, secondly, the efficiency scores are used as the response variables of a certain machine learning model). To name some studies within this line of research: Hong et al. (1999) introduced the hybrid methodology utilizing DEA along with machine learning technique of decision trees to generate classification rules in the context of

² In addition to DEA and FDH, the measurement of dynamic efficiency within adjustment cost framework also involves the stochastic frontier analysis (SFA) approach, for example in the studies by Rungsuriyawiboon and Stefanou (2007) or Minviel and Sipiläinen (2021).

system integration projects. [Sohn and Moon \(2004\)](#) defined a decision tree-based model using DEA results for the effective technology commercialization. [Emrouznejad and Anouze \(2010\)](#) proposed to use the classification and regression tree to estimate the factors associated with DEA-based efficiency scores for the banking sector. [Yeh et al. \(2010\)](#) used the DEA efficiency as predictive variable of business failure, integrating rough set theory with support vector machines technique to increase the accuracy of this prediction. [Azadeh et al. \(2011\)](#) proposed to combine DEA, artificial neural network, and rough set theory to evaluate the impact of personnel efficiency attributes on total efficiency. In [Kubler et al. \(2016\)](#), a state-of-the-art survey about fuzzy analytic hierarchy process applications was carried out. [Rebai et al. \(2020\)](#) applied DEA in the first stage to assess school efficiency and then in the second stage this study used regression trees and random forests to identify the variables associated with school efficiency. [Özcan and Kayıkçıoğlu \(2021\)](#) evaluated the efficiency of a speaker recognition systems through DEA and classification methods such as artificial neural networks. Finally, [Gholizadeh et al. \(2022\)](#) proposed a model to optimize an electrical discharge machine’s parameters through DEA in a fuzzy context and an artificial neural network.

Only a few papers took on the challenge of the development of methods for the measurement of efficiency where machine learning techniques are integrated into the formal production framework. With this regard, [Kuosmanen and Johnson \(2010, 2017\)](#) reinterpreted DEA as a non-parametric least-squares regression subject to shape constraints on the production frontier and sign constraints on residuals, introducing the Corrected Concave Non-parametric Least Squares (CCNLS). [Parmer and Racine \(2013\)](#) proposed non-parametric kernel-based frontier estimators that deliver a smooth production frontier consistent with basic production axioms. [Daouia et al. \(2016\)](#) developed a novel method based on quadratic and cubic splines with shape constraints (monotone and/or concave frontier smoothing) that can be used for the production function estimation. [Esteve et al. \(2020\)](#) adapted the machine learning technique of CART for the estimation of efficient frontiers. The developed method estimates efficient frontiers through step functions, competing against the standard FDH approach. This method was further extended in [Aparicio et al. \(2021\)](#) through the introduction of the convexity assumption, which competes against the standard DEA. [Valero-Carreras et al. \(2021\)](#) introduced the method known as Support Vector Frontiers, which is based on the adaptation of the machine learning technique of Support Vector Machines with the purpose of estimating production functions fulfilling the usual microeconomic postulates. This method was further broadened to a multiple input–output setting in the study by [Valero-Carreras et al. \(2022\)](#). [Tsionas \(2022\)](#) proposed the smooth monotone concave probabilistic regression trees for the estimation of efficiency and productivity. [Guerrero et al. \(2022\)](#) introduced a technique called DEA-based Machines which is a machine learning-type extension of DEA method, which allows estimating polyhedral technologies following the Structural Risk Minimization principle. [Olesen and Ruggiero \(2018, 2022\)](#) introduced the use of hinging hyperplanes as a flexible non-parametric representation of a production function. Finally, [Esteve et al. \(2023\)](#) adapted the machine learning technique of Random Forest to determine super-efficiency in the context of the FDH method. All the aforementioned studies were conceived in the static framework of production and did not consider the dynamics of firms’ production decisions.

The literature review undertaken in this section leads us to conclude that none of the previous studies attempted to link the estimation of the dynamic efficient frontier with machine learning techniques. And this is a gap in the literature that we aim to fill in with the current study through the introduction of a new method linking dynamic efficiency measurement with machine learning technique that allows estimating the actual production frontier. The new approach is built on the recently developed methods of Efficiency Analysis Trees ([Esteve et al., 2020](#)) and Convexified Efficiency Analysis Trees ([Aparicio et al., 2021](#)), mixing dynamic efficiency measurement with machine learning techniques.

3. Background

This section provides an overview of FDH, DEA, Dynamic DEA, EAT and CEAT.

3.1. FDH and DEA

Let us consider n Decision-Making Units (DMUs) that utilize m inputs to generate s outputs. They can be denoted as: $(x_i, y_i), i = 1, \dots, n$. We also assume that $x_i = (x_{1i}, \dots, x_{mi}) \in R_+^m, i = 1, \dots, n$, and $y_i = (y_{1i}, \dots, y_{si}) \in R_+^s, i = 1, \dots, n$. The relative efficiency of each DMU₀ in the sample is assessed with reference to the production technology, which is defined as: $\psi = \{(x, y) \in R_+^{m+s} : x \text{ can produce } y\}$. FDH and DEA are the most common non-parametric techniques when it comes to measuring the efficiency of the production units. The technology estimators generated by these two methods satisfy certain classical assumptions, such as free disposability of inputs and outputs, convexity and minimal extrapolation in the case of DEA and only free disposability and minimal extrapolation in the case of FDH (see [Banker et al., 1984](#)).

Numerous measures for determining technical efficiency for DMUs within a full input–output space within a DEA and FDH context have been proposed in the literature. Moreover, the directional distance function (DDF) ([Chambers et al., 1998](#)) stands out for its properties (duality, units invariance, translation invariance, and flexibility). The interpretation of DDF is easier, since the fixed directional vector g represents how many g ’s are necessary to reach the frontier of the technology. A non-negative value indicates that the assessed unit lies inside the reference technology, whereas a negative value indicates that the evaluated unit is located above the reference frontier. Regarding its flexibility, DDF measures technical efficiency following any direction to the frontier, as long as inputs are reduced, and outputs are increased. The directional distance function, given a directional vector $g_0 = (g_0^x, g_0^y) \in R_+^{m+s}$ for an evaluated point (x_0, y_0) , is defined as follows:

$$\overrightarrow{D}(x_0, y_0; g_0^x, g_0^y) = \max\{\beta_0 \in R : (x_0 - \beta_0 g_0^x, y_0 + \beta_0 g_0^y) \in \psi\} \quad (1)$$

The optimization model of the FDH technique for estimating the value of the directional distance function is defined as follows:

$$\begin{aligned} \overrightarrow{D}^{FDH}(x_0, y_0; g_0^x, g_0^y) = \max & \quad \beta_0 \\ \text{s.t.} & \quad \sum_{i=1}^n \lambda_i x_{ji} \leq x_{j0} - \beta_0 g_{j0}^x, \quad j = 1, \dots, m \\ & \quad \sum_{i=1}^n \lambda_i y_{ri} \geq y_{r0} + \beta_0 g_{r0}^y, \quad r = 1, \dots, s \\ & \quad \sum_{i=1}^n \lambda_i = 1, \\ & \quad \lambda_i \in \{0, 1\}, \quad i = 1, \dots, n \end{aligned} \quad (2)$$

In the DEA case, the optimization model is as follows:

$$\begin{aligned} \overrightarrow{D}^{DEA}(x_0, y_0; g_0^x, g_0^y) = \max & \quad \beta_0 \\ \text{s.t.} & \quad \sum_{i=1}^n \lambda_i x_{ji} \leq x_{j0} - \beta_0 g_{j0}^x, \quad j = 1, \dots, m \\ & \quad \sum_{i=1}^n \lambda_i y_{ri} \geq y_{r0} + \beta_0 g_{r0}^y, \quad r = 1, \dots, s \\ & \quad \sum_{i=1}^n \lambda_i = 1, \\ & \quad \lambda_i \geq 0, \quad i = 1, \dots, n \end{aligned} \quad (3)$$

Notice that the difference between the above two models consists of the nature of the lambda decision variables. While in the case of the FDH model, these variables are binary, in the case of the DEA model, the lambdas are continuous. Additionally, [Simar and Zelenyuk \(2011\)](#) introduced stochastic versions of previous FDH and DEA models.

3.2. Dynamic FDH and DEA

Capital investments are not considered in the previous models as they are based on a conventional static approach. The directional distance function of FDH and static DEA do not outperform the dynamic directional distance function and, moreover, the dynamic technology represents a production technology with adjustment costs (Kapelko et al., 2014; Silva et al., 2015; Silva et al., 2021). More notation is necessary in this section to present the dynamic model.

We assume that each $(x_i, y_i), i = 1, \dots, n$, consumes m variable inputs³ $x_i = (x_{1i}, \dots, x_{mi})$, and also uses a vector of f gross investments in quasi-fixed inputs $I_i = (I_{1i}, \dots, I_{fi})$, and a vector of f quasi-fixed inputs $k_i = (k_{1i}, \dots, k_{fi})$, for producing s outputs $y_i = (y_{1i}, \dots, y_{si})$. The dynamic production technology transforms variable inputs and gross investments into outputs at a given level of quasi-fixed inputs.

The dynamic directional function in the input–output space in FDH can be defined as follows (Kapelko et al., 2021):

$$\begin{aligned} \overrightarrow{D}^{FDH}(x_0, I_0, y_0, k_0; g_0^x, g_0^y, g_0^I) = \text{Max} & \quad \beta_0 \\ \text{s.t.} & \quad \sum_{i=1}^n \lambda_i x_{ji} \leq x_{j0} - \beta_0 g_{j0}^x, \quad j = 1, \dots, m \\ & \quad \sum_{i=1}^n \lambda_i y_{ri} \geq y_{r0} + \beta_0 g_{r0}^y, \quad r = 1, \dots, s \\ & \quad \sum_{i=1}^n \lambda_i (I_{hi} - \delta_h k_{hi}) \geq (I_{h0} - \delta_h k_{h0}) + \beta_0 g_{h0}^I, \quad h = 1, \dots, f \\ & \quad \sum_{i=1}^n \lambda_i = 1, \\ & \quad \lambda_i \in \{0, 1\}, \quad j = 1, \dots, n \end{aligned} \tag{4}$$

In the case of DEA, the dynamic directional distance function in the input–output–investment space is defined as follows (Silva et al., 2021; Aparicio and Kapelko, 2019; and Ang and Lansink, 2018):

$$\begin{aligned} \overrightarrow{D}^{DEA}(x_0, I_0, y_0, k_0; g_0^x, g_0^y, g_0^I) = \text{Max} & \quad \beta_0 \\ \text{s.t.} & \quad \sum_{i=1}^n \lambda_i x_{ji} \leq x_{j0} - \beta_0 g_{j0}^x, \quad j = 1, \dots, m \\ & \quad \sum_{i=1}^n \lambda_i y_{ri} \geq y_{r0} + \beta_0 g_{r0}^y, \quad r = 1, \dots, s \\ & \quad \sum_{i=1}^n \lambda_i (I_{hi} - \delta_h k_{hi}) \geq (I_{h0} - \delta_h k_{h0}) + \beta_0 g_{h0}^I, \quad h = 1, \dots, f \\ & \quad \sum_{i=1}^n \lambda_i = 1, \\ & \quad \lambda_i \geq 0, \quad j = 1, \dots, n \end{aligned} \tag{5}$$

where δ_h represents depreciation rates that will be specific for each type of investment, and g_0^x represents a directional vector of the variable inputs, g_0^y represents a directional vector for outputs and g_0^I represents the directional vector for gross investments. We can define the dynamic distance function by contracting variable inputs, while expanding outputs and gross investments at the same time. β_0 measures the degree of dynamic technical inefficiency in the entire input–output space of DMU_0 .

Fig. 1 illustrates the gap identified in the literature on dynamic

inefficiency measurement. In the figure, FDH and DEA frontiers are drawn for a particular data cloud in a scenario under the dynamic approach. Each observation (firm) is identified by a blue point. Additionally, the true frontier associated with the data generating process that is behind the generation of the observations is represented by a red line. FDH and DEA frontiers are as close to the data as possible because of the minimal extrapolation principle (see Banker et al., 1984). This principle is mainly the cause of the overfitting problem suffered by FDH and DEA. As can be seen from the figure, there is a gap between the true frontier and the estimated frontiers. In the case of the directional distance function, regardless of the selected direction (and we show three different alternatives in the figure), the gap yields imprecise estimates of technical inefficiency, that is, overly optimistic inefficiency scores, when FDH or DEA are used. This problem encourages the search for an accurate estimate of the efficient frontier under the dynamic approach. In the next section, we will introduce a possible solution to this weakness based on machine learning.

3.3. EAT and CEAT

EAT is based on CART (Breiman et al., 1984) for regression. It is a new machine learning-based technique capable of satisfying microeconomic principles, such as free disposability. EAT shares similarities with

FDH, although EAT overcomes the overfitting problem associated with FDH, due to applying a process grounded on cross-validation and pruning of the tree structure generated by the regression tree approach. Fig. 2 is a graphical representation of these two techniques, where a single output is generated from a single input. Both approaches yield a non-decreasing step-function as the estimate of the technology. However, while FDH is as close to the data cloud as possible, the EAT technique can be located from above the FDH estimate in an attempt to approximate the actual production frontier.

We will now introduce the main steps of the EAT algorithm that will allow a tree structure to be built. Let us consider a node t to be split into two new nodes: t_L and t_R (the first node of the tree will always contain the data). Moreover, this node houses a subset of the original sample $\aleph =$

³ Dynamic models distinguish between variable and quasi-fixed inputs.

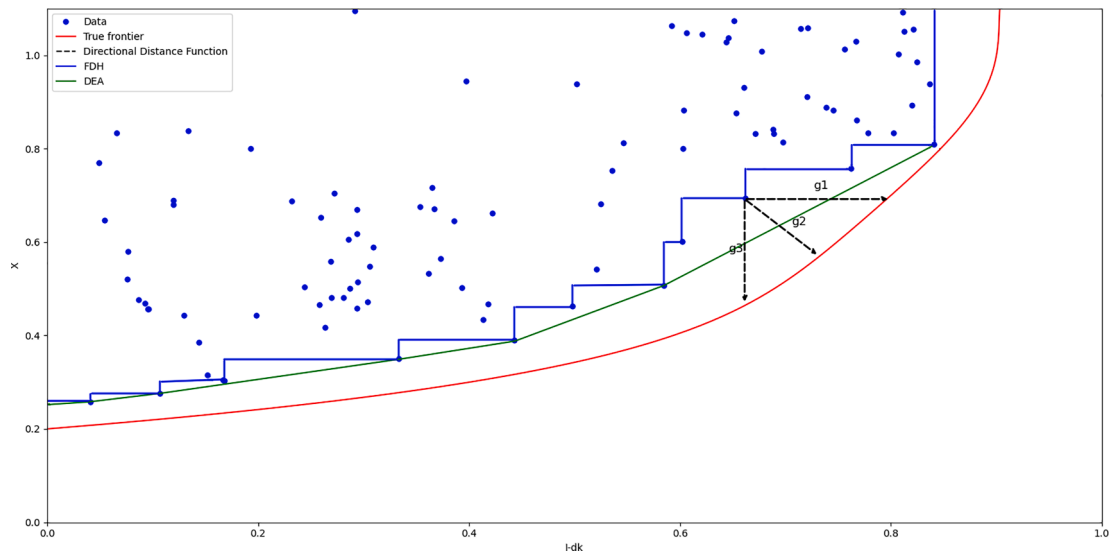


Fig. 1. A simple graphic illustration of some gaps of the standard literature.

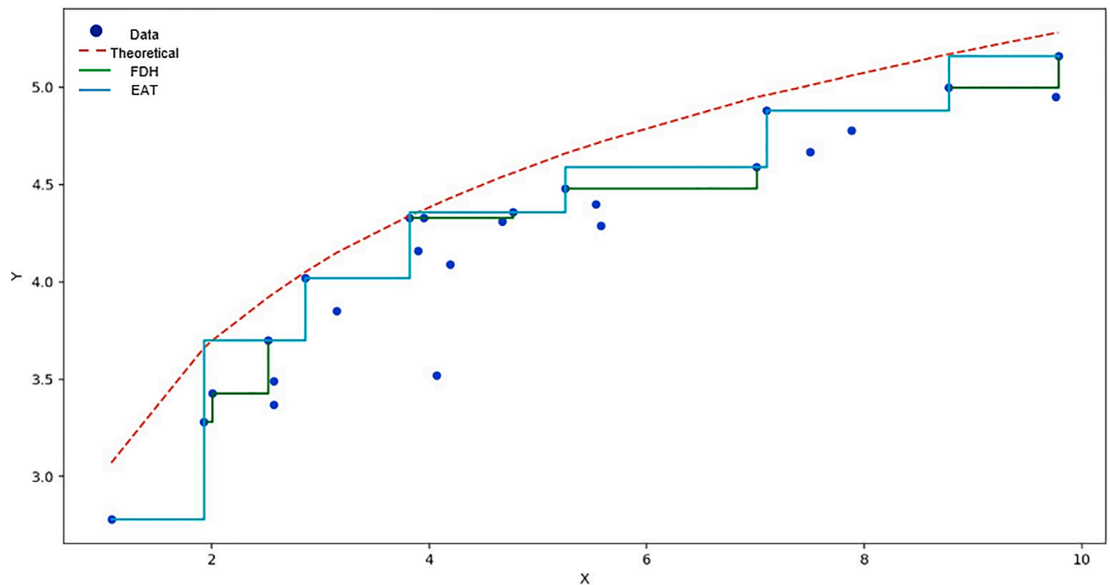


Fig. 2. Graphical example of the frontier estimates provided by EAT and FDH.

$\{(x_i, y_i)\}_{i=1, \dots, n}$. We then have that an input variable $j, j = 1, \dots, m$ and a threshold $s_j \in S_j$, where S_j is the set of possible thresholds for variable j , will be selected so that the sum of the Mean Squared Error (MSE) calculated for the data associated with the left child node t_L and right child node t_R , i.e., the data that meet the condition $x_j < s_j$ and $x_j \geq s_j$, respectively, is minimized. Additionally, in the EAT algorithm, the set S_j is defined from the observed values of the input j in the data sample. Formally, the idea behind the splitting process consists of selecting the (x_j, s_j) combination that minimizes $R(t_L) + R(t_R) = \frac{1}{n} \sum_{(x_i, y_i) \in t_L} \sum_{r=1}^s (y_{ri} - y_r(t_L))^2 + \frac{1}{n} \sum_{(x_i, y_i) \in t_R} \sum_{r=1}^s (y_{ri} - y_r(t_R))^2$, where $y_r(t)$ denotes the estimation of the r -th output of node t . Moreover, a node is terminal in the EAT algorithm when a certain stopping rule is satisfied, such as $n(t) \leq n_{\min} = 5$. The deep tree built is denoted as $T_{\max}(\mathbb{N})$. The key is how to define $y_r(t)$ in each node, during the growing process of the tree structure, for satisfying free disposability and for the output estimation to envelop the data from above. In this sense, an important notion that was introduced by Esteve et al. (2020) is the (input) Pareto-dominance between nodes, which is a way of transferring the concept of

Pareto-dominance between input vectors in the standard efficiency literature to decision trees (see Fig. 2). After executing the split, a region in the input space is determined by the conditions $\{x_j < s_j\}$ or $\{x_j \geq s_j\}$. This region is denoted as the “support” of the corresponding node t and, formally is defined as: $\text{supp}(t) = \{x \in R_+^m : a_j^t \leq x_j < b_j^t, j = 1, \dots, m\}$, where the parameters a_j^t and b_j^t are generated from those thresholds selected during the splitting process. Furthermore, on the premise of node support, (input) Pareto-dominance between nodes can be defined. Before showing this definition, we first need to introduce some notation. In this respect, let Q be the total number of splits executed, $T_q(\mathbb{N})$ be the tree built following the q -th split, and $\tilde{T}_q(\mathbb{N})$ be the set of leaf nodes in $T_q(\mathbb{N}), q = 1, \dots, Q$. Then, the (input) Pareto-dominant nodes of node t can be defined as $I_{T_q(\mathbb{N})}(t) = \{t' \in \tilde{T}_q(\mathbb{N}) \setminus \{t\} : \exists x \in \text{supp}(t), \exists x' \in \text{supp}(t') \text{ such that } x' \leq x\}$. In words, an element in $I_{T_q}(t)$ is a node containing at least one input vector in its corresponding support, so that at least one input vector belonging

to the support of node t can be Pareto dominated. Also, let $t^* \in \tilde{T}_q(\mathbb{N})$ be the node to be split at the step q in the algorithm and let $T(q|t^* \rightarrow t_L, t_R)$ be the tree structure associated with this specific split. So, the way to estimate $y_r(t)$ satisfying the free disposability property for each node $t^* \in \tilde{T}_q(\mathbb{N})$ and in such a way that the data cloud is enveloped from above, for the right child node case, the estimation of its corresponding parent node, i.e., $y_r(t_R) = y_r(t^*)$, $r = 1, \dots, s$, while, for the left child node case:

$$y_r(t_L) = \max\{\max\{y_{ri} : (x_i, y_i) \in t_L\}, y_r(I_{T(q|t^* \rightarrow t_L, t_R)}(t_L))\}, \quad r = 1, \dots, s, \quad (6)$$

where $y_r(I_{T(q|t^* \rightarrow t_L, t_R)}(t_L)) = \max\{y_r(t') : t' \in I_{T(q|t^* \rightarrow t_L, t_R)}(t_L)\}$ and $y_r(t')$ is the estimation of the output y_r at node $t' \in \tilde{T}(q|t^* \rightarrow t_L, t_R)$, $r = 1, \dots, s$.

Finalizing the splitting process, the algorithm yields a deep tree $T_{\max}(\mathbb{N})$ that suffers from the overfitting problem like FDH. To overcome this problem, Esteve et al. (2020) proposed to prune the tree following the error-complexity measure $R_\alpha(T(\mathbb{N}))$ of Breiman et al. (1984). This measure depends on the accuracy of the tree, i.e., the sum of the MSE at each leaf node, the number of leaf nodes and the parameter α , which compensates the values of these errors. Formally it is defined as $R_\alpha(T(\mathbb{N})) = R(T(\mathbb{N})) + \alpha|\tilde{T}(\mathbb{N})|$. $R_\alpha(T(\mathbb{N}))$ is minimized by the pruning process based on cross validation. Concluding the pruning process of $T_{\max}(\mathbb{N})$, we obtain the tree $T^*(\mathbb{N})$ that does not suffer from the overfitting problem.

Finally, $d_{T^*(\mathbb{N})}(x)$ denotes the multidimensional estimator derived from $T^*(\mathbb{N})$, i.e., $d_{T^*(\mathbb{N})}(x) = \sum_{t \in \tilde{T}^*(\mathbb{N})} y_r(t) I(x \in t)$, for all $r = 1, \dots, s$, while $I(\cdot)$ is the indication function. From this estimator, the technology derived from the EAT is defined as $\hat{\psi}_{T^*(\mathbb{N})} = \{(x, y) \in R_+^{m+s} : y \leq d_{T^*(\mathbb{N})}(x)\}$. Esteve et al. (2020) proved that the set $\hat{\psi}_{T^*(\mathbb{N})}$ can be equivalently written as follows:

$$\hat{\psi}_{T^*(\mathbb{N})} = \left\{ (x, y) \in R_+^{m+s} : x \geq \sum_{t \in \tilde{T}^*(\mathbb{N})} \lambda_t a^t, y \leq \sum_{t \in \tilde{T}^*(\mathbb{N})} \lambda_t d_{T^*}(a^t), \sum_{t \in \tilde{T}^*(\mathbb{N})} \lambda_t = 1, \lambda_t \in \{0, 1\}, \forall t \right\}. \quad (7)$$

The above estimator of the technology only assumes free disposability. Due to this, the EAT technique competes against the FDH method. Nevertheless, an estimator can also be built for competing against the DEA approach by convexifying $\hat{\psi}_{T^*(\mathbb{N})}$. This is called the Convexificated Efficiency Analysis Trees (CEAT) estimator of the underlying technology:

$$\text{conv}(\hat{\psi}_{T^*(\mathbb{N})}) = \left\{ (x, y) \in R_+^{m+s} : x \geq \sum_{t \in \tilde{T}^*(\mathbb{N})} \lambda_t a^t, y \leq \sum_{t \in \tilde{T}^*(\mathbb{N})} \lambda_t d_{T^*}(a^t), \sum_{t \in \tilde{T}^*(\mathbb{N})} \lambda_t = 1, \lambda_t \geq 0, \forall t \right\}. \quad (8)$$

In the same way, as in FDH and DEA, it is possible to define the estimation of the DDF based on the EAT algorithm, by substituting ψ by $\hat{\psi}_{T^*(\mathbb{N})}$ in: $\overrightarrow{D}^{EAT}(x_0, y_0; g_0^x, g_0^y) = \max\{\beta_0 \in R : (x_0 - \beta_0 g_0^x, y_0 + \beta_0 g_0^y) \in \hat{\psi}_{T^*(\mathbb{N})}\}$. Additionally, in line with Aparicio et al. (2021, Proposition 3), we know that $\overrightarrow{D}^{EAT}(x_0, y_0; g_0^x, g_0^y)$ can be calculated through the following Mixed-Integer Linear Program (MILP)⁴.

$$\begin{aligned} \overrightarrow{D}^{EAT}(x_0, y_0; g_0^x, g_0^y) = \max & \quad \beta_0 \\ \text{s.t.} & \quad \sum_{t \in \tilde{T}^*(\mathbb{N})} \lambda_t a_j^t \leq x_{j0} - \beta_0 g_{j0}^x, \quad j = 1, \dots, m \\ & \quad \sum_{t \in \tilde{T}^*(\mathbb{N})} \lambda_t d_{rT^*}(a^t) \geq y_{r0} + \beta_0 g_{r0}^y, \quad r = 1, \dots, s \\ & \quad \sum_{t \in \tilde{T}^*(\mathbb{N})} \lambda_t = 1, \\ & \quad \lambda_t \in \{0, 1\}, \quad t \in \tilde{T}^*(\mathbb{N}) \end{aligned} \quad (9)$$

In contrast to (2), which works with the observed data, model (9) uses the non-observed database $\hat{\mathbb{N}} = \{(a^t, d_{T^*}(a^t))\}_{t \in \tilde{T}^*}$, which comes from building an optimal tree structure $T^*(\mathbb{N})$ through the EAT algorithm.

Finally, let us show the model that should be solved by applying the CEAT technique.

$$\begin{aligned} \overrightarrow{D}^{CEAT}(x_0, y_0; g_0^x, g_0^y) = \max & \quad \beta_0 \\ \text{s.t.} & \quad \sum_{t \in \tilde{T}^*(\mathbb{N})} \lambda_t a_j^t \leq x_{j0} - \beta_0 g_{j0}^x, \quad j = 1, \dots, m \\ & \quad \sum_{t \in \tilde{T}^*(\mathbb{N})} \lambda_t d_{rT^*}(a^t) \geq y_{r0} + \beta_0 g_{r0}^y, \quad r = 1, \dots, s \\ & \quad \sum_{t \in \tilde{T}^*(\mathbb{N})} \lambda_t = 1, \\ & \quad \lambda_t \geq 0, \quad t \in \tilde{T}^*(\mathbb{N}) \end{aligned} \quad (10)$$

Finally, we would like to highlight a limitation related to the approach based on EAT, which is associated with its intrinsic complexity. The EAT approach is linked to a very intensive computational procedure, which is related to the algorithm that must be previously executed before solving the corresponding Mixed Integer Linear Programming model. This feature contrasts sharply with the simplicity of the standard Free Disposal Hull technique, which is entirely based on Mixed Integer Linear Programming. Something similar can be said regarding the comparison between CEAT and DEA. From a computational point of view, DEA, which is only based upon Linear Programming, is clearly simpler than CEAT. Additionally, and regarding the sample size required to properly build the EAT model, it is worth mentioning that the typical stopping rule pointed out by Breiman et al. (1984) is usually applied, that is, the sample size of each terminal node in the tree must be five or more. However, this is the standard condition when the objective of the regression tree is estimating the mean of the response variable. In the case of being interested in quantiles, it seems that this number should be bigger (Breiman et al., 1984, Hastie et al. 2009), probably ten or more. Hence, given a certain number of terminal nodes, EAT, which tries to predict extreme behaviours of the response variable (the output), could need at least twice the sample size than the standard CART for regression. Nevertheless, a computational study should confirm this conclusion, which is beyond the scope of this paper.

4. Dynamic EAT and CEAT

In this section, we introduce a new technique based on the adaptation of the dynamic model to the EAT and CEAT contexts.

Previous models did not take capital investments into account and were based on the static approach. However, the dynamic approach as opposed to the static approach takes on relevance when dynamic factors, such as investment, need to be considered in data analysis. It has to be said that since the dynamic directional distance function is flexible and displays good properties, it has come to the forefront as the ideal approach for measuring dynamic technical efficiency.

In the same context as in Section 3.2., the dynamic directional distance function in the full input–output space based on the EAT technique can be determined through the following optimization model:

⁴ See Lam (2010) and Orkcu et al. (2015), where other MILP models have been implemented in the context of DEA.

$$\begin{aligned}
 \overrightarrow{D}^{EAT}(x_0, I_0, y_0, k_0; g_0^x, g_0^y, g_0^I) = \text{Max} & \quad \beta_0 \\
 \text{s.t.} & \quad \sum_{t \in \tilde{T}^*(\mathbb{N})} \lambda_t a_j^t \leq x_{j0} - \beta_0 g_{j0}^x, \quad j = 1, \dots, m \\
 & \quad \sum_{t \in \tilde{T}^*(\mathbb{N})} \lambda_t d_{rT^*}(a^t) \geq y_{r0} + \beta_0 g_{r0}^y, \quad r = 1, \dots, s \\
 & \quad \sum_{t \in \tilde{T}^*(\mathbb{N})} \lambda_t w_{hT^*}(a^t) \geq (I_{h0} - \delta_h k_{h0}) + \beta_0 g_{h0}^I, \quad h = 1, \dots, f \\
 & \quad \sum_{t \in \tilde{T}^*(\mathbb{N})} \lambda_t = 1, \\
 & \quad \lambda_t \in \{0, 1\}, \quad t \in \tilde{T}^*(\mathbb{N})
 \end{aligned} \tag{11}$$

Next, let us explain how the above model was derived. When using the EAT technique is the objective, then, before applying the corresponding algorithm that yields the tree structure $T^*(\mathbb{N})$, it is very important to identify which variables work as response variables for the regression technique and which do not. These particular variables will be estimated from the predictor variables (in our context, the inputs of the model) through several steps by splitting the nodes of a tree according to the EAT algorithm, as was explained in Section 3.3. To understand the derivation of model (11), first, take a look at model (4) in Section 3.2, which corresponds to the dynamic directional distance function under FDH. In that mathematical programming model, the third type of constraint, linked to gross investments in quasi-fixed inputs, and that characterizes the dynamic model in comparison with the static model, has an inequality sign associated with an output-type restriction: “ \geq ”. Accordingly, when the EAT is applied, the input data is utilized for estimating both outputs and the composite variables $z_h := (I_h - \delta_h k_h)$, $h = 1, \dots, f$, which are combinations between the gross investments, the quasi-fixed inputs and the depreciation rates. Due to that, the original EAT algorithm must be modified to provide estimations of these two types of variables in each node that is split. Additionally, as happens with the original algorithm, the technique must guarantee that free disposability is satisfied and, at the same time, the estimate takes values greater or equal than the observations in the target variables (y_r ,

$r = 1, \dots, s$, and z_h , $h = 1, \dots, f$). This is achieved by using the same estimation formula for the outputs and a similar adaptation for the variables z_h , as follows. For the right child node that would appear by splitting a parent node t^* , we have $z_h(t_R) = z_h(t^*)$, $h = 1, \dots, f$. Put in words, we must use the same estimation given for the parent node. For the left child node case, we must use:

$$z_h(t_L) = \max\{\max\{z_{hi} : (x_i, y_i, z_i) \in t_L\}, z_h(I_{T(q|t^* \rightarrow t_L, t_R)}(t_L))\} \tag{12}$$

where $z_h(I_{T(q|t^* \rightarrow t_L, t_R)}(t_L)) = \max\{z_h(t') : t' \in I_{T(q|t^* \rightarrow t_L, t_R)}(t_L)\}$ and $z_h(t')$ is the estimation of the variable z_h at node $t' \in \tilde{T}(q|t^* \rightarrow t_L, t_R)$, $h = 1, \dots, f$. Notice that the last formula utilizes the notion of (input) Pareto dominance between nodes that was explained in Section 3.3.

After applying the splitting process of nodes, we get a tree structure that suffers, in general, from overfitting and must be pruned by resorting to cross-validation (Breiman et al., 1984). The optimal tree derived from this process is denoted as $T^*(\mathbb{N})$. Finally, the way of estimating the outputs is as $d_{rT^*(\mathbb{N})}(x) = \sum_{t \in \tilde{T}^*(\mathbb{N})} y_r(t) I(x \in t)$, for all $r = 1, \dots, s$, with $I(\cdot)$ being the indication function, that is, we must use the same formula utilized for this type of variables in the original EAT algorithm. In the case of the variables z_h , $h = 1, \dots, f$, we must use the corresponding adaptation of the previous formula, i.e., $w_{hT^*(\mathbb{N})}(x) = \sum_{t \in \tilde{T}^*(\mathbb{N})} z_h(t) I(x \in t)$, $h = 1, \dots, f$.

The optimal tree structure $T^*(\mathbb{N})$ is also associated with a disjoint partition of the input space. Each leaf node t of that tree is linked to a region in the input space, called the support of the node and characterized by two corner points: a^t and b^t . In Fig. 3, we showed an example of partition of the input space where the corner points for each support are also highlighted. In particular, the corner points a^t are key for the characterization of the technology that can be derived from the EAT algorithm (see expression (7)) and are used for modelling any technical efficiency measure, such as the directional distance function.

Finally, by analogy with model (9) in Section 3.3, the model to be used for computing the dynamic directional distance function through the adaptation of the EAT would be model (11). In this model, δ_h indicates the depreciation rate, and g_0^x is a directional vector for variable inputs, g_0^y is a directional vector for outputs and g_0^I is a directional vector for gross investments. In addition, β_0 is the degree of dynamic technical inefficiency in the full input–output–investment space of DMU₀.

Finally, a convex estimator of the technology can be obtained from model (11) by relaxing the binary nature of the decision variables λ_t . In this way, the dynamic directional distance function estimated by CEAT is based on the model defined as follows:

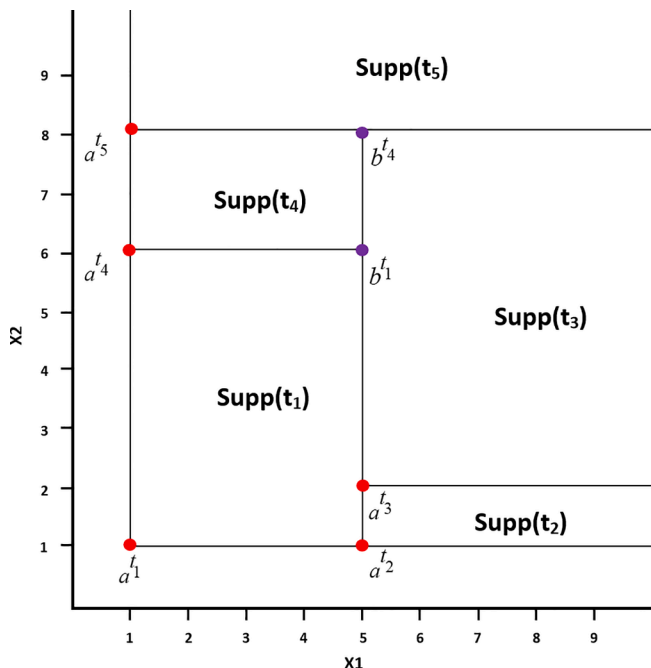


Fig. 3. Example of Pareto-dominance.

$$\begin{aligned}
 \vec{D}^{CEAT}(x_0, I_0, y_0, k_0; g_0^x, g_0^y, g_0^I) = \text{Max} & \quad \beta_0 \\
 \text{s.t.} & \quad \sum_{t \in \tilde{T}(\mathbb{N})} \lambda_t a_j^t \leq x_{j0} - \beta_0 g_{j0}^x, \quad j = 1, \dots, m \\
 & \quad \sum_{t \in \tilde{T}(\mathbb{N})} \lambda_t d_{rT^*}(\mathbf{a}^t) \geq y_{r0} + \beta_0 g_{r0}^y, \quad r = 1, \dots, s \\
 & \quad \sum_{t \in \tilde{T}(\mathbb{N})} \lambda_t w_{hT^*}(\mathbf{a}^t) \geq (I_{h0} - \delta_h k_{h0}) + \beta_0 g_{h0}^I, \quad h = 1, \dots, f \\
 & \quad \sum_{t \in \tilde{T}(\mathbb{N})} \lambda_t = 1, \\
 & \quad \lambda_t \geq 0, \quad t \in \tilde{T}(\mathbb{N})
 \end{aligned} \tag{13}$$

5. Empirical application

5.1. Dataset

Our data set comprises annual observations of dairy firms taken in five EU countries for the years 2014 and 2018. The choice of these two years for the analysis stems from the fact that in April 2015 dairy quotas that imposed constraints on milk production in the dairy sector were abolished. Obviously, the quota imposition and removal affected not only dairy farms, but also dairy processors (Hirsch et al., 2020). Therefore, by choosing 2014 and 2018, we were able to analyze the year with full quotas and the year of the quota-free system, hence we were able to assess the impact of quota removal on the inefficiency at the firm level (under the assumption of ceteris paribus, i.e., supposing that the remaining factors that could affect the dairy market kept constant). We focused the analysis on the main dairy producers in the EU by region: Italy and Spain for Southern Europe, France and Germany for Western Europe, and Poland for Eastern Europe⁵. ORBIS dataset (Bureau van Dijk) was the source of our data⁶.

The following variable specification for inefficiency estimation was used: two variable inputs, one quasi-fixed input, one gross investment, and one output, which were all assessed using accounting data. The two variable inputs were material and labor costs (denoted further as materials and labor), which were depreciated by the producer price index for non-durable consumer goods and labor cost index in industry, respectively. The quasi-fixed input (capital) measure was taken as the value of fixed assets in year τ and was depreciated using the producer price index for capital goods. Gross investments in fixed assets in year τ

⁵ Looking at turnover and production of the dairy manufacturing sector of each country, in a relative way, with respect to the total EU dairy turnover and production, the leaders of this sector are: France, Germany, Italy, Netherlands, Spain, the UK and Poland (Eurostat 2021a). The remaining EU countries represent the negligible contributions to the total EU turnover and production. Because of data availability (the lack of data for some input-output variables), we were not able to include the Netherlands and the UK in our sample. In addition, nowadays, UK is outside EU, which gives another argument for not including this country in our sample of EU countries.

⁶ We are aware that the use of a dataset from the dairy manufacturing industry alone cannot justify the effectiveness of the method. However, in Esteve et al. (2020), a complete simulation study was carried out to compare the performance of the EAT technique with respect to the standard non-parametric frontier estimates. Esteve et al. (2020) concluded that EAT clearly outperforms standard non-parametric methods when the estimation of technical efficiency is the concern, overcoming the overfitting problems identified with techniques as FDH and DEA. In particular, it was shown that the technique based on machine learning can reduce the mean squared error by between 13% and 70% and the bias by between 10% and 49%. In this regard, we believe that this simulation study justifies the effectiveness of the new methodology, which is applied for the first time in a real dataset under the dynamic approach.

were gauged as the value of fixed assets in year $\tau + 1$, minus the value of fixed assets in year τ , plus the value of depreciation in year $\tau + 1$, and were depreciated using the producer price index for capital goods. Output was quantified by revenue, which was depreciated using the producer price index for food manufacturing. All aforementioned price indices were country-specific and were downloaded from Eurostat (2021b) database. All variables were extracted from the ORBIS database in local currencies (which were Euros except for Poland, for which the currency was Zloty). The variables were adjusted using the purchasing power parity (PPP) of the local currency to the US dollar for the unbiased comparison of the variables between countries⁷.

The final dataset was obtained by removing missing and outlier observations. Simar (2003) approach was applied to detect outliers. Our final sample consisted of a balanced panel of 1,465 dairy-manufacturing firms in five EU countries observed in 2014 and 2018. Table 1 reports descriptive statistics for the inputs, investments, and output by country for the data amalgamated for these two years and for each year separately. The data in Table 1 shows that the German dairy-manufacturing sample exceeded all other countries with regard to average output in both 2014 and 2018, whereas the smallest value of output was produced by an average firm in Spain for both 2014 and 2018. Furthermore, the data indicates the increase in average values of all variables across all countries between 2014 and 2018. The statistics in Table 1 also show a significantly large variation in the sample, as evidenced by the large values of standard deviations, compared to their respective averages.

5.2. Results

The values of the directional vectors used in computations were: $g_{j0}^x = x_{j0} - \min_{i=1, \dots, n; \tau=2014, 2018} \{x_{ji}^\tau\}$, $j = 1, \dots, m$, for variable inputs, $g_{r0}^y = \max_{i=1, \dots, n; \tau=2014, 2018} \{y_{ri}^\tau\} - y_{r0}$, $r = 1, \dots, s$, for outputs, and $g_{h0}^I = 0.2 \cdot \max_{i=1, \dots, n; \tau=2014, 2018} \{k_{hi}^\tau\} - 0.2 \cdot k_{h0}$, $h = 1, \dots, f$, for investments; where the superscript τ for inputs, outputs and investments denotes the year of the corresponding observation. Additionally, the directional vector for investments contains 20 percent of the size of the capital stock, which approximates the size of investments in firms. Previous literature on the dynamic efficiency relied also on the 20 percent of the capital stock as a directional vector for investments due to the high heterogeneity in the investment variable (see, for example, Kapelko et al., 2014; Dakpo and Lansink, 2019). Following Portela et al. (2004), this makes the

⁷ Given that, in this paper, each country is studied in a separate way, and we use a model that is units invariant, there might be no need to convert the local currencies using PPP. Nevertheless, since we also aim to eventually compare the inefficiencies between models and countries, the application of PPP is necessary for the meaningful comparison in line with the standard literature on international comparisons (see, for example, Jorgenson and Yip, 2001; Jorgenson, 2018). Only the PPP of the local currencies to US dollar was available in the database.

Table 1

Descriptive statistics (averages and standard deviations) of input–output variables per country, for 2014, 2018 and 2014–2018 (thousands of PPP, as of 2011).

Year / Country	No of firms	Output	Materials	Labor	Capital	Investments
2014						
France	205	18,348.15 (62,727.36)	12,318.09 (41,981.00)	2,210.97 (9,088.49)	3,812.37 (13,983.80)	572.96 (2,044.39)
Germany	35	323,661.04 (917,051.37)	264,625.97 (734,400.02)	19,659.09 (61,209.09)	34,798.34 (105,548.30)	13,178.38 (42,691.63)
Italy	752	7,552.02 (11,665.78)	55.99 (86.87)	735.09 (1,145.91)	1,661.63 (3,342.40)	325.70 (1,230.63)
Poland	107	90,256.10 (231,036.13)	71,881.81 (189,992.76)	4,779.93 (10,074.07)	53,819.76 (299,959.08)	1,947.29 (12,777.49)
Spain	366	6,089.77 (25,821.38)	4,175.09 (18,807.81)	779.24 (2,682.05)	1,617.61 (5,301.24)	247.39 (1,189.14)
2018						
France	205	24,342.26 (69,653.01)	15,951.07 (45,592.28)	2,567.54 (8,835.05)	4,771.34 (17,033.80)	1,014.59 (3,838.41)
Germany	35	449,244.47 (1228446.39)	362,067.85 (988,981.25)	26,079.77 (75,058.10)	55,485.05 (165,593.60)	14,315.09 (35,503.12)
Italy	752	10,068.48 (15,563.63)	7,298.48 (11,287.32)	939.93 (1,473.91)	1,990.06 (3,693.43)	445.19 (1,081.93)
Poland	107	125,635.52 (341,701.44)	100,747.28 (284,559.28)	5,345.35 (12,731.53)	64,798.75 (288,364.92)	4,614.65 (15,257.75)
Spain	366	7,966.24 (28,469.58)	5,231.02 (20,255.65)	964.26 (2,511.94)	1,882.80 (5,177.61)	550.21 (4,313.89)
Average for 2014 and 2018						
France	410	21,345.20 (66,267.60)	14,134.58 (43,808.01)	2,389.25 (8,953.48)	4,291.85 (15,571.94)	793.77 (3,079.32)
Germany	70	386,452.76 (1,077,962.13)	313,346.91 (866,098.10)	22,869.43 (68,063.34)	45,141.70 (138,238.72)	13,746.73 (38,980.90)
Italy	1,504	8,810.25 (13,806.40)	3,677.23 (8,762.73)	837.51 (1,323.67)	1,825.84 (3,524.95)	385.44 (1,159.82)
Poland	214	107,945.81 (291,519.80)	86,314.54 (241,805.88)	5,062.64 (11,456.48)	59,309.25 (293,579.21)	3,280.97 (14,102.80)
Spain	732	7,028.01 (27,175.38)	4,703.05 (19,538.91)	871.753 (2,598.26)	1,750.20 (5,237.89)	398.80 (3,165.62)

Standard deviations are in parentheses.

Table 2

Average values for inefficiencies in dynamic DEA, dynamic CEAT, dynamic FDH and dynamic EAT, for 2014, 2018 and 2014–2018.

Year/Country	Convex		Non-convex	
	Dynamic DEA	Dynamic CEAT	Dynamic FDH	Dynamic EAT
2014				
France	0.1658	0.3979	0.0136	0.0672
Germany	0.0438	0.3007	0*	0.0613
Italy	0.2165	0.3055	0.0271	0.0851
Poland	0.1562	0.4802	0.0096	0.1238
Spain	0.4684	0.5865	0.0532	0.1088
2018				
France	0.1548	0.3607	0.0063	0.0607
Germany	0.0347	0.4718	0.0007	0.0542
Italy	0.2028	0.3488	0.0254	0.0654
Poland	0.4724	0.5795	0.1666	0.3108
Spain	0.2385	0.4951	0.0229	0.1014
Average for 2014 and 2018				
France	0.1603	0.3793	0.0100	0.0640
Germany	0.0393	0.3863	0.0004	0.0578
Italy	0.2097	0.3272	0.0263	0.0753
Poland	0.3143	0.5299	0.0881	0.2173
Spain	0.3535	0.5408	0.0381	0.1051

*Average result equal to zero due to rounding.

directional vector translation invariant, hence also the directional distance function (Aparicio et al, 2016). Consequently, we were able to deal with the variables that reach zero or negative values (Pastor and Aparicio, 2015). In our case, we can keep the firms with negative values of investments in the sample. Additionally, due to the definition of our directional vector, the distance between any unit and the corresponding frontier is determined by projecting the DMU towards a common (ideal) point for all observations and years. The calculations were conducted for two traditional dynamic models, one assuming convexity (DEA) and the second one not assuming convexity (FDH), and for the two new models proposed in this study, that is, dynamic CEAT and dynamic EAT. These were carried out for each year (2014 and 2018) and each country

Table 3

Number of efficient firms for dynamic DEA, dynamic CEAT, dynamic FDH and dynamic EAT, for 2014 and 2018.

Year/Country	Efficient - Convex		Efficient - Non-convex	
	Dynamic DEA	Dynamic CEAT	Dynamic FDH	Dynamic EAT
2014				
France	17 (8.29%)	6 (2.93%)	166 (80.98%)	82 (40.00%)
Germany	8 (22.86%)	4 (11.43%)	27 (77.14%)	23 (65.71%)
Italy	19 (2.53%)	6 (0.80%)	433 (57.58%)	179 (23.80%)
Poland	13 (12.15%)	3 (2.80%)	88 (82.24%)	36 (33.64%)
Spain	6 (1.64%)	2 (0.55%)	218 (59.56%)	90 (24.59%)
2018				
France	15 (7.32%)	2 (0.98%)	185 (90.24%)	89 (43.41%)
Germany	9 (25.71%)	4 (11.43%)	23 (65.71%)	21 (60.00%)
Italy	24 (3.19%)	6 (0.80%)	453 (60.24%)	210 (27.93%)
Poland	13 (12.15%)	4 (3.74%)	64 (59.81%)	28 (26.17%)
Spain	11 (3.01%)	5 (1.37%)	257 (70.22%)	115 (31.42%)

The percentage of all observations in a given country and year is presented in parentheses.

separately. Then we also computed the changes in inefficiencies between 2014 and 2018 for each of the four models to provide some insights into the possible effects of dairy quota removal.

We start with the comparison of inefficiencies for the estimated models. Table 2 summarizes the averages for the calculated inefficiencies per time period and country. Looking firstly at results for convex models (that is the second and third column of Table 2), we can conclude that average values for dynamic inefficiency using CEAT were larger than DEA counterparts, regardless of the year and country under analysis. This stems from the fact that DEA suffers from overfitting, while CEAT aims at estimating the real dynamic frontier which generated the data. Overall, for 2014–2018, the largest differences between

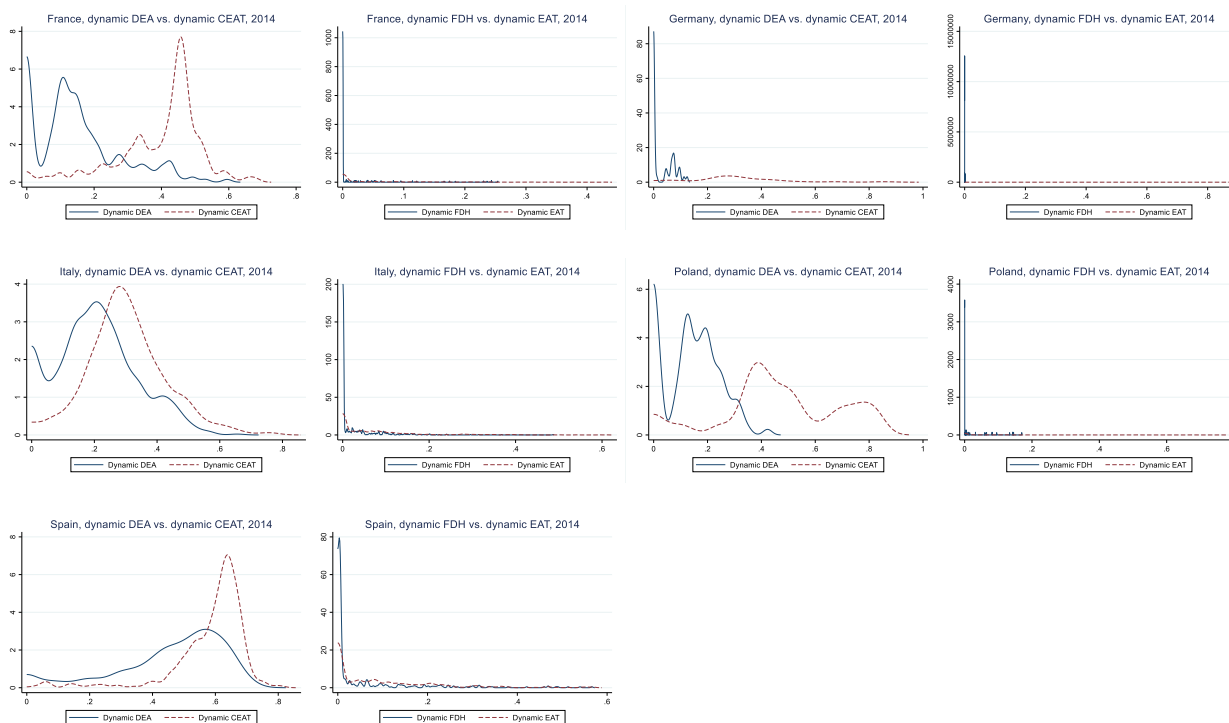


Fig. 4. Kernel density plots for different dynamic models in 2014, by country.

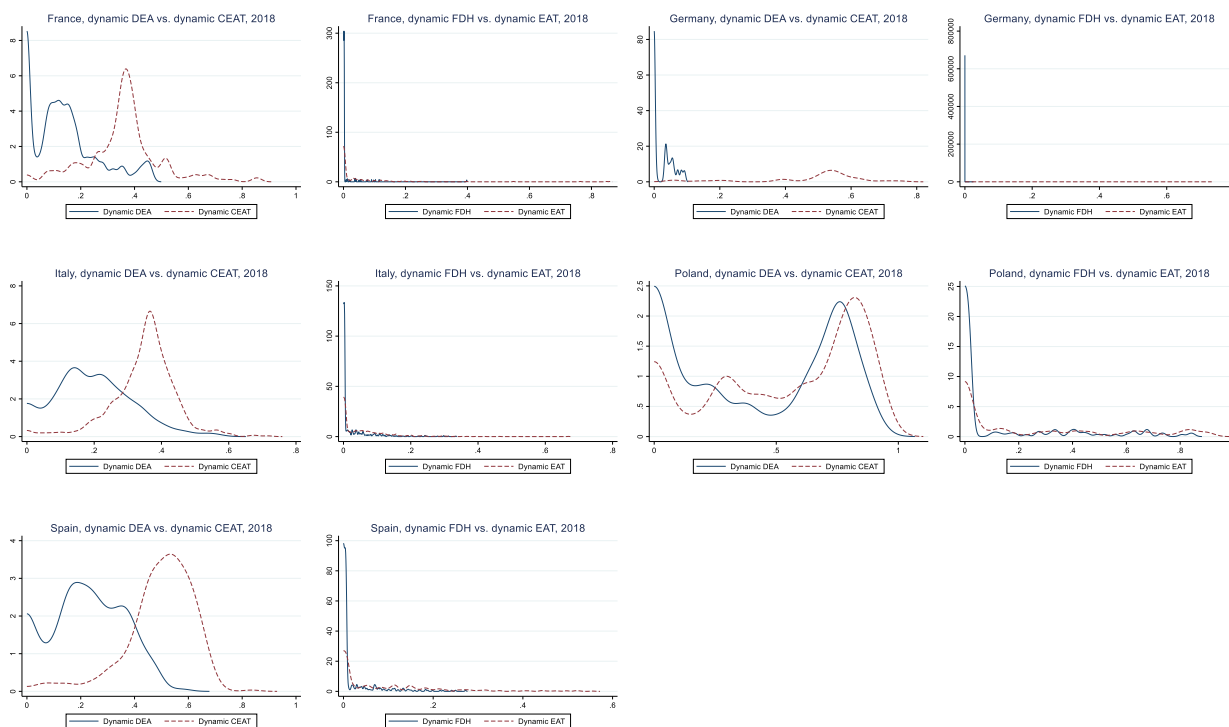


Fig. 5. Kernel density plots for different dynamic models in 2018, by country.

models were found for Germany and France, on average. Furthermore, on average across 2014 and 2018, the largest inefficiency was observed for dairy companies in Spain while assuming both dynamic DEA and CEAT, and the lowest for firms in Germany when assuming dynamic CEAT and for Italian firms when assuming dynamic CEAT. It seems, therefore, that German firms that were the largest with respect to their

input-output-investment profiles (see Table 2) obtained the best efficiency outcomes. For example, 0.3535 of inefficiency reported for Spanish firms using the dynamic DEA model indicates that this was the distance between the level of inputs, outputs and investment at the actual point and its target ideal point, given the dynamic production frontier calculated by DEA and the directional vector defined as the

range of possible improvement of unit under analysis (Portela et al., 2004). Assuming the model of dynamic CEAT and being restricted by the theoretical dynamic frontier from which the observations were generated, Spanish firms' distance to the ideal point had a magnitude of 0.5408. Hence, dairy firms in Spain reached the ideal point at 65% and 46%, assuming the dynamic DEA frontier and dynamic theoretical frontier, respectively and given the directional vector. Conversely, firms in Germany on average for 2014 and 2018 reached an inefficiency level of only 0.0393 when solving the DEA model; hence they achieved the ideal point at 96%, given the directional vector. When assuming the CEAT frontier, Italian firms achieved the best performance, with an inefficiency level of 0.3272 which indicates that the path to the ideal point was accomplished at 67% of the value of the directional vector. However, further research is necessary to assess the exact magnitude and sources of the differences in inefficiencies between countries.

As regards non-convex models (that is the fourth and fifth column of Table 2), not surprisingly we came across smaller inefficiencies in comparison to convex models observed for all periods and countries. Moreover, the average inefficiencies of the EAT model were systematically larger than their FDH counterparts across all periods and countries. Similar to the DEA model, FDH suffers from overfitting, while EAT aims at estimating the dynamic theoretical frontier, which might explain these findings. In an attempt to assess the differences between countries, our findings indicate that, on average, across 2014 and 2018 the distance to the ideal point was larger for Polish dairy firms, while German firms were at the opposite end of the spectrum, assuming both non-convex models. Specifically, on average, Polish firms reached the ideal point at 91% and 78% for dynamic FDH and EAT models, respectively, while for German firms these numbers were 99% and 94%, given the directional vector. Hence, negligible improvements were required for German firms with regard to inputs, outputs and investments in order to reach both FDH and theoretical frontier. Nevertheless, the value of the directional vector should be taken into account in order to assess the exact magnitude of the changes required.

Overall, the findings on the differences in dynamic inefficiencies obtained using different models are consistent with Proposition 2 (iii) in Aparicio et al. (2021) formulated in the static context of production that states that EAT (or CEAT) technology envelops FDH (or DEA) technology. Hence, also using dynamic measures we should expect inefficiency indicators in EAT and CEAT to be either equal or larger than their FDH and DEA counterparts, respectively. Accordingly, these findings are in line with empirical results in Aparicio et al. (2021), however again these were obtained using static measures of efficiency, hence they cannot be directly compared with our dynamic models.

We now turn our attention to the results on the number of efficient firms across all models, which is summarized in Table 3. Comparing dynamic DEA with dynamic CEAT, and dynamic FDH with dynamic EAT, we should notice that the number of efficient dairy companies was substantially smaller under the new dynamic models based on machine learning techniques than those corresponding to the traditional dynamic models; this was true for all time periods and countries. Hence, it can be concluded that dynamic EAT and CEAT allowed for a better discrimination among observations than their traditional counterparts, that is, DEA- and FDH- based dynamic models. This is especially evident with the findings for French dairy manufacturing companies, which were in

overwhelming majority efficient under the FDH approach (80.98% of the sample in 2014 and 90.24% in 2018). On the contrary, under the EAT approach, these numbers were substantially lower: 40.00% in 2014 and 43.41% in 2018. Likewise, comparing convex and non-convex dynamic models, the number of efficient dairy firms under DEA and CEAT was substantially smaller than in non-convex models (FDH and EAT). Therefore, the discriminating power improved when moving from non-convex to convex models. Overall, these findings suggest the advantage of the new models within each group of dynamic methods (convex versus non-convex) as well as the superiority of CEAT among all four models in terms of discriminating power. This obviously has important implications in practice, for example in terms of appropriate ranking of firms.

Let us now have a closer look at whole distributions of inefficiency scores for the models analyzed. Figs. 4 and 5 depict the kernel distributions by country for the pairs of models: dynamic DEA-dynamic CEAT and dynamic FDH-dynamic EAT, for 2014 and 2018, respectively. When plotting these distributions, we followed the procedure proposed by Simar and Zelenyuk (2006). What immediately can be observed is that DEA and CEAT distributions were different for all countries and years. It is also clear that DEA distributions tended to be concentrated at the lower values of inefficiency as compared to the CEAT counterparts. The differences between FDH and EAT were not so clear in figures. The most visible results were for Italy and Spain in 2014 and Italy, Poland and Spain in 2018. It is apparent in these cases that FDH scores were concentrated more around 0 than EAT scores. Hence, most of firms operated on the dynamic frontier of best practices. For the rest of the cases the differences cannot be visually assessed since FDH scores were for majority of firms equal to zero, which distorted the scale in the graphs and, as a result, graphs could not differentiate between distributions. The results obtained by kernel densities were further confirmed by the statistical test proposed by Simar and Zelenyuk (2006), which is based on the Li (1996) test. In particular, different pairs of distributions (within-model perspective, that is, dynamic CEAT vs. dynamic DEA, and dynamic EAT vs. dynamic FDH, and between-model point of view, that is, convex models versus non-convex models) were proved to be different in all countries and years.

Finally, to conclude our analysis and provide some empirical insights into dairy industry dynamics and the possible effect of dairy quota removal, we focused on the dynamic inefficiency change between 2014 and 2018. Table 4 presents the average change in dynamic inefficiencies for all four models and for all five countries, computed as the value of the final year minus the value in the initial year. The results are conclusive across all models for dairy firms in France, Poland and Spain, showing that dynamic inefficiency decreased in France and Spain between 2014 and 2018, but increased for Poland. We confirmed the statistical significance of these differences by running the Simar and Zelenyuk (2006) test. Therefore, on average dairy firms in France and Spain significantly improved their performance from 2014 to 2018, while the opposite was observed for companies in Poland. The change in performance of firms from 2014 till 2018 can be due to quota abolishment, among many other factors that could contribute to this change. The literature is not conclusive on the relationship between quotas and efficiency. From the theoretical point of view, quotas were believed to create economic inefficiency in contrast to a free market system (Fulginiti and Perrin, 1993), but increase in efficiency was also predicted, especially when markets are subject to tradable quotas (Boots et al., 1997; Alston, 1981; Colman, 2000; and Areal et al., 2012a). Empirical studies of specific countries did not report conclusive results as well (Alvarez et al. 2006; Sauer, 2010; Areal et al., 2012a; Areal et al., 2012b; Garcia-Covarrubias et al., 2020; and Hansen et al., 2021). Our results on the improvement in efficiency in the quota-free period in France and Spain are consistent with the studies by Garcia-Covarrubias et al. (2020), which for Ireland observed an increase in efficiency in post-quota period, Alvarez et al. (2006) that for Spain found that quotas were not allocated to efficient farms and, therefore, they did not contribute to increasing efficiency of

Table 4
Change in inefficiencies (dynamic DEA, dynamic CEAT, dynamic FDH and dynamic EAT) between 2014 and 2018, average values reported.

Country	Dynamic DEA	Dynamic CEAT	Dynamic FDH	Dynamic EAT
France	-0.0110	-0.0372	-0.0073	-0.0065
Germany	-0.0091	0.1711	0.0007	-0.0070
Italy	-0.0138	0.0432	-0.0017	-0.0196
Poland	0.3162	0.0993	0.1570	0.1870
Spain	-0.2300	-0.0914	-0.0302	-0.0074

the sector, and Sauer (2010), which for Denmark suggested that abolition of the milk quota system has positive effects on milk farms' productivity. The negative effect on efficiency in a quota-free period found for Poland is in line with findings by Areal et al. (2012a), Areal et al. (2012b) that for the UK concluded that participation in a quota system was linked to being efficient as well as by Hansen et al. (2021) that reported a positive effect of quota filling on the revenue efficiency in Norway. Our results could be explained by the recent findings by Hirsch et al. (2020), who analyzed the flexibility of EU dairy processors to adjust production to fluctuating economic conditions, including the abolition of the quota system. This study found that the flexibility is the highest in Poland and low flexibility characterizes firms in France and Spain, while a significant negative relationship is found between flexibility and efficiency given the trade-off that exists between these two characteristics. Therefore, high flexibility in Poland and low flexibility in France and Spain could explain efficiency worsening in the quota-free system in Poland and its improvement in France and Spain. Our results for Italy and Germany were not consistent across models, hence we cannot formulate precise conclusions regarding the effects of quota removal in the dairy sector of these countries.

6. Conclusions and future work

This paper proposed to incorporate machine learning techniques into the measurement of dynamic inefficiency of firms. This allowed to advance the research on dynamic efficiency measurement from a descriptive analysis of the sample data to the estimation of the actual production frontier which enabled to avoid the common problem of traditional dynamic models related with an overestimation of efficiency scores. Bridging the gap between dynamic inefficiency and machine learning was based on the recently developed EAT and CEAT models (Esteve et al., 2021; Aparicio et al., 2021). EAT and CEAT, through the application of classification and regression trees with pruning and cross-validation, allowed to build frontier models that avoid the overfitting problem and as a result approximated the theoretical frontier. Linking CEAT and EAT with dynamic inefficiency measurement resulted in the development of two models that were coined dynamic EAT and dynamic CEAT. Their characteristics and comparison with standard dynamic FDH and dynamic DEA were further shown through an empirical application to the recent dataset of the main dairy producing firms in EU countries.

The main empirical results of this study indicated that inefficiency in the dynamic CEAT or EAT was higher than their corresponding values calculated through the dynamic DEA or FDH. Not surprisingly, we also found that dynamic EAT and dynamic FDH yielded lower inefficiency scores than their convex versions of dynamic CEAT and dynamic DEA. Furthermore, we noticed that the discriminating power of dynamic DEA drastically improved when switching to dynamic CEAT; likewise, dynamic EAT allowed for a better discrimination than that of dynamic FDH. This result has important practical implications, since using models with lower discriminating power such as dynamic DEA and dynamic FDH does not always allow to appropriately rank units, therefore limiting the managerial insights or their ability to be complemented with other techniques such as second stage regression analysis. Switching to new models of dynamic CEAT and dynamic EAT increases the discriminating power allowing for better ranking of units, therefore it is important for firms' managers supporting their decision-making process. In particular, the usage of the new models can help managers for a better comparison with competitors and finding appropriate benchmarks for learning about the best practices. In turn, the managers are better informed for making important production decisions. Finally, in the paper we also provided some insights into the development of inefficiency in the period before and after dairy quota removal (2014 vs 2018), concluding that firms in France and Spain improved their performance in the quota-free period in contrast to the quota-regulated period, while the opposite was observed for firms in Poland. Dairy quota removal impacted dairy processors only indirectly

as opposed to dairy farms. Nevertheless, the results obtained might imply that managers of firms in France and Spain were restricted by quotas when quotas' restrictions were in place, and therefore took a benefit from the quota-free trade, while the opposite was true for the firms in Poland.

There are several challenges involved in the proposed research that provide rich opportunities for future research. From an empirical point of view, we assumed that the change in inefficiencies between 2014 and 2018 was due to the removal of quotas. But obviously there could be many other variables that impacted this change. Therefore, a future study could be conducted to analyze the factors responsible for the observed changes in inefficiencies. Moreover, we restricted our analysis to only two years since we aimed to illustrate the functioning and characteristics of the new methods and their comparison with traditional models. Obviously, more years of analysis would be required to reach more detailed empirical conclusions, which is also an interesting avenue for future research. From a methodological point of view, future research study could further extend the new methods of dynamic EAT and CEAT to allow for an appropriate comparison of the differences in inefficiencies between countries, for example through the application of metafrontier techniques (Battese et al., 2004; O'Donnell et al., 2008) or the adaptation of the so-called Camanho-Dyson index (Camanho and Dyson, 2006). A limitation of the new methodology is its computational complexity in comparison with FDH and DEA. Future research should further analyse these issues by providing improvements of the original algorithms with respect to the execution time of the models. Furthermore, to test the discriminating power of new methods more thoroughly, ranking methods could be applied and developed in this context. Finally, the promising methodological extensions could involve the estimation of productivity change over time as well as economic efficiency based on new dynamic models.

CRedit authorship contribution statement

Juan Aparicio: Methodology, Investigation, Writing – original draft, Writing – review & editing, Funding acquisition. **Miriam Esteve:** Methodology, Investigation, Writing – original draft, Visualization, Software, Data curation, Writing – review & editing. **Magdalena Kapelko:** Methodology, Investigation, Writing – original draft, Writing – review & editing, Funding acquisition.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Acknowledgments

We thank five anonymous reviewers for providing constructive comments and help in improving the contents and presentation of this paper. The authors thank the grant PID2019-105952 GB-I00 funded by Ministerio de Ciencia e Innovación/ Agencia Estatal de Investigación /10.13039/501100011033. Additionally, J. Aparicio thanks the grant PROMETEO/2021/063 funded by the Valencian Community (Spain). Moreover, M. Esteve gratefully acknowledges the financial support from the Spanish Ministry of Science, Innovation and Universities under Grant FPU17/05365. The calculations for the adapted Li test were made at the Wrocław Centre for Networking and Supercomputing (www.wccs.wroc.pl), grant no. 286.

References

- Alston, J. M. (1981). A note on the effects of non-transferable quotas on supply functions. *Review of Marketing and Agricultural Economics*, 49(430–2016-31590), 189–197.
- Alvarez, A., Arias, C., & Orea, L. (2006). Explaining differences in milk quota values: The role of economic efficiency. *American Journal of Agricultural Economics*, 88(1), 182–193.
- Ang, F., Mortimer, S. M., Areal, F. J., & Tiffin, R. (2018). On the opportunity cost of crop diversification. *Journal of Agricultural Economics*, 69(3), 794–814.
- Ang, F., & Lansink, A. O. (2018). Decomposing dynamic profit inefficiency of Belgian dairy farms. *European Review of Agricultural Economics*, 45(1), 81–99.
- Aparicio, J., Pastor, J. T., & Vidal, F. (2016). The directional distance function and the translation invariance property. *Omega*, 58, 1–3.
- Aparicio, J., & Kapelko, M. (2019). Accounting for slacks to measure dynamic inefficiency in data envelopment analysis. *European Journal of Operational Research*, 278(2), 463–471.
- Aparicio, J., Esteve, M., Rodríguez-Sala, J. J., & Zofio, J. L. (2021). *The Estimation of Productive Efficiency Through Machine Learning Techniques: Efficiency Analysis Trees*. In *Data-Enabled Analytics* (pp. 51–92). Cham: Springer.
- Areal, F. J., Tiffin, R., & Balcombe, K. (2012). Farm technical efficiency under a tradable milk quota system. *Journal of Dairy Science*, 95(1), 50–62.
- Areal, F. J., Tiffin, R., & Balcombe, K. G. (2012). Provision of environmental output within a multi-output distance function approach. *Ecological Economics*, 78, 47–54.
- Azadeh, A., Saberi, M., Moghaddam, R. T., & Javanmardi, L. (2011). An integrated data envelopment analysis–artificial neural network–rough set algorithm for assessment of personnel efficiency. *Expert Systems with Applications*, 38(3), 1364–1373.
- Baležentis, T. (2016). Dynamic efficiency in Lithuanian cereal farms. *Management Theory and Studies for Rural Business and Infrastructure Development*, 38(2), 114–127.
- Baležentis, T., & Oude Lansink, A. (2020). Measuring dynamic biased technical change in Lithuanian cereal farms. *Agribusiness*, 36(2), 208–225.
- Banker, R. D., Charnes, A., & Cooper, W. W. (1984). Some models for estimating technical and scale inefficiencies in data envelopment analysis. *Management Science*, 30(9), 1078–1092.
- Battese, G. E., Rao, D. S., & O’Donnell, C. J. (2004). A metafrontier production function for estimation of technical efficiencies and technology gaps for firms operating under different technologies. *Journal of Productivity Analysis*, 21(1), 91–103.
- Boots, M., Lansink, A. O., & Peerlings, J. (1997). Efficiency loss due to distortions in Dutch milk quota trade. *European Review of Agricultural Economics*, 24(1), 31–46.
- Breiman, L., Friedman, J., Olshen, R., & Stone, C. (1984). *Classification and Regression Trees*. Belmont, CA: Wadsworth International Group.
- Camanho, A. S., & Dyson, R. G. (2006). Data envelopment analysis and Malmquist indices for measuring group performance. *Journal of Productivity Analysis*, 26(1), 35–49.
- Chambers, R. G., Chung, Y., & Färe, R. (1998). Profit, directional distance functions, and Nerlovian efficiency. *Journal of Optimization Theory and Applications*, 98(2), 351–364.
- Charnes, A., Cooper, W. W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. *European Journal of Operational Research*, 2(6), 429–444.
- Colman, D. (2000). Inefficiencies in the UK milk quota system. *Food Policy*, 25(1), 1–16.
- Dakpo, K. H., & Lansink, A. O. (2019). Dynamic pollution-adjusted inefficiency under the by-production of bad outputs. *European Journal of Operational Research*, 276(1), 202–211.
- Daouia, A., Noh, H., & Park, B. U. (2016). Data envelope fitting with constrained polynomial splines. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 78(1), 3–30.
- Depriens, D., Simar, L., & Tulkens, H. (1984). Measuring Labor Inefficiency in Post Offices. In M. Marchand, P. Pestieau, & H. Tulkens (Eds.), *the Performance of Public Enterprises: Concepts and Measurements* (pp. 243–267). Amsterdam: North-Holland.
- Eisner, R., & Strotz, R. H. (1963). *Determinants of business investment*. New York: Prentice-Hall.
- Emrouznejad, A., & Anouze, A. L. (2010). Data envelopment analysis with classification and regression tree—a case of banking efficiency. *Expert Systems*, 27(4), 231–246.
- Engida, T. G., Rao, X., & Lansink, A. G. O. (2020). A dynamic by-production framework for analyzing inefficiency associated with corporate social responsibility. *European Journal of Operational Research*, 287(3), 1170–1179.
- Epstein, L. G. (1981). Duality theory and functional forms for dynamic factor demands. *The Review of Economic Studies*, 48(1), 81–95.
- Esteve, M., Aparicio, J., Rabasa, A., & Rodríguez-Sala, J. J. (2020). Efficiency analysis trees: A new methodology for estimating production frontiers through decision trees. *Expert Systems with Applications*, 162, Article 113783.
- Esteve, M., Aparicio, J., Rodríguez-Sala, J. J., & Zhu, J. (2023). Random Forests and the measurement of super-efficiency in the context of Free Disposal Hull. *European Journal of Operational Research*, 304(2), 729–744.
- Eurostat (2021a). Structural business statistics. <https://ec.europa.eu/eurostat/web/structural-business-statistics/data/database>.
- Eurostat (2021b). Short-term business statistics. <https://ec.europa.eu/eurostat/web/short-term-business-statistics/data/database>.
- Fallah-Fini, S., Triantis, K., & Johnson, A. L. (2014). Reviewing the literature on non-parametric dynamic efficiency measurement: State-of-the-art. *Journal of Productivity Analysis*, 41(1), 51–67.
- Fukuyama, H., & Weber, W. L. (2017). Measuring bank performance with a dynamic network Luenberger indicator. *Annals of Operations Research*, 250(1), 85–104.
- Fulginiti, L., & Perrin, R. (1993). The theory and measurement of producer response under quotas. *The Review of Economics and Statistics*, 97–106.
- García-Covarrubias, L., Laepple, D., Dillon, E., & Thorne, F. (2020). The role of hired labor in transient and persistent technical efficiency on Irish dairy farms. Working paper. <https://ideas.repec.org/p/ags/aeaa20/304395.html>.
- Gholizadeh, H., Fathollahi-Fard, A. M., Fazlollahtabar, H., & Charles, V. (2022). Fuzzy data-driven scenario-based robust data envelopment analysis for prediction and optimisation of an electrical discharge machine’s parameters. *Expert Systems with Applications*, 193, Article 116419.
- Guerrero, N. M., Aparicio, J., & Valero-Carreras, D. (2022). Combining Data Envelopment Analysis and Machine Learning. *Mathematics*, 10(6), 909.
- Hansen, B. G., Haga, H., & Lindblad, K. B. (2021). Revenue efficiency, profitability, and profitability potential on organic versus conventional dairy farms—Results from comparable groups of farms. *Organic Agriculture*, 11(3), 351–365.
- Hastie, T., Tibshirani, R., Friedman, J. H., & Friedman, J. H. (2009). *The elements of statistical learning: data mining, inference, and prediction* (Vol. 2., 1–758).
- Hirsch, S., Mishra, A., Möhring, N., & Finger, R. (2020). Revisiting firm flexibility and efficiency: Evidence from the EU dairy processing industry. *European Review of Agricultural Economics*, 47(3), 971–1008.
- Hong, H. K., Ha, S. H., Shin, C. K., Park, S. C., & Kim, S. H. (1999). Evaluating the efficiency of system integration projects using data envelopment analysis (DEA) and machine learning. *Expert Systems with Applications*, 16(3), 283–296.
- Jorgenson, D. W. (2018). Production and welfare: Progress in economic measurement. *Journal of Economic Literature*, 56(3), 867–919.
- Jorgenson, D. W., & Yip, E. (2001). Whatever happened to productivity growth? *New Developments in Productivity Analysis*, 63, 509–540.
- Kapelko, M. (2017). Dynamic versus static inefficiency assessment of the Polish meat-processing industry in the aftermath of the European Union integration and financial crisis. *Agribusiness*, 33(4), 505–521.
- Kapelko, M. (2019). Measuring productivity change accounting for adjustment costs: Evidence from the food industry in the European Union. *Annals of Operations Research*, 278(1), 215–234.
- Kapelko, M., Lansink, A. O., & Stefanou, S. E. (2021). Measuring dynamic inefficiency in the presence of corporate social responsibility and input indivisibilities. *Expert Systems with Applications*, 176, Article 114849.
- Kapelko, M., Oude Lansink, A., & Stefanou, S. E. (2014). Assessing dynamic inefficiency of the Spanish construction sector pre-and post-financial crisis. *European Journal of Operational Research*, 237(1), 349–357.
- Kapelko, M., Oude Lansink, A., & Stefanou, S. E. (2017). Input-specific dynamic productivity change: Measurement and application to European dairy manufacturing firms. *Journal of Agricultural Economics*, 68(2), 579–599.
- Kubler, S., Robert, J., Derigent, W., Voisin, A., & Le Traon, Y. (2016). A state-of-the-art survey & testbed of fuzzy AHP (FAHP) applications. *Expert Systems with Applications*, 65, 398–422.
- Kuosmanen, T., & Johnson, A. L. (2010). Data envelopment analysis as nonparametric least-squares regression. *Operations Research*, 58(1), 149–160.
- Kuosmanen, T., & Johnson, A. (2017). Modeling joint production of multiple outputs in StoNED: Directional distance function approach. *European Journal of Operational Research*, 262(2), 792–801.
- Lam, K. F. (2010). In the determination of weight sets to compute cross-efficiency ratios in DEA. *Journal of the Operational Research Society*, 61(1), 134–143.
- Li, Q. (1996). Nonparametric testing of closeness between two unknown distribution functions. *Econometric Reviews*, 15(3), 261–274.
- Mariz, F. B., Almeida, M. R., & Aloise, D. (2018). A review of dynamic data envelopment analysis: State of the art and applications. *International Transactions in Operational Research*, 25(2), 469–505.
- Martínez-Victoria, M., Maté-Sánchez-Val, M., & Lansink, A. O. (2019). Spatial dynamic analysis of productivity growth of agri-food companies. *Agricultural Economics*, 50(3), 315–327.
- Minviel, J. J., & Sipiläinen, T. (2021). A dynamic stochastic frontier approach with persistent and transient inefficiency and unobserved heterogeneity. *Agricultural Economics*, 52(4), 575–589.
- Nemoto, J., & Goto, M. (2003). Measurement of dynamic efficiency in production: An application of data envelopment analysis to Japanese electric utilities. *Journal of Productivity Analysis*, 19(2), 191–210.
- O’Donnell, C. J., Rao, D. S., & Battese, G. E. (2008). Metafrontier frameworks for the study of firm-level efficiencies and technology ratios. *Empirical Economics*, 34(2), 231–255.
- Olesen, O. B., & Ruggiero, J. (2018). An improved Afriat–Diewert–Parkan nonparametric production function estimator. *European Journal of Operational Research*, 264(3), 1172–1188.
- Olesen, O. B., & Ruggiero, J. (2022). The hinging hyperplanes: An alternative nonparametric representation of a production function. *European Journal of Operational Research*, 296(1), 254–266.
- Lansink, A. O., Stefanou, S., & Serra, T. (2015). Primal and dual dynamic Luenberger productivity indicators. *European Journal of Operational Research*, 241(2), 555–563.
- Özcan, Z., & Kayıkçıoğlu, T. (2021). Evaluating MFCC-based speaker identification systems with data envelopment analysis. *Expert Systems with Applications*, 168, Article 114448.
- Parmeter, C. F., & Racine, J. S. (2013). Smooth constrained frontier analysis. In *Recent advances and future directions in causality, prediction, and specification analysis* (pp. 463–488). New York, NY: Springer.
- Pastor, J. T., & Aparicio, J. (2015). *Translation invariance in data envelopment analysis*. In *Data envelopment analysis* (pp. 245–268). Boston, MA: Springer.
- Portela, M. S., Thanassoulis, E., & Simpson, G. (2004). Negative data in DEA: A directional distance approach applied to bank branches. *Journal of the Operational Research Society*, 55(10), 1111–1121.
- Rebai, S., Yahia, F. B., & Essid, H. (2020). A graphically based machine learning approach to predict secondary schools performance in Tunisia. *Socio-Economic Planning Sciences*, 70, Article 100724.

- Rungsuriyawiboon, S., & Stefanou, S. E. (2007). Dynamic efficiency estimation: An application to US electric utilities. *Journal of Business & Economic Statistics*, 25(2), 226–238.
- Sauer, J. (2010). Deregulation and dairy production systems: A Bayesian distance function approach. *Journal of Productivity Analysis*, 34(3), 213–237.
- Setiawan, M. (2019). Dynamic productivity growth and its determinants in the Indonesian food and beverages industry. *International Review of Applied Economics*, 33(6), 774–788.
- Silva, E., & Stefanou, S. E. (2003). Nonparametric dynamic production analysis and the theory of cost. *Journal of Productivity Analysis*, 19(1), 5–32.
- Silva, E., & Stefanou, S. E. (2007). Dynamic efficiency measurement: Theory and application. *American Journal of Agricultural Economics*, 89(2), 398–419.
- Silva, E., Lansink, A. O., & Stefanou, S. E. (2015). The adjustment-cost model of the firm: Duality and productive efficiency. *International Journal of Production Economics*, 168, 245–256.
- Simar, L. (2003). Detecting outliers in frontier models: A simple approach. *Journal of Productivity Analysis*, 20(3), 391–424.
- Simar, L., & Wilson, P. W. (1998). Sensitivity analysis of efficiency scores: How to bootstrap in nonparametric frontier models. *Management Science*, 44(1), 49–61.
- Simar, L., & Wilson, P. W. (2000a). A general methodology for bootstrapping in nonparametric frontier models. *Journal of Applied Statistics*, 27(6), 779–802.
- Simar, L., & Wilson, P. W. (2000b). Statistical inference in nonparametric frontier models: The state of the art. *Journal of Productivity Analysis*, 13(1), 49–78.
- Simar, L., & Zelenyuk, V. (2006). On testing equality of distributions of technical efficiency scores. *Econometric Reviews*, 25(4), 497–522.
- Simar, L., & Zelenyuk, V. (2011). Stochastic FDH/DEA estimators for frontier analysis. *Journal of Productivity Analysis*, 36(1), 1–20.
- Sohn, S. Y., & Moon, T. H. (2004). Decision tree based on data envelopment analysis for effective technology commercialization. *Expert systems with Applications*, 26(2), 279–284.
- Thaker, K., Charles, V., Pant, A., & Gherman, T. (2022). A DEA and random forest regression approach to studying bank efficiency and corporate governance. *Journal of the Operational Research Society*, 73(6), 1258–1277.
- Tone, K., & Tsutsui, M. (2010). Dynamic DEA: A slacks-based measure approach. *Omega*, 38(3–4), 145–156.
- Tone, K., & Tsutsui, M. (2014). Dynamic DEA with network structure: A slacks-based measure approach. *Omega*, 42(1), 124–131.
- Treadway, A. B. (1970). Adjustment costs and variable inputs in the theory of the competitive firm. *Journal of Economic Theory*, 2(4), 329–347.
- Tsionas, M. G. (2022). Efficiency estimation using probabilistic regression trees with an application to Chilean manufacturing industries. *International Journal of Production Economics*, 108492.
- Tsionas, M. G., Malikov, E., & Kumbhakar, S. C. (2020). Endogenous dynamic efficiency in the intertemporal optimization models of firm behavior. *European Journal of Operational Research*, 284(1), 313–324.
- Tsolas, I. E., Charles, V., & Gherman, T. (2020). Supporting better practice benchmarking: A DEA-ANN approach to bank branch performance assessment. *Expert Systems with Applications*, 160, 113599.
- Valero-Carreras, D., Aparicio, J., & Guerrero, N. M. (2021). Support vector frontiers: A new approach for estimating production functions through support vector machines. *Omega*, 104, Article 102490.
- Valero-Carreras, D., Aparicio, J., & Guerrero, N. M. (2022). Multi-output Support Vector Frontiers. *Computers & Operations Research*, 143, Article 105765.
- Yeh, C. C., Chi, D. J., & Hsu, M. F. (2010). A hybrid approach of DEA, rough set and support vector machines for business failure prediction. *Expert Systems with Applications*, 37(2), 1535–1541.