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The standard total factor productivity index and its decomposition

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ABSTRACT

The Malmquist productivity index is one of the best known and most widely used measures in the economic literature to quantify and decompose changes in productivity of multi-input multi-output production processes over time. Two main approaches are used to calculate this index: the adjacent Malmquist index and the base period Malmquist index. No base period is required to calculate the adjacent Malmquist index, but it fails to comply with the circularity property. The base period Malmquist index uses the technology of a base period and is circular, but the base period choice is arbitrary. There is, therefore, a trade-off between the choice of one or another version of the Malmquist index. The aim of this paper is to introduce a new total factor productivity index that is simultaneously circular and does not need to resort to a base period or *ad hoc* reference. To this end, we propose a new multi-input multi-output reference production technology for use as a standard for measuring and decomposing total factor productivity changes. The standard production technology is conceptually attractive because its parameterization is versatile and adaptable to the evolution of a set of firms performing any multi-input multi-output production process. Additionally, the new approach can bring about a true total factor productivity index, which can be decomposed into an output change and an input change.

1. Introduction

In the early 1980s, Caves et al. (1982) introduced the Malmquist productivity index (MPI), also called the adjacent Malmquist index (Adj-MI), as an adaptation to production theory of the index originally defined by Malmquist (1953) for consumer theory. As opposed to other classical alternatives in economics like the Törnqvist, Laspeyres, Paasche and Fisher indices, which all rely on price information, the MPI is a non-price dependent approach for determining total factor productivity changes (TFPC) over time. Later, Färe et al. (1992, 1994a) showed how to implement the Adj-MI under data envelopment analysis (DEA). DEA is a non-parametric technique strongly based on mathematical linear programming for estimating production frontiers and technical efficiency. Moreover, Färe et al. (1992) suggested an initial decomposition of the Adj-MI into a catching-up effect (efficiency change) and a frontier-shift effect (technical change) in order to derive direct drivers of productivity change. Later, other alternative decompositions were developed (see, for example, Färe et al, 1994b, Ray and Desli, 1997, Lovell, 2003 and Zofío, 2007).

Under the axiomatic test approach to index number theory, the definition of an appropriate index for capturing productivity change requires the establishment of a set of properties (also called tests) that the index formula should satisfy. In the 1920s, Fisher (1922) proposed a sizeable number of such, economically and mathematically intuitive, tests, which any approach had to pass to be considered a suitable index number. Later, Eichhorn and Voeller (1976) provided a summary of the main tests published in the literature. More up-to-date summaries are Balk (1995), Althin (2001), Balk (2008, Chapter 3) and Diewert and Fox (2017). Within this axiomatic test approach, the circular test (circularity) is a desirable property for any productivity change index. Simply speaking, a productivity change index (I) computed at three points in time, t, $t+1$ and $t+2$, passes the circular test when ${}^{t}I^{t+1} \cdot {}^{t+1}I^{t+2} = {}^{t}I^{t+2}$.

The theoretical formulation of the Adj-MI by Färe et al. (1992) only considers two periods of time. In real applications, where the available panel data covers more than two periods, the Adj-MI is repeatedly determined for each pair of consecutive periods of time, as in Färe et al. (1992). In this case, the Adj-MI is calculated as the geometric average of two terms (each using a different period as a reference technology) and does not satisfy the circularity test. Therefore, Berg et al. (1992) were the first to introduce in their celebrated paper a version of the MPI that

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satisfied circularity: the base period Malmquist productivity index (BP-MI). In their definition, the index is not an average of two terms. Instead, it is the ratio of a pair of distance functions calculated from a common reference technology across all the considered years. In the case of the BP-MI, the reference technology is empirically determined.

Other researchers have also taken this avenue in order to determine productivity change in applications or propose new theoretical solutions in the context of productivity measurement under the satisfaction of the circularity property. Regarding other approaches for achieving circularity, worthy of note are, for example, Elteto and Koves (1964) and Szulc (1964), who proposed the EKS method, which is the geometric mean of the ratios of all bilateral indexes, where each entity is taken in turn as the base technology. Pastor and Lovell (2005), who introduced a circular global Malmquist index based on an empirically determined reference technology for all the considered periods. In this case, the empirically determined reference technology does not correspond to the first period of the series, but, instead, matches the convex hull of all the estimated technologies in the periods under evaluation. Additionally, Pastor et al. (2011) defined the biennial Malmquist productivity index, as a refinement of the previous global Malmquist index and Afsharian and Ahn (2015) proposed an alternative Malmquist-type index, the overall Malmquist index, defined as the smallest aggregate technology set. Other recent contributions are Aparicio and Santín (2018), who adapted the so-called Camanho-Dyson index (Camanho & Dyson, 2006) in order to compare the performance of groups of decision-making units (DMUs) over time subject to the circularity property, and Camanho et al. (2021) and Walheer (2022), who defined the global counterpart of the MPI for group contexts. All in all, the Adj-MI and the BP-MI have attracted the interest of many scholars in economics for empirical applications over the last decades[1].

Since its introduction (Caves et al., 1982), the Adj-MI has been extensively applied in empirical research and is often referred to as a total factor productivity index (TFPI). However, from a theoretical standpoint, it does not fulfill the criteria of a true TFPI—namely, being expressible as the ratio of an output quantity index to an input quantity index. O'Donnell (2012) (see also Ang and Kerstens, 2017) rigorously demonstrated that the standard formulation of the Adj-MI lacks multiplicative completeness, a feature that is critical for the clear interpretation of productivity. This limitation has spurred criticism and alternative proposals in the literature (for example, the so-called Hicks–Moorsteen productivity index introduced by Bjurek 1996). Nevertheless, the Adj-MI continues to enjoy widespread application because it operates independently of price data, offers a straightforward decomposition into efficiency and technical change, and is easily computed using Data Envelopment Analysis. These practical advantages have entrenched the index in many areas of applied economics, even though its theoretical underpinnings continue to be debated.

In this paper, we suggest a new approach to overcome the two weaknesses of the Malmquist productivity index pointed out above: i) the trade-off between circularity or independency of the considered reference time period and ii) the issue of whether or not the Malmquist index is a TFPI. To simultaneously solve the above problems, we propose the use of a reference production technology or simply a standard to be systematically used in production economics for enhancing the measurement of productivity changes.

Although several innovative approaches have been developed to address the limitations of the adjacent and base-period Malmquist indices—such as the global Malmquist index (Pastor & Lovell, 2005), the biennial Malmquist index (Pastor et al., 2011), the overall Malmquist index (Afsharian & Ahn, 2015), and group-oriented circular indices (Camanho et al., 2021)—none of these methods simultaneously satisfy both the circularity and the base-period-independence properties while also providing a multiplicatively complete TFPI to carry out robust multilateral comparisons of DMUs over time. These methods often rely on empirical constructs (e.g., convex hulls or chained bilateral comparisons) that vary with the dataset and hinder cross-study comparability. Moreover, their formulations do not generally admit a decomposition into separate output and input quantity changes. In contrast, our proposed approach introduces a parametrically defined standard reference technology that eliminates the need for *ad hoc* base periods, ensures circularity, can be decomposed into efficiency change, global technical change and local technical change and allows the index to be expressed as the ratio of output to input quantity changes, making it a true TFPI. This dual solution to the two main limitations of existing Malmquist-type indices underlines the relevance and necessity of our contribution.

The paper is organized as follows. Section 2 briefly introduces the background about the two versions of the Malmquist index and the recent decomposition of the technical change introduced by Aparicio and Santín (2024). In Section 3, we define the new TFPI based on standard reference technology, together with its decomposition, and the steps for applying this methodology in practice. Section 3 also provides a numerical example to illustrate the theoretical ideas. In Section 4, we exemplify the new decomposition by applying the approach to the group of 42 Swedish pharmacies operating from 1980 to 1989 previously used in Färe et al. (1992), Balk and Althin (1996) and then in Althin (2001) for the purposes of comparison with the Adj-MI and the BP-MI. Finally, Section 5 outlines the conclusions and points out some future research lines.

2. Background

In this section, we briefly introduce the notation for the Adj-MI and the BP-MI. Additionally, we introduce the formal definition of a TFPI and briefly discuss the locality of the technical change component of the Adj-MI and its decomposition in global technical change and local technical change, introduced by Aparicio and Santín (2024). Although other techniques could be used to estimate the corresponding technologies and technical efficiency, our calculations are based on data envelopment analysis (DEA) (see Charnes et al., 1978 and Banker et al., 1984) because DEA has the flexibility to handle multi-input multi-output production processes.

2.1. The adjacent Malmquist index and the base-period Malmquist index

Let us consider a panel of $j = 1, ..., J$ decision-making units (DMUs) and, at least, two different time periods, t and $t + 1$. In this context, x_{ji}^{τ} denotes the quantity of the i-th input, $i = 1, ..., m$, consumed by the j-th DMU, $j = 1,...,J$, in the period τ, $\tau = t, t + 1$. And y_{jr}^{τ} denotes the quantity of the r-th output, $r = 1,...,n$, produced by the j-th DMU, $j = 1,...,J$, in the period τ, $\tau = t, t + 1$. In vectorial notation, we use $\left(x_j^{\tau}, y_j^{\tau}\right)$ to denote the input-output bundle corresponding to the j-th DMU, $j = 1, ..., J$. Additionally, we need to define two technologies to establish the relationship between inputs $x = (x_1, ..., x_m) \in R_+^m$ and outputs $y = \left(y_1, ..., y_n\right) \in R_+^n$. Within this framework, the *contemporaneous* benchmark technology is defined as $T^{\tau} = \{(x^{\tau}, y^{\tau}) : x^{\tau} \text{can produce} y^{\tau}\}$, $\tau = t, t + 1$.

As previously mentioned, there are very different approaches to estimate a technology from a sample of observed bundles of inputs and outputs. One, classified as a non-parametric methodology, is DEA. Within this approach, T^{τ} is estimated assuming Constant Returns to Scale (CRS) as follows (see Charnes et al., 1978):

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[1] Note that, based on a Google Scholar web search conducted on May 2025, we found that, measured by the number of citations, the Adj-MI is more successful than the BP-MI. Caves et al. (1982) accumulates 6,510 citations for the Adj-MI, whereas its famous decomposition proposed by Färe et al. (1994b) has 7,808 citations. On the other hand, Berg et al. (1992) receives a "mere" 909 citations.

$$T^{\tau} = \left\{ \begin{array}{l} (x^{\tau}, y^{\tau}) \in R_{+}^{m+n} : y_r^{\tau} \leq \sum_{j=1}^{J} \lambda_j y_{jr}^{\tau}, \forall r = 1, \ldots, n, \\ x_i^{\tau} \geq \sum_{j=1}^{J} \lambda_j x_{ji}^{\tau}, \forall i = 1, \ldots, m, \lambda_j \geq 0, \forall j = 1, \ldots, J \end{array} \right\} \tag{1}$$

Given the input-output bundle (x_o^l, y_o^l) observed in period l, l taking the values t or $t+1$, the Shephard output distance function (Shephard, 1970) with respect to the technology T^{τ}, $\tau = t, t+1$, is generically defined as:

$$D^{\tau}(x_o^l, y_o^l) = \inf\{\gamma > 0 : (x_o^l, y_o^l / \gamma) \in T^{\tau}\}. \tag{2}$$

An equivalent representation of the technology, which is useful when an output-oriented distance function is utilized, is associated with the partially oriented notion of output production possibility set $P^{\tau}(x^{\tau}) = \{y^{\tau} \in R_{+}^{n} : (x^{\tau}, y^{\tau}) \in T^{\tau}\}$. In this way, the Shephard output distance function in (2) can be equivalently rewritten as:

$$D^{\tau}(x_o^l, y_o^l) = \inf\{\gamma > 0 : (y_o^l / \gamma) \in P^{\tau}(x_o^l)\}. \tag{3}$$

Under DEA, the value of the Shephard output distance function $D^{\tau}(x_o^l, y_o^l)$ is calculated as follows:

$$\begin{array}{lll} [D^{\tau}(x_o^l, y_o^l)]^{-1} = & \max & \phi \\ & s.t. & \\ & \sum_{j=1}^{J} \lambda_j x_{ji}^{\tau} \leq x_{oi}^{l}, & i = 1, \ldots, m \\ & \sum_{j=1}^{J} \lambda_j y_{jr}^{\tau} \geq \phi y_{or}^{l}, & r = 1, \ldots, n \\ & \lambda_j \geq 0, & j = 1, \ldots, J \end{array} . \tag{4}$$

The (simple) Adj-MI grounded on the technology of period τ is defined as follows.

$$M^{\tau}(x_o^t, y_o^t, x_o^{t+1}, y_o^{t+1}) = \frac{D^{\tau}(x_o^{t+1}, y_o^{t+1})}{D^{\tau}(x_o^t, y_o^t)}, \quad \tau = t, t+1. \tag{5}$$

Given that the choice of either of the above two indices, expression (5) for $\tau = t, t+1$, is arbitrary with respect to the measurement of productivity change over time, and possibly leading to different results Caves et al. (1982) suggested taking the geometric mean of both expressions, that is,

$$M(x_o^t, y_o^t, x_o^{t+1}, y_o^{t+1}) = \sqrt{M^t(x_o^t, y_o^t, x_o^{t+1}, y_o^{t+1}) \cdot M^{t+1}(x_o^t, y_o^t, x_o^{t+1}, y_o^{t+1})}. \tag{6}$$

Eq. (7) is also known as the Färe, Grosskopf, Lindgren and Roos (1989) adjacent Malmquist output-based productivity index. This index was originally decomposed into efficiency change and technical change as follows (Färe et al., 1992):

$$M(x_o^t, y_o^t, x_o^{t+1}, y_o^{t+1}) = EC_o^{t,t+1} \cdot TC_o^{t,t+1}, \tag{7}$$

where $EC_o^{t,t+1} = \frac{D^{t+1}(x_o^{t+1}, y_o^{t+1})}{D^t(x_o^t, y_o^t)}$ and $TC_o^{t,t+1} = \left[\frac{D^t(x_o^t, y_o^t)}{D^{t+1}(x_o^t, y_o^t)} \cdot \frac{D^t(x_o^{t+1}, y_o^{t+1})}{D^{t+1}(x_o^{t+1}, y_o^{t+1})} \right]^{1/2}$.

However, the above index does not satisfy circularity (see, for example, Pastor and Lovell, 2007). A desirable property of any productivity change index covering a long period of time is that it is possible to chain it. This is known as circularity in index number theory. Chaining is out of the question if the reference technology changes over time, which is the case with the Adj-MI. Fortunately, Berg et al. (1992) proposed an index that compares adjacent period data using a technology from a unique base period. This BP-MI satisfies circularity, generates a single measure of productivity change, and can also be decomposed into efficiency change and technical change, as demonstrated by Berg et al. In this regard, the 'chain' version of the Malmquist productivity index, known as the BP-MI, is defined as follows (Berg et al., 1992):

$$M^b(x_o^t, y_o^t, x_o^{t+1}, y_o^{t+1}) = \frac{D^b(x_o^{t+1}, y_o^{t+1})}{D^b(x_o^t, y_o^t)}, \tag{8}$$

where b denotes the base time period (usually the first period in a series). Additionally, (8) can be decomposed into a term interpreted as efficiency change and a component interpreted as technical change:

$$M^b(x_o^t, y_o^t, x_o^{t+1}, y_o^{t+1}) = EC_o^{t,t+1} \cdot TC(b)_o^{t,t+1}, \tag{9}$$

where $EC_o^{t,t+1} = \frac{D^{t+1}(x_o^{t+1}, y_o^{t+1})}{D^t(x_o^t, y_o^t)}$ and $TC(b)_o^{t,t+1} = \frac{D^t(x_o^t, y_o^t)/D^b(x_o^t, y_o^t)}{D^{t+1}(x_o^{t+1}, y_o^{t+1})/D^b(x_o^{t+1}, y_o^{t+1})}$.

In the decomposition, the expression of the efficiency change component is the same as the expression of the same term in (7), whereas the frontier shift over time is measured in (9) by the distance between technology t and $t+1$, albeit as a relative distance to the base reference technology T^b. However, and in contrast to the Adj-MI, the values of Berg et al.'s index (1992) are not independent of the selected base period.

2.2. Global and local technical changes

Regarding the two decompositions of the Malmquist indices in both (7) and (9), there is a coincident component that measures efficiency change and another term that captures the frontier shift. As these expressions are defined in (7) and (9), technical change is determined locally, exclusively assessing the input-output bundles of the evaluated company. Therefore, under the Adj-MI, the geometric average of individual technical changes is calculated as a way of capturing the overall pattern for the whole industry in a single value.

In contrast, as Aparicio and Santín (2024) have recently shown, technical change is expected to be a global phenomenon, not simply the average of local measures or technical change (see also Balk and Althin, 1996, Asmild and Tam, 2007, Otsuki, 2013). When assessing productivity change over time, there is often a misinterpretation of technical change, calculated as the geometric average of technological changes between two periods using firm-specific data. Yet, the shift in the frontier over time reflects a global phenomenon tied to relative technological progress or regress across all frontiers.

Aparicio and Santín (2024) addressed this gap by introducing a new decomposition of the adjacent Malmquist index's technical change into two components, the global technical change (GTC), which represents a Hick's neutral technical change that is shared by all production units, and the local technical change (LTC), which captures how each firm experiences the GTC. To carry out this decomposition these authors resort to the use of a unit-hypercube of dimension H; $H = m+n$. In mathematics, a hypercube is an extension into a H-dimensional space of a square for two dimensions, a cube for three dimensions, or a tesseract for four dimensions. This geometric shape is always compact and convex. It is a regular polytope with sides perpendicular to each other, all sides equal in length, and all angles measuring right angles. Each vertex of a hypercube is equidistant from every other adjacent vertex. The number of vertices in a hypercube is 2^H. Specifically, we will utilize the concept of a unit hypercube, which is a hypercube where the line segments between connected vertices measure one unit in length.

For embedding all DMUs information about inputs and outputs within a unit-hypercube, Aparicio and Santín (2024) exploit the units-invariance property inherent in radial DEA models (see Lovell and Pastor, 1995). As inputs and outputs are frequently measured in different units and orders of magnitude, the inputs and outputs of all DMUs are normalized by the maximum observed value of each dimension. After normalization, the transformed inputs and outputs naturally belong to the range of zero to one, and the efficiency scores calculated using DEA for the original and the transformed data coincide. The next step is to generate a set of K uniformly distributed random values[2] for each dimension within the range of zero to one. These generated values from different variables are combined eventually to artificially produce a uniformly distributed set of K virtual DMUs within the unit-hypercube of dimension H, denoted as $\aleph = \left\{\left(x_k^H, y_k^H\right)\right\}_{k=1}^{K^{m+n}}$, where $\aleph$ denotes the set of inputs and outputs used by the K synthetic DMUs while $\left(x_k^H, y_k^H\right)$ corresponds to the input-output bundle of the k-th DMU, $k = 1, \dots, K$.

Once the K synthetic DMUs have been created, it is possible to measure their distances with respect to the two frontiers in t and $t+1$. The geometric average of the technical changes for the K virtual DMUs is the measure of the GTC, while the LTC reflects how each real DMU experiences the GTC individually. Therefore, the TC in (8) can be further decomposed as follows:

$$M\left(x_o^t, y_o^t, x_o^{t+1}, y_o^{t+1}\right) = EC_o^{t,t+1} \cdot TC_o^{t,t+1} = EC_o^{t,t+1} \cdot GTC_H^{t,t+1} \cdot LTC_o^{t,t+1}, \tag{10}$$

where

$$GTC_H^{t,t+1} = \left(\prod_{k=1}^{K^{m+n}} \frac{D^t\left(x_k^H, y_k^H\right)}{D^{t+1}\left(x_k^H, y_k^H\right)}\right)^{\frac{1}{K^{m+n}}} \quad \text{and} \quad LTC_o^{t,t+1} = \frac{TC_o^{t,t+1}}{GTC_H^{t,t+1}}.$$

In (10), $M\left(x_o^t, y_o^t, x_o^{t+1}, y_o^{t+1}\right)$, $EC_o^{t,t+1}$ and $TC_o^{t,t+1}$ are calculated as in (7). However, according to Aparicio and Santín (2024) the traditional technical change $TC_o^{t,t+1}$is decomposed into two components: a global technical change $GTC_H^{t,t+1}$, where H denotes the dimension of the unit-hypercube used for its calculus, i.e. the sum of the number of inputs and the number of outputs, and a local technical change $LTC_o^{t,t+1}$. The component $GTC_H^{t,t+1}$captures the overall frontier shift between the two periods, avoiding any potential bias that may arise when DMUs are not uniformly distributed across the two production frontiers. The local technical change $LTC_o^{t,t+1}$in (10) measures the relative position of each real DMU at t and $t + 1$ in relation to the synthetic average$GTC_H^{t,t+1}$.

3. The standard total factor productivity index

Over the last few decades, the MPI has been under the spotlight as a measure of productivity change over time for several reasons. One is related to the fact that it does not need information on market prices, which makes this index suitable for benchmarking public services produced by public production units. For this reason, the MPI has been widely applied in sectors like education (De Witte & López-Torres, 2017; Arbona et al. 2022), health (Hollingsworth, 2008; Kohl et al. 2019), the judicial system (Falavigna et al. 2018; Pereira et al. 2025) or long-term care (Salinas-Jiménez et al. 2003) among others. Although much has already been written about this index, no approach has, to the best of our knowledge, so far succeeded in simultaneously satisfying the following properties: circularity, independency of a considered reference period for index calculation and the determination of a TFPI. Moreover, the decomposition of this new index incorporates the possibility of considering the technical change term as the mixture of two components, a global and a local frontier shift. In this section, we introduce a solution aimed at filling this gap of the literature.

3.1. Standard reference technology

Our approach is fundamentally based upon the definition of a standard reference technology (from now on 'the standard'), that is, a common reference or base technology for use as cornerstone in any empirical problem to measure a TFPI and its change over time. We are aware that many standards could be defined as possible alternatives. However, once a standard has been accepted by most practitioners in a field, its associated advantages far outweigh any possible disadvantages. In our productivity context, the main benefit of using standard reference technology will be an increase in the comparability of the results achieved by different researchers, overcoming one of the main issues associated with non-parametric frontier approaches. Additionally, as far as the direct benefits of the new TFPI defined based on our proposal of a standard reference technology are concerned, the properties of circularity, independence of the considered reference period for index calculation and the determinateness test will be naturally satisfied.

Moreover, due to the features of the standard that we propose, the new approach is rewritten as a ratio of an output quantity change index to an input quantity change index. In this regard, the new index could be reinterpreted as a multiplicatively complete TFPI. This is something that the Adj-MI or the BP-MI cannot guarantee (O'Donnell, 2012). The Adj-MI and the BP-MI do not satisfy this property except in trivial or highly stylized settings. For example, multiplicative completeness holds when the technology is restricted to a single input and a single output, and the distance functions used in the index construction are computed under constant returns to scale (see Grifell-Tatjé and Lovell, 1995). However, these assumptions are not realistic in most empirical problems. Additionally, as a direct consequence of its application, the new approach is also able to be decomposed in the traditional efficiency change, an average global technical change and a local technical change.

Our approach is particularly grounded on the expression of the BP-MI, see (8), where the base technology b is a parametric synthetic reference technology set as standard rather than a particular set corresponding to the first, the last or any period or combination of time periods in the empirical series. In this respect, given the difficulty for dealing parametrically with multi-input multi-output production processes, we select the following simple formulation of a multi-output multi-input transformation function, introduced by Färe and Primont (1995, p. 25 and p. 155), which is based on the notion of distance function:

$$D_{FP}(x_o, y_o) = \frac{f(y)}{g(x)} = \beta_0 \left(\sum_{r=1}^{n} \beta_r y_{or}^{\delta}\right)^{\frac{1}{\delta}} \Bigg/ \alpha_0 \left(\sum_{i=1}^{m} \alpha_i x_{oi}^{\rho}\right)^{\frac{1}{\rho}}. \tag{11}$$

Note that according to O'Donnell (2012) the definition of (11) is a TFPI understood as the ratio of an output quantity index $f(y)$ to an input quantity index $g(x)$. Indeed, the numerator in (11) is an output aggregator function defined as a constant elasticity of transformation function (CET) (Kumbhakar, 1996; Grosskopf et al. 1997; Fernández et al. 2000). Likewise, the denominator in (11) is an input aggregator function expressed as a constant elasticity of substitution (CES) function (Uzawa, 1962; Färe et al., 1994c, p. 53; Coelli et al. 2005, p. 211).

From (11), we can naturally define the reference technology $S := \left\{(x, y) \in R_+^{m+n} : D_{FP}(x, y) \le 1\right\}$. Next, we prove that if this technology is defined through the output production possibility sets $P_S(x) = \left\{y \in R_+^n : (x,y) \in S\right\} = \left\{y \in R_+^n : D_{FP}(x,y) \le 1\right\}, \forall x \in R_+^m$, then $D_{FP}\left(x_o, y_o\right)$ matches the definition of the Shephard output distance function in (3).

Proposition 1. Let $P_S(x) = \left\{y \in R_+^n : D_{FP}(x, y) \le 1\right\}, \forall x \in R_+^m$. If $\beta_0\left(\sum_{r=1}^n \beta_r y_{or}^{\delta}\right)^{\frac{1}{\delta}} / \alpha_0\left(\sum_{i=1}^m \alpha_i x_{oi}^{\rho}\right)^{\frac{1}{\rho}} > 0$, then $D_{FP}\left(x_o, y_o\right) = \inf\left\{\gamma > 0 : \left(y_o / \gamma\right) \in P_S(x)\right\}$.

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[2] The GTC is calculated using a numerical method that consists of projecting K synthetic DMUs to the two frontiers whose average distance is to be evaluated. Therefore, theoretically the greater the number of artificial DMUs generated, the better our estimate of the GTC for the sector between periods should be. However, since we will use a geometric average to aggregate all these calculations, there should be a point at which it is no longer worthwhile to increase the size of this artificial data set. Aparicio and Santín (2024) use $K = 1,000,000$ in their empirical application. Their choice is based on the demonstration that for K values between 10,000 and 10,000,000, the geometric mean of the GTC estimates changes only in the fourth decimal place.

Proof. Based on the expression of the Shephard output distance function (3), we can take the following steps:

$$\inf\{\gamma > 0 : (y_o/\gamma) \in P_S(x_o)\} = \inf\{\gamma > 0 : D_{FP}(x_o, y_o/\gamma) \le 1\} =$$

$$\inf\left\{\gamma > 0 : \beta_0\left(\sum_{r=1}^{n}\beta_r\left(\frac{y_{or}}{\gamma}\right)^{\delta}\right)^{\frac{1}{\delta}} \Big/ \alpha_0\left(\sum_{i=1}^{m}\alpha_i x_{oi}^{\rho}\right)^{\frac{1}{\rho}} \le 1\right\} =$$

$$\inf\left\{\gamma > 0 : \gamma \ge \beta_0\left(\sum_{r=1}^{n}\beta_r y_{or}^{\delta}\right)^{\frac{1}{\delta}} \Big/ \alpha_0\left(\sum_{i=1}^{m}\alpha_i x_{oi}^{\rho}\right)^{\frac{1}{\rho}}\right\} =$$

$$\beta_0\left(\sum_{r=1}^{n}\beta_r y_{or}^{\delta}\right)^{\frac{1}{\delta}} \Big/ \alpha_0\left(\sum_{i=1}^{m}\alpha_i x_{oi}^{\rho}\right)^{\frac{1}{\rho}} = D_{FP}(x_o, y_o).$$

■

Next, we prove that the output production possibility sets $P_S(x)$ satisfy the set of usual axioms in microeconomics under a specification of the parameters in (11) (for more details, see Färe and Primont, 1995, p. 27).

Proposition 2. Let $x \in R_+^m$ and $\rho > 0$. Let $\beta_0 > 0$, $\beta_r \ge 0$, $r = 1,\ldots,n$, $\alpha_0 > 0$, $\alpha_i \ge 0$, $i = 1, \ldots, m$, and $\delta \ge 1$. Then, $P_S(x)$ fulfills the following axioms:

(A1) $0_s \in P_S(x)$.
(A2) If $y \in P_S(x)$ and $0 < \theta \le 1$, then $\theta y \in P_S(x)$ (weak disposability of outputs).
(A3) If $y \in P_S(x)$ and $y' \le y$, then $y' \in P_S(x)$ (strong disposability of outputs).
(A4) $P_S(x)$ is a bounded set (scarcity).
(A5) $P_S(x)$ is a closed set.
(A6) $P_S(x)$ is a convex set.

Proof. We will use that

$$P_S(x) = \{y \in R_+^n : D_{FP}(x,y) \le 1\} = \left\{y \in R_+^n : \beta_0\left(\sum_{r=1}^{n}\beta_r y_r^{\delta}\right)^{\frac{1}{\delta}} \le \alpha_0\left(\sum_{i=1}^{m}\alpha_i x_i^{\rho}\right)^{\frac{1}{\rho}}\right\} = \left\{y \in R_+^n : h(y) \le \alpha_0\left(\sum_{i=1}^{m}\alpha_i x_{oi}^{\rho}\right)^{\frac{1}{\rho}}\right\},$$

where $h(y) := \beta_0\left(\sum_{r=1}^{n}\beta_r y_r^{\delta}\right)^{\frac{1}{\delta}}$. Then, (A1) is true because $h(0_n) = 0$ and $\alpha_0\left(\sum_{i=1}^{m}\alpha_i x_{oi}^{\rho}\right)^{\frac{1}{\rho}} \ge 0$.

(A2) $h(\theta y) = \beta_0\left(\sum_{r=1}^{n}\beta_r(\theta y_r)^{\delta}\right)^{\frac{1}{\delta}} = \theta\beta_0\left(\sum_{r=1}^{n}\beta_r y_r^{\delta}\right)^{\frac{1}{\delta}} = \theta h(y) \le h(y) \le \alpha_0\left(\sum_{i=1}^{m}\alpha_i x_{oi}^{\rho}\right)^{\frac{1}{\rho}}$, which implies that $\theta y \in P_S(x)$.
(A3) is true because $h(y') \le h(y)$.
(A4) holds because $0 \le y_r \le \frac{\alpha_0\left(\sum_{i=1}^{m}\alpha_i x_{oi}^{\rho}\right)^{\frac{1}{\rho}}}{\beta_0\beta_r^{\frac{1}{\delta}}}$, for all $r = 1,\ldots,s$.
(A5) is satisfied because $h(y)$ is a continuous function.

Finally, to prove (A6), notice that *h(y)* is a quasi-convex function. This last claim is true because $\beta_0 > 0$ and $\left(\sum_{r=1}^{n}\beta_r y_r^{\delta}\right)^{\frac{1}{\delta}} = f[g(y)]$, with $g(y) = \sum_{r=1}^{n}\beta_r y_r^{\delta}$ and $f(z) = z^{\frac{1}{\delta}}$. *f(z)* is a monotone increasing function and *g(y)* is a convex function because it is the sum of convex functions, since $\beta_r \ge 0$, $r = 1,\ldots,s$, and $\delta \ge 1$. Then, we can apply Theorem 13.8(b) in Madden (1986) (because a convex function is also a quasi-convex function) and we get that $f[g(y)]$ is a quasi-convex function. Therefore, *h(y)* is also a quasi-convex function. As a consequence, any lower contour set is convex. In particular, $P_S(x)$ is a lower contour set. ■

Furthermore, S is a conical technology and, therefore, it exhibits (global) constant returns to scale. To demonstrate this point, notice that, by (11), $D_{FP}(\sigma x_o, \sigma y_o) = D_{FP}(x_o, y_o)$ for any $\sigma > 0$ and, consequently, if $(x_o, y_o) \in S$, then $\sigma(x_o, y_o) \in S$ due to the definition of S. In this way, we have that $S = \sigma S$ for all $\sigma > 0$, which is the definition of global CRS (see Färe and Primont, 1995, p. 23).

Additionally, (11) can be plugged into the Malmquist expression in (5) to determine productivity change.

By plugging (11) into (5), we get the definition of the so-called standard total factor productivity change index (STFPCI) between periods *t* and *t+1*, denoted by $S(x_o^t, y_o^t, x_o^{t+1}, y_o^{t+1})$, as follows:

$$S(x_o^t, y_o^t, x_o^{t+1}, y_o^{t+1}) = \frac{D_{FP}(x_o^{t+1}, y_o^{t+1})}{D_{FP}(x_o^t, y_o^t)}, \tag{12}$$

which, by applying (11), can be explicitly expressed as follows:

$$S(x_o^t, y_o^t, x_o^{t+1}, y_o^{t+1}) = \frac{\beta_0\left(\sum_{r=1}^{n}\beta_r\left(y_{or}^{t+1}\right)^{\delta}\right)^{\frac{1}{\delta}} \Big/ \alpha_0\left(\sum_{i=1}^{m}\alpha_i\left(x_{oi}^{t+1}\right)^{\rho}\right)^{\frac{1}{\rho}}}{\beta_0\left(\sum_{r=1}^{n}\beta_r\left(y_{or}^{t}\right)^{\delta}\right)^{\frac{1}{\delta}} \Big/ \alpha_0\left(\sum_{i=1}^{m}\alpha_i\left(x_{oi}^{t}\right)^{\rho}\right)^{\frac{1}{\rho}}} = . \tag{13}$$

$$= \frac{\left[\left(\sum_{r=1}^{n}\beta_r\left(y_{or}^{t+1}\right)^{\delta}\right) \Big/ \left(\sum_{r=1}^{n}\beta_r\left(y_{or}^{t}\right)^{\delta}\right)\right]^{\frac{1}{\delta}}}{\left[\left(\sum_{i=1}^{m}\alpha_i\left(x_{oi}^{t+1}\right)^{\rho}\right) \Big/ \left(\sum_{i=1}^{m}\alpha_i\left(x_{oi}^{t}\right)^{\rho}\right)\right]^{\frac{1}{\rho}}}$$

A TFPC index is formally defined in the literature as the ratio of an output quantity change index to an input quantity change index. According to this definition, the Adj-MI and the BP-MI are not TFPC indices (an exception is the Hicks–Moorsteen productivity index, see Bjurek, 1996), implying that they cannot always be interpreted as measures of actual productivity change over time (O'Donnell, 2012). However, the new STFPI can be easily expressed as a ratio of an output quantity change index to an input quantity change index, which means that it can be really understood as an actual TFPC index.

At this point, the new index could be naturally decomposed through a direct application of (9) in efficiency change and technical change, where the reference technology associated with the base period *b* is substituted by the standard reference technology *S*. But in addition, following Aparicio and Santín (2024) we also decompose the technical change in (9), now $TC(S)_o^{t,t+1}$, in two parts. First, a $GTC(S)_H^{t,t+1}$ that represents the frontier shift over time as a global phenomenon affecting the studied industry rather than an average of how the observed DMUs locally experience technical change. Second, our decomposition also identifies a $LTC(S)_o^{t,t+1}$ devoted to capturing how a company locally experiences technological change over time. In the three terms the *S* in parentheses denotes the standard reference technology.

Let us assume that $\aleph = \left\{\left(x_k^H, y_k^H\right)\right\}_{k=1}^{K^{m+n}}$ is the set of *K* artificial production units generated at random over the $H = m+n$ dimensions of the unit-hypercube. Our proposal for decomposing the STFPI is as follows:

$$S(x_o^t, y_o^t, x_o^{t+1}, y_o^{t+1}) = EC_o^{t,t+1}\cdot TC(S)_o^{t,t+1} = EC_o^{t,t+1}\cdot GTC(S)_H^{t,t+1}\cdot LTC(S)_o^{t,t+1}, \tag{14}$$

where

$$EC_o^{t,t+1} = \frac{D^{t+1}(x_o^{t+1}, y_o^{t+1})}{D^t(x_o^t, y_o^t)},$$

$$TC(S)_o^{t,t+1} = \left[\left(\frac{D_{FP}(x_o^{t+1}, y_o^{t+1})}{D^{t+1}(x_o^{t+1}, y_o^{t+1})}\right)\cdot\left(\frac{D^t(x_o^t, y_o^t)}{D_{FP}(x_o^t, y_o^t)}\right)\right]$$

Now, the $TC(S)_o^{t,t+1}$ in (14) can be further decomposed in a $GTC(S)_H^{t,t+1}$ and a $LTC(S)_o^{t,t+1}$ as follows:

$$GTC(S)_H^{t,t+1} = \left(\prod_{k=1}^{K^{m+n}} \frac{D^t\left(x_k^H, y_k^H\right)}{D^{t+1}\left(x_k^H, y_k^H\right)} \right)^{\frac{1}{K^{m+n}}},$$

$$LTC(S)_o^{t,t+1} = \left[\left(\frac{D_{FP}\left(x_o^{t+1}, y_o^{t+1}\right)}{D^{t+1}\left(x_o^{t+1}, y_o^{t+1}\right)} \right) \cdot \left(\frac{D^t\left(x_o^t, y_o^t\right)}{D_{FP}\left(x_o^t, y_o^t\right)} \right) \cdot \left(\prod_{k=1}^{K^{m+n}} \frac{D^{t+1}\left(x_k^H, y_k^H\right)}{D^t\left(x_k^H, y_k^H\right)} \right)^{\frac{1}{K^{m+n}}} \right].$$

In (14), we identify the term $EC_o^{t,t+1}$ as the typical component for measuring efficiency change between periods t and $t+1$ that also appears in (7), (9), and (10). Following Aparicio and Santin (2024), the traditional technical change $TC(b)_o^{t,t+1}$ in (9) referred to the standard reference technology is now $TC(S)_o^{t,t+1}$ a term that can be decomposed into two components: a global technical change $GTC(S)_H^{t,t+1}$ and a local technical change$LTC(S)_o^{t,t+1}$. The component $GTC(S)_H^{t,t+1}$ captures the frontier shift of the corresponding sector over time. To this end, $GTC(S)_H^{t,t+1}$ averages the global frontier shift between t and $t+1$ evaluated for every synthetic point $\left(x_k^H,\ y_k^H\right)$, $k=1,\ldots,K$ within the unit-hypercube of dimension H. In (14), the local technical change $LTC(S)_o^{t,t+1}$ is the measurement of the relative position of each DMU at $t+1$ with respect to the synthetic average global technology change. Note, in contrast, that the traditional Adj-MI and BP-MI exclusively calculate local technical change components.

Regarding the axiomatic properties of the STFPI, this new approach is assessed resorting to the tests shown in Frisch (1936) and more recently in Althin (2001). Most of the properties met by the STFPI, for example, circularity, are directly inherited from the BP-MI. Other properties, like independency of base period, are satisfied due to the use of a standard as a reference technology for all the calculations and any panel data. Next, we summarize all the tests that the new approach satisfies by stating Proposition 3.

Before presenting Proposition 3, we recall the main axiomatic properties commonly used to evaluate the theoretical soundness of productivity indices. The tests include:

Identity: The index should equal one when inputs and outputs remain unchanged across periods.

Time reversal: Interchange the role of period's t and $t+1$, then the new index should equal the reciprocal of the original index.

Circularity: Chaining indices over multiple periods should yield the same result as direct comparison between the initial and final periods.

Commensurability: The index should be invariant to the units of measurement of inputs and outputs.

Determinateness: If any output or input equals zero, the index should result in a positive real number. Following Frisch (1936), in this scenario, the index should take strictly positive values.

Inverse Proportionality of inputs given CRS: If period t+1 inputs are multiplied with a scalar α, then the new index should equal α^{-1} times the old index.

Proportionality of outputs: If period $t+1$ outputs are multiplied with a scalar β, then the new index should equal β times the old index.

Independence of base period: The result should not depend on the arbitrary choice of base year.

Proposition 3. The STFPI passes the tests of identity, time reversal, circularity, commensurability, determinateness, inverse proportionality of inputs, proportionality of outputs and independency of base period.

Proof. The STFPI satisfies the tests of identity, time reversal, circularity, and commensurability because it uses a reference technology as the base period. Regarding independency of base period, it is trivially satisfied due to the use of a standard technology. By expression (13), the STFPI satisfies both inverse proportionality of inputs and proportionality of outputs. Regarding determinateness, following Frisch (1936), if only one individual quantity becomes zero, then the STFPI is well-defined and, consequently, it does not become zero, infinite, or indeterminate. ■

Table 1 compares the traditional adjacent and base period versions of the Malmquist index with respect to the STFPI approach and the satisfaction of the usual tests[3]. In this regard, with respect to this list of properties, the results reported in the columns labelled Adj-MI and BP-MI in Table 1 are familiar. Regarding the results in Table 1, we find that the new index outperforms the most common MPI indices in the literature. The STFPI meets all the tests except proportionality of inputs (as applies to the classical approaches), although all three indices satisfy the test of inverse proportionality of inputs under CRS (because the output distance function is homogeneous of degree -1 in inputs if and only if the technology exhibits CRS – see Färe, 1988, p. 52). This makes economic sense since productivity is expected to be half if inputs are doubled. The proportionality of outputs is another property that all three indices satisfy without special conditions, thanks to the fact that the output distance function is homogeneous of degree +1 for outputs (see, for example, Färe and Primont, 1995, p. 17). Finally, on top of the satisfaction of all these desirable properties, we should add that the standard-based approach is also a multiplicatively complete TFPC index, to which neither the Adj-MI or BP-MI can lay claim.

3.2. Steps for using the standard reference technology in practice

Before indicating how to use the new index in practice, a previous indispensable step is to set values for the parameters that appear in (11) in order to define a specific standard technology. This technology will be used to compute all distance functions and productivity measures. For the sake of simplicity, our proposal is based on geometric grounds and sets the parameters that define the output and the input side, the numerator and the denominator in (11), as the (n-1) sphere centered at the origin of the coordinate or, in other words, the surface of the n-ball in the positive orthant centered at the origin of the coordinate for the output side and its inverse for the input side. All in all, this implies defining the following parameter values:

$$\begin{aligned} &\beta_r = 1,\ \forall r = 1, \ldots, n, \\ &\alpha_i = 1,\ \forall i = 1, \ldots, m, \\ &\quad \delta = 2,\ \rho = 2, \\ &\beta_0 = \frac{1}{\sqrt{n}}, \alpha_0 = \frac{1}{\sqrt{m}} \end{aligned} \tag{15}$$

The parameters proposed in the definition of the standard reference technology—namely, the weights and elasticity values in (15) for the output and input aggregator functions defined in (11)—are chosen to reflect a neutral and symmetric productivity benchmark. On the output side, the CET function implies a balanced trade-off among outputs, while on the input side, the CES function captures symmetric substitutability among inputs. Setting all weights equally and choosing unitary elasticity values yields a symmetric frontier where each variable contributes proportionally and no *a priori* preference is imposed on any input or output. This parameterization defines a geometrically[5] regular production set, which acts as an external standard rather than a fitted empirical frontier. While alternative configurations could be justified in context-

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[3] Under output-oriented distance functions, the indices pass the inverse proportionality test of inputs given CRS. In contrast, if we use input-orientation (as assumed in Althin, 2001), then the indices pass the proportionality test of outputs given CRS.

[4] Regarding the determinateness test in the case of our index, problems only do arise when all the components of any of the input or output vectors observed in periods t or $t+1$ for DMU$_0$ are zero. However, this case does not make economic sense in a real production process.

[5] Note that on the output side a (n-1)-sphere centered at the origin of the coordinate evokes the production possibilities frontier displayed in practically all efficiency measurement handbooks, whereas on the input side the inverse of the (m-1)-sphere evokes the shape of a classical isoquant curve that in this case is tangent to the axis.

Table 1
Index tests for different productivity indices[4].

Test	Adj-MI	BP-MI	STFPI
T1: Identity	Yes	Yes	Yes
T2: Time reversal	Yes	Yes	Yes
T3: Circularity	No	Yes	Yes
T4: Commensurability	Yes	Yes	Yes
T5: Determinateness	No	No	Yes
T6: Inverse Proportionality of inputs	Yes	Yes	Yes
T7: Proportionality of outputs	Yes	Yes	Yes
T8: Independent of base period	Yes	No	Yes

specific applications, our aim is to establish a universally applicable productivity comparator that ensures theoretical consistency, base-period independence, and multiplicative completeness. This standardization enables cross-study comparability of productivity change.

Additionally, in empirical problems, inputs and outputs are quantitative variables measured in different units and often expressed in very different orders of magnitude. This is not a problem in traditional applications of the BP-MI where the base technology is defined *ad hoc* using exactly the same variables and units of measurement as the inputs and outputs managed by the production units in the industry to be evaluated. However, under the new framework, the standard reference technology is parametrically defined in pure numbers with no units of measurement. As it makes no sense to add apples and oranges, our first task in order to solve the empirical problem on our hands is to normalize the initial empirical database with the twofold aim of erasing the units of measurement and preventing variables measured with higher values weight more in the productivity and the productivity change calculus in expressions (11) and (13).

To do this, we propose dividing all the output and input variables by the highest value observed for each variable in the panel data. By doing this for every variable, all values in all periods will be naturally bounded between zero and one, where the maximum observed values will be equal to one for all outputs and inputs in the transformed database. For instance, if the maximum value over all the periods in the database for the *r*-th output is 20 tons of apples, then we will divide all observed values of this *r* output by 20 tons of apples. In this way, the result would be a (pure) number between zero and one free of units of measurement. This normalization renders all inputs and outputs unit-free within the [0,1] range, removing any influence from their original measurement scale. Since the standard reference technology is defined independently of units, the STFPI remains dimensionless and comparable across applications. Rescaling any variable in the original data would not affect the normalized dataset or the resulting index. Moreover, the distance functions used in the decomposition are computed with radial DEA models, which are unit-invariant (Lovell & Pastor, 1995). Therefore, neither the index nor its decomposition is affected by differences in units or scaling.

Note that in general, $GTC_H^{t,t+1} \neq GTC(S)_H^{t,t+1}$ and $LTC_o^{t,t+1} \neq LTC(S)_o^{t,t+1}$. This is because in Aparicio and Santín (2024) $GTC_H^{t,t+1}$ and $LTC_o^{t,t+1}$ are calculated after normalizing the data every two consecutive periods while in this framework the normalization is done for all periods. Once the original database is transformed, we can directly plug the resulting data into (13) to output the STFPI[6].

3.3. A numerical example

To illustrate how the index works in practice, we resort to the numerical example introduced by Aparicio and Santín (2024). Let us assume an industry composed of 11 DMUs, that using a CRS technology produces two outputs (y_2 and y_1) using one input (x) in two time periods *t* and *t*+*1*. The data, together with the outputs and inputs normalized by the maximum observed outputs and input values in the two periods are listed in Table 2.

Fig. 1 represents the production frontiers for the two periods using the normalized data. The piece-wise linear form of the non-parametric frontier in a DEA model under CRS reveals that *A, B, C, D* and *E* are efficient in period *t*, while only *A, C, D* and *E* remain fully efficient in period *t*+1. Regarding Table 2 and Fig. 1, we also observe that DMUs *C* and *I* have the same input-output information in both periods. It is clear from Fig. 1 that the productivity of firm *A* is lower at *t*+*1* than in period *t*, although it still belongs to the production frontier. The opposite applies for firms *D* and *E*, which are efficient in both periods but have higher output values in period *t*+*1* with respect to *t*.

The next step is to use Eqs. (7), (9) and (14) to compute[7] and analyze the differences between the three productivity indices: the two traditional Malmquist indices plus the new standard total factor productivity index, alongside the components of these indices described above. Table 3 reports these results.

Looking at Table 3, note firstly that the efficiency change $EC_o^{t,t+1}$ is calculated in the same way and is the same for all three indices. Secondly, on average, all three indices and their technical changes are very similar. Paired correlation coefficients among the three indices exceed 0.96. Mean technical changes are below one in all three cases: 0.9756, 0.9657 and 0.9441 for the Adj-MI, the BP-MI and the STFPI, respectively, leading to the conclusion that, on average, the firms of this sector suffered a technical regress between the two periods. It is worth mentioning here that, in the case of the STFPI, the abovementioned average technical change $TC(S)_o^{t,t+1} = 0.9441$ is the component in (14) that evolves from the technical change of BP-MI in (9), where the standard technology substitutes the base period and here is showed for comparison purposes. However, the new STFPI allows a further valuable decomposition of the technical change $TC(S)_o^{t,t+1}$ between *t* and *t*+*1* into two components: an average global technical change $GTC(S)_H^{t,t+1}$ and a local technical change$LTC(S)_o^{t,t+1}$.

In this example, the $GTC(S)_H^{t,t+1} = 1.0254$ shows that the production frontier at *t*+1 has experienced, on average, a Hicks-neutral shift upwards of 2.54% with respect to the technology in *t*. The *GTC* is global in the sense that it accounts for the technical change that has occurred on average across the two production frontiers between the two periods and not only the geometric mean of those distances between *t* and *t*+1 calculated locally for the observed firms.

Furthermore, the local technical change $LTC(S)_o^{t,t+1}$ is the measurement of the relative position of each firm at *t*+1 with respect to the synthetic average technology change. This information is highly useful for avoiding misleading results, for example, both traditional indices might conclude by averaging the observed technical changes (0.9756 and 0.9657 for the adj-MI and the BPMI respectively) that the industry had a technical regress. The new decomposition may suggest that this industry made technical progress globally, although most firms, 9 out of 11, were producing closer to the region where the technology was experiencing technical regress.

Finally, as discussed earlier, the STFPCI can be used to calculate an output change over an input change as in Bjurek's (1996) index, which is out of the question using either the Adj-MI or the BP-MI. In this example,

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[6] Eq. (12) suggests that we might project the normalized empirical data in *t* and *t*+*1* against the surface of the reference technology where $D_{FP}(x_o, y_o) = 1$. This would be equivalent to the traditional procedure that we follow for the BP-MI in (8). The difference is that there are no parameters for the base technology in (8) because the parametric technology in this case is unknown and must be estimated. Within the new framework, however, we do know the function and the parameters of the standard reference technology so we can simply plug the normalized data into (13).

[7] To replicate this numerical example, we provide the R code used for calculating the results of this section in Annex A.

Table 2
Production data for eleven DMUs in two periods.

	Inputs and Outputs						*Normalized Inputs and Outputs*[*]					
	t			*t + 1*			*t*			*t + 1*		
DMU	y_2	y_1	x	y_2	y_1	x	y_2	y_1	x	y_2	y_1	x
A	1	6.5	1	3	16.5	3	0.0667	0.3611	0.2	0.2	0.9167	0.6
B	9	18	3	2	5	1	0.6	1	0.6	0.1333	0.2778	0.2
C	5	5	1	5	5	1	0.3333	0.2778	0.2	0.3333	0.2778	0.2
D	12	6	2	15	8	2	0.8	0.3333	0.4	1	0.4444	0.4
E	13	2	2	9	2	1	0.8667	0.1111	0.4	0.6	0.1111	0.2
F	6	16	4	1.5	5	1	0.4	0.8889	0.8	0.1	0.2778	0.2
G	10	15	5	5	10	5	0.6667	0.8333	1	0.3333	0.5556	1
H	8	8	2	7	9	2	0.5333	0.4444	0.4	0.4667	0.5	0.4
I	9	12	3	12	16	4	0.6	0.6667	0.6	0.8	0.8889	0.8
J	3	13.5	3	3	15	4	0.2	0.75	0.6	0.2	0.8333	0.8
K	2	2	1	3	6	2	0.1333	0.1111	0.2	0.2	0.3333	0.4
Max[*]	13	**18**	5	**15**	16.5	**5**	0.8667	1	1	1	0.9167	1

[*] The information in the second column 'Inputs and Outputs' in Table 2 is normalized by maximum outputs and input values to define all variables in the [0,1] interval.

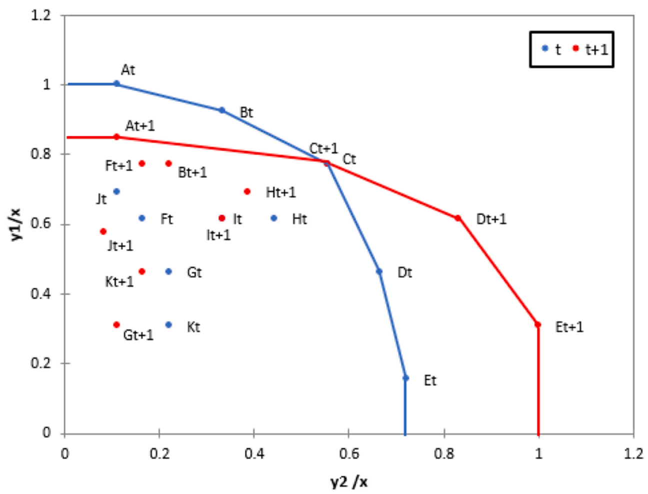

Fig. 1. Production frontiers for 11 DMUs in two time periods.

the output and input changes are straightforward obtained by plugging the normalized output and input values of Table 2 into (13) imposing the parameters in (15), and the results, the output change divided by the input change match the STFPI in Table 3. Table 4 shows these results demonstrating that the STFPI is a multiplicatively complete TFP index.

4. Empirical application

In this empirical application, we use the same panel data of 42 Swedish pharmacies between 1980 and 1989 as was previously analyzed in the seminal papers by Färe et al. (1992), Balk and Althin (1996) and Althin (2001). This set of pharmacies produce four outputs employing four inputs. The four inputs are 'Labor input for pharmacist' (X1); 'Labor input for technical staff' (X2); 'Equipment services' (X3) and 'Building services' (X4). Both labor inputs (X1 and X2) are measured in number of hours per year, X3 is measured using the annual depreciation of pharmacy equipment measured in 1980 prices and X4 is assumed to be proportional to the available floor space, measured in square meters. The four outputs are 'Drug deliveries to hospitals' (O1); 'Prescription drugs for outpatient care' (O2); 'Medical appliances for the handicapped' (O3) and 'Over the counter goods' (O4). The first three outputs (O1, O2 and O3) are measured in number of times, whereas the fourth (O4) is measured in 1980 prices[8].

Table 5 shows the productivity changes and their decomposition for the Adj-MI, the BP-MI, using the first year (1980) as the fixed reference technology, and the new STFPCI. The modeled technologies assume constant returns to scale and strong disposability of inputs, and distances are calculated using an output orientation. The global technical change is calculated using $K = 1{,}000{,}000$ synthetic DMUs. The results provided are the geometric means of the 42 analyzed pharmacies[9].

To interpret the results, it is worth noting that a value of one means no productivity change, a number greater than one means productivity progress and less than one is equivalent to productivity regress. By construction, efficiency changes coincide for all three indices. Therefore, the differences among indices stem from the calculus of technical change. Another straight result is that, as expected, the BP-MI and the STFPI satisfy the circularity test, and this implies that the accumulated productivity and technical changes are equal to the direct productivity and technical changes of the first period (1980) relative to the last period (1989). However, the $GTC_H^{t,t+1}$and the $LTC_o^{t,t+1}$in Aparicio and Santín (2024) do not pass the circularity test because both terms are obtained every two adjacent periods as the $TC_o^{t,t+1}$ in the adjacent Malmquist index. Table 6 shows that Pearson's correlation coefficients for the nine average standard total factor productivity and traditional technical change values for this industry are positive, greater than 0.92 and statistically significant. In fact, correlations between the two traditional indices are slightly lower than when they are correlated with the STFPI.

The new index provides richer information than the traditional approaches. Firstly, the accumulated average global technical change for this industry over the 1980-1989 period (1.1401) is much smaller than the resulting technical change calculated as the average of observed DMUs by the Adj-MI (1.2703) and the BP-MI (1.6214). Around half of the observed technical change is driven by the local technical change (1.1330) of a bunch of DMUs, which on average push the technology up above the average global technical change. This new decomposition is useful for disentangling how the technical change evolved in every

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[8] Färe et al. (1992) provide more details about the database, the variables, descriptive statistics and disaggregated adjacent Malmquist index and its decomposition over time for the set of 42 pharmacies.

[9] Annex B provides individualized results for the set of 42 pharmacies in every period including the aggregate output (Table B1), aggregate input (Table B2), STFPI (Table B3), STFPCI (Table B4), output changes (Table B5), input changes (Table B6), efficiency changes (Table B7) and local technical changes (Table B8). Likewise, Annex C shows the R code used and how to retrieve the data for calculating the results of this section.

Table 3
Numerical example. Total factor productivity indices and their components.

	Adjacent MI			Base Period MI*				Standard TFPCI**			
DMU	Adj-MI	$EC_o^{t,t+1}$	$TC_o^{t,t+1}$	BP-MI	$EC_o^{t,t+1}$	$TC(b)_o^{t,t+1}$	STFPI	$EC_o^{t,t+1}$	$TC(S)_o^{t,t+1}$	$GTC(S)_H^{t,t+1}$	$LTC(S)_o^{t,t+1}$
A	0.8490	1	0.8490	0.8519	1	0.8518	0.8517	1	0.8517	1.0254	0.8306
B	0.8192	0.9333	0.8777	0.8148	0.9333	0.8730	0.7926	0.9333	0.8493	1.0254	0.8282
C	1	1	1	1	1	1	1	1	1	1.0254	0.9752
D	1.2697	1	1.2697	1.2667	1	1.2667	1.2627	1	1.2627	1.0254	1.2314
E	1.3960	1	1.3960	1.4074	1	1.4074	1.3967	1	1.3967	1.0254	1.3621
F	1.2337	1.4229	0.8670	1.2286	1.4229	0.8635	1.2115	1.4229	0.8515	1.0254	0.8304
G	0.6393	0.7083	0.9025	0.6250	0.7083	0.8824	0.6071	0.7083	0.8571	1.0254	0.8358
H	1.0691	1.0972	0.9744	1.0417	1.0972	0.9494	0.9852	1.0972	0.8979	1.0254	0.8756
I	1	1.0606	0.9429	1	1.0606	0.9429	1	1.0606	0.9429	1.0254	0.9195
J	0.8300	0.9711	0.8548	0.8289	0.9711	0.8537	0.8281	0.9711	0.8527	1.0254	0.8316
K	1.3307	1.4167	0.9393	1.2500	1.4167	0.8824	1.1199	1.4167	0.7905	1.0254	0.7709
GM	1.0126	1.0379	0.9756	1.0023	1.0379	0.9657	0.9799	1.0379	0.9441	1.0254	0.9207

* The base technology corresponds to the period *t*.

** The $GTC(S)_H^{t,t+1}$ is calculated defining K=1,000,000. Note that, by construction, for two periods $GTC_H^{t,t+1} = GTC(S)_H^{t,t+1}$, however, $LTC_o^{t,t+1} \neq LTC(S)_o^{t,t+1}$. Therefore, the $LTC_o^{t,t+1}$ in Aparicio and Santín (2024) can by retrieved by $LTC_o^{t,t+1} = TC_0^{t,t+1}/GTC(S)_H^{t,t+1}$.

Table 4
The STFPI and its decomposition into output and input changes.

	Periods *t* and *t+1*			Period *t*			Period *t+1*		
DMU	STFPCI	Output Change	Input Change	STFPI	Aggregate Output	Aggregate Input	STFPI	Aggregate Output	Aggregate Input
A	0.8517	2.555	3	1.2983	0.2597	0.2	1.1057	0.6634	0.6
B	0.7926	0.2642	0.3333	1.3744	0.8246	0.6	1.0894	0.2179	0.2
C	1	1	1	1.5341	0.3068	0.2	1.5341	0.3068	0.2
D	1.2627	1.2627	1	1.5321	0.6128	0.4	1.9345	0.7738	0.4
E	1.3967	0.6984	0.5	1.5446	0.6178	0.4	2.1574	0.4315	0.2
F	1.2115	0.3029	0.25	0.8616	0.6892	0.8	1.0438	0.2088	0.2
G	0.6071	0.6071	1	0.7546	0.7546	1	0.4581	0.4581	1
H	0.9852	0.9852	1	1.2273	0.4909	0.4	1.2091	0.4836	0.4
I	1	1.3333	1.3333	1.0570	0.6342	0.6	1.0570	0.8456	0.8
J	0.8281	1.1041	1.3333	0.9148	0.5489	0.6	0.7575	0.6060	0.8
K	1.1199	2.2397	2	0.6136	0.1227	0.2	0.6872	0.2749	0.4
GM	0.9799	0.9103	0.9289	1.1073	0.4740	0.4281	1.0850	0.4315	0.3977

period. For example, according to the Adj-MI and BP-MI, technical change was measured in the period 80-81 as 0.9698 and 1.0334, respectively, leading to inconclusive results. However, the STFPI decomposition concludes that, on average, the industry made slight technical progress equal to $GTC(S)_H^{1980,1981} = 1.0272$, mainly led by pharmacies 8, 17, 33, 35, 37, 41 and 42 (see Table B8 in Annex B) where the $LTC(S)_o^{t,t+1} > 1$ (they behave as DMUs C and D in Fig. 1). Nevertheless, the rest of pharmacies were located in the region where the production frontier suffered a technical regress (as the one experienced by DMUs A, B, F, G, H, I, J and K in Fig. 1), and they pull back the technology shift enough $LTC(S)_o^{1980,1981} = 0.9636$ to offset the average positive global effect resulting in an observed technical change below one $TC(S)_o^{1980,1981} = 0.9899$.

Other valuable information that was previously discussed in Section 3.1 is that normalized output and input values can be plugged into (13) using the STFPI in order to calculate how it decomposes into an aggregate output and an aggregate input change. Table 7 provides this decomposition.

The measures of output change and input change can be useful from a managerial point of view, especially in sectors like the public services, where there are no prices for many variables and individual information about each DMU is important for decision-making by policymakers. Output changes that are greater than one indicates an increase in the aggregate outputs from period t to period $t+1$, while values of less than one denotes a decline. The values related to input changes can be interpreted in the same manner with respect to the inputs. From Table 6, we observe that the clear decline in productivity in the 83-84 period is explained by the fact that, on average, the decreases in outputs (0.9071) were greater than the reduction in inputs (0.9490). Years 81-82, 84-85, 86-87 and 87-88 were especially interesting because, on average, the industry managed to increase productivity by producing more (output change greater than one) with fewer resources (input change less than one). Interestingly, the opposite result, producing less with more, never happened in the above period, although the productivity gains in periods 82-83 and 85-86 were due to positive output changes that were greater than the positive input increases.

5. Conclusions

In this paper, we introduce a new standard total factor productivity index based on defining a standard reference technology for all required calculations. As is the case in other fields, the acceptance of a common standard improves the comparability of calculations, ease of use and satisfaction of interesting properties. In our case, we defined the standard reference technology based on the geometrical grounds of a hypersphere. This standard does not need to be estimated from data and is grounded on a particular parametric specification of a general multi-output multi-input production technology approach previously defined by Färe and Primont (1995) based on the notion of distance function.

The use of the standard reference technology contributes to benchmarking literature in different directions. First, the index is much easier to obtain than previous productivity indices. In this case, it is sufficient to normalize the original information by the maximum observed values of each input and output and plugging the obtained values in the parametric formula that defines the STFPI in (11) using the parameters

Table 5
Average results for the Adj-MI, BP-MI and STFPCI and their decompositions.

	Base Period-Malmquist Index (BP-MI) (Berg et al. 1992)			Adjacent-Malmquist Index (Adj-MI) (Färe et al., 1992)			Aparicio and Santín (2024)*		Standard Total Factor Productivity Change Index (STFPCI)**				
Year	BP-MI	$EC_o^{t,t+1}$	$TC(b)_o^{t,t+1}$	Adj-MI	$EC_o^{t,t+1}$	$TC_o^{t,t+1}$	$GTC_H^{t,t+1}$	$LTC_o^{t,t+1}$	STFPCI	$EC_o^{t,t+1}$	$TC(S)_o^{t,t+1}$	$GTC(S)_H^{t,t+1}$	$LTC(S)_o^{t,t+1}$
80-81	1.0561	1.0220	1.0334	0.9911	1.0220	0.9698	1.0496	0.9239	1.0116	1.0220	0.9899	1.0272	0.9636
81-82	1.1072	0.9353	1.1838	1.0740	0.9353	1.1483	1.1733	0.9787	1.0763	0.9353	1.1508	1.1967	0.9616
82-83	1.0586	1.0548	1.0036	1.0246	1.0548	0.9713	0.9557	1.0164	1.0402	1.0548	0.9862	0.9525	1.0353
83-84	0.9923	0.9875	1.0049	0.9424	0.9875	0.9544	0.8980	1.0628	0.9572	0.9875	0.9694	0.9046	1.0716
84-85	1.0495	1.0039	1.0454	1.0435	1.0039	1.0395	1.0134	1.0258	1.0379	1.0039	1.0339	1.0205	1.0131
85-86	1.0470	0.9868	1.0610	1.0189	0.9868	1.0325	1.0147	1.0176	1.0082	0.9868	1.0217	1.0075	1.0141
86-87	1.1035	1.0015	1.1018	1.0665	1.0015	1.0649	1.0229	1.0410	1.0824	1.0015	1.0808	1.0217	1.0578
87-88	1.0365	1.0133	1.0229	1.0435	1.0133	1.0298	1.0008	1.0290	1.0261	1.0133	1.0127	0.9990	1.0137
88-89	1.0572	1.0056	1.0513	1.0513	1.0056	1.0455	1.0192	1.0258	1.0317	1.0056	1.0260	1.0256	1.0004
Mean	1.0559	1.0007	1.0552	1.0277	1.0007	1.0269	1.0141	1.0127	1.0296	1.0007	1.0288	1.0147	1.0140
Accum.	1.6320	1.0065	1.6214	1.2786	1.0065	1.2703	1.1339	1.1203	1.3001	1.0065	1.2917	1.1401	1.1330
Direct	1.6320	1.0065	1.6214	1.1922	1.0065	1.1844	1.1664	1.0155	1.3001	1.0065	1.2917	1.1401	1.1330

* $TC_o^{t,t+1} = GTC_H^{t,t+1} \cdot LTC_o^{t,t+1}$.

** $TC(S)_o^{t,t+1} = GTC(S)_H^{t,t+1} \cdot LTC(S)_o^{t,t+1}$.

Table 6
Pearson correlation coefficients of average productivity change and technical change values.

	Productivity indices				Technical changes		
	Adj-MI	BP-MI	STFPCI		$TC_o^{t,t+1}$	$TC(b)_o^{t,t+1}$	$TC(S)_o^{t,t+1}$
Adj-MI	1	0.8292	0.9479	$TC_o^{t,t+1}$	1	0.9246	0.9766
BP MI	0.8292	1	0.9246	$TC(b)_o^{t,t+1}$	0.9246	1	0.9662
STFPCI	0.9479	0.9246	1	$TC(S)_o^{t,t+1}$	0.9766	0.9662	1

Table 7
Productivity results for the STFPCI and its decomposition into output and input changes.

Year	STFPCI	Output Change	Input Change
80-81	1.0116	0.9880	0.9766
81-82	1.0763	1.0589	0.9838
82-83	1.0402	1.0782	1.0365
83-84	0.9572	0.9071	0.9477
84-85	1.0379	1.0323	0.9946
85-86	1.0082	1.0372	1.0287
86-87	1.0824	1.0683	0.9870
87-88	1.0261	1.0178	0.9919
88-89	1.0317	0.9982	0.9675
Average	1.0283	1.0194	0.9901
Accumulated	1.3001	1.1890	0.9146
Direct	1.3001	1.1890	0.9146

in (15) for calculating total factor productivity for each period. The second advantage is that being a multiplicatively complete index on a parametric basis, any manager, practitioner or evaluator can compare the performance of a DMU with respect to other DMUs or even with respect to itself over time. Thirdly, the index facilitates the comparison of production processes using the same quantity inputs and quantity outputs. If an industry is already using a STFPI, the arrival of new DMUs within the industry or the comparison with other industries can be done by simply normalizing the new information with the maximum values that were used when the benchmarking began. Adding information from new periods or new DMUs does not change the results from previous periods.

Fourth, the index fulfils all the desirable properties or tests for an index number. To date, there has been a trade-off between circularity and the base period independency test. In particular, if we opted for the BP-MI instead of the Adj-MI, the gain in terms of circularity was offset by reference period dependency. On the other hand, if we utilized the Adj-MI instead of the BP-MI, our approach passed the base period independency test, but the circularity property did not hold. The STFPI satisfies both, circularity and independency from the choice of a base period. Additionally, the new approach also satisfies the determinateness test thanks to the formulation of the standard technology that now is additive instead of multiplicative. This is something to which neither of the above Malmquist indices can lay claim. Another interesting property satisfied by the STFPI is that it can be identified as a true total factor productivity index, that is, it can be expressed as the ratio of an aggregated change in outputs and an aggregated change in inputs. In contrast, the most common versions of the Malmquist productivity index do not fulfill this property.

Regarding the decomposition of the new approach into typical drivers, efficiency change and technical change, we follow the ideas established by Balk and Althin (1996), Asmild and Tam (2007), Otsuki (2013) and, recently, by Aparicio and Santín (2024), where the technical change component is regarded as a global phenomenon affecting the frontier shift of the entire sector. In particular, our approach, as that proposed by Aparicio and Santin (2024), is based on the decomposition of traditional technical change into two subcomponents: a global technical change across the industry and how a company locally experiences technical change over time depending on the particular position of this production unit with respect to technologies at *t* and *t+1*. We identified both drivers in our approach by exploiting a grid of synthetic DMUs generated within a unit-hypercube. Once synthetic data are generated, the remaining distances can be calculated using different DEA models and resorting to the parametric standard technology.

To facilitate the application and interpretation of this new approach, we also described a numerical example with eleven DMUs using one input to produce two outputs in two periods outlining all necessary steps for calculating all the measurements under DEA. Furthermore, in our empirical application, we use the same panel data of 42 Swedish pharmacies between 1980 and 1989 that was previously analyzed in the seminal papers by Färe et al. (1992), Balk and Althin (1996) and Althin (2001). Results show that the STFPI provides meaningful results to carry out the benchmarking of the evaluated DMUs. Moreover, the results are sound when they are compared to the two previous Malmquist index versions.

This paper should open up a good number of research avenues for the future. The standard can be extended to other Malmquist-type indices, like the Camanho and Dyson (2006) index for dealing with productivity gaps between groups of DMUs and its time-based version proposed by Aparicio and Santín (2018). Another challenge is to extend the decomposition of the global and local technical changes for production technologies operating under variable returns to scale over time.

CRediT authorship contribution statement

Juan Aparicio: Methodology, Formal analysis, Investigation, Conceptualization, Writing – original draft, Funding acquisition. **Daniel Santín:** Methodology, Formal analysis, Writing – original draft, Conceptualization, Visualization, Investigation, Data curation.

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Annex A. R code to replicate the numerical example in Section 3.3

```
library(Benchmarking) library(uniformly)
#To ensure replication we set a seed set.seed(18072007)
# Table 2. Production data for eleven DMUs in two periods. xt<-matrix(c(1,3,1,2,2,4,5,2,3,3,1), ncol = 1) xt1<-matrix(c(3,1,1,2,1,1,5,2,4,4,2), ncol = 1) y2t<-matrix(c(1,9,5,12,13,6,10,8,9,3,2), ncol = 1) y1t<-matrix(c(6.5,18,5,6,2,16,15,8,12,13.5,2), ncol = 1) y2t1<-matrix(c(3,2,5,15,9,1.5,5,7,12,3,3), ncol =1) y1t1<-matrix(c(16.5,5,5,8,2,5,10,9,16,15,6), ncol = 1)
# Build a matrix with all values of each output and input in the 2 periods. y2=matrix(c(y2t, y2t1), nrow=11,ncol=2) y1=matrix(c(y1t, y1t1), nrow=11,ncol=2) x=matrix(c(xt, xt1), nrow=11,ncol=2)
# Seek the maximum values for each variable in the 2 periods. y2max=max(y2) y1max=max(y1) xmax=max(x)
# Normalize the variables in the 2 periods by the maximum observed values. y2tn=y2t/y2max y2t1n=y2t1/y2max y1tn=y1t/y1max y1t1n=y1t1/y1max xtn=xt/xmax xt1n=xt1/xmax ytn<-matrix(c(y2tn, y1tn), nrow=11,ncol=2) yt1n<-matrix(c(y2t1n, y1t1n), nrow=11,ncol=2)
# Table3. Calculate the adjacent Malmquist Productivity Index.
# Note that the notation in the paper uses Shephard’s measures while
# the library ’Benchmarking’ uses Farrell’s measures. For this reason, we run
# the ’input orientation’ for obtaining the ’output orientation’ interpretation. mpi <- malmq(xtn,ytn,ID0 = NULL,xt1n,yt1n,ID1 = NULL, RTS="crs",
ORIENTATION = "in")
# Adjacent Malmquist index adjMPI=mpi$m
# Efficiency change
EC=mpi$ec
# Technical change
TC=mpi$tc
# Base Period Malmquist Index
# Period t is the base technology. We need to run the DEAs in Eq. 8 deat1vst=dea(xt1n, yt1n, RTS="crs", ORIENTATION="in", XREF=xtn, YREF=ytn) deat=dea(xtn, ytn, RTS="crs", ORIENTATION="in", XREF=NULL, YREF=NULL)
# The Base Period Malmquist index (BP-MI) and its geometric mean
MIb=deat1vst$eff/deat$eff
AVMIb=prod(MIb)ˆ(1/11)
# In the BP-MI the efficiency change is EC
# The technical change in the BP-MI and its geometric mean
TCb=(deat1vst$eff/deat$eff)*(1/EC)
AVTCb=prod(TCb)ˆ(1/11)
# Table 3 & 4
# The Standard Total Factor Prodctivity Index and its decomposition
# m = Number of inputs; n = Number of outputs; m=1 n=2
# The STFPI in t ytnum=sqrt(rowSums(ytnˆ2))*(1/sqrt(n)) xtdenom=sqrt(rowSums(xtnˆ2))*(1/sqrt(m))
STFPIt=ytnum/xtdenom
# The STFPI in t+1 yt1num=sqrt(rowSums(yt1nˆ2))*(1/sqrt(n)) xt1denom=sqrt(rowSums(xt1nˆ2))*(1/sqrt(m))
STFPIt1=yt1num/xt1denom
# The Standard Total Factor Productivity Change
STFPItt1=STFPIt1/STFPIt
AVSTFPI=prod(STFPItt1)ˆ(1/11)
### Table 4. Output and input changes between t and t+1
OUTCH=yt1num/ytnum
INPCH=xt1denom/xtdenom
### STFPIC decomposition in Table 3.
```

```
### GLOBAL TECHNICAL CHANGE
# Generate a big K number of synthetic DMUs inside the unit-hypercube
# of dimension H. H=m+n:m= number of inputs; n = number of outputs.
# The library 'uniformly' allows to generate the unit hypercube H in one step.
# H = unit-hypercube size
# K = Number of synthetic DMUs within the unit-hypercube H for calculating GTC
H=m+n
K=1000000 simH<-runif_in_cube(K, H)
Hh<-abs(simH)
Xh<-Hh[,1]
Yh<-Hh[,2:3]
# Project the K synthetic DMUs inside hypercube H against the
# empirical technologies in t and t+1. Computation time depends on K hcubet=dea(Xh, Yh, RTS="crs", ORIENTATION="in", XREF=xtn,
YREF=ytn) hcubet1=dea(Xh, Yh, RTS="crs", ORIENTATION="in", XREF=xt1n, YREF=yt1n)
### GLOBAL TECHNICAL CHANGE.
# The GTC is the geometric mean of all the TCs computed for the K synthetic DMUs
Globalt=10^(mean(log10(hcubet$eff),na.rm=TRUE))
Globalt1=10^(mean(log10(hcubet1$eff),na.rm=TRUE))
GTC=Globalt/Globalt1
### LOCAL TECHNICAL CHANGE
# The LTC for the Standard is obtained as
LTCS<-STFPItt1/(GTC*mpi$ec)
AVLTCS=prod(LTCS)^(1/11)
### Technical Change for the Standard (TCs)
TCS=GTC*LTCS
AVTCS=prod(TCS)^(1/11)
```

Annex. B

Table B1
Aggregate output levels for 42 Swedish pharmacies between 1980 and 1989.

ID	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989
1	0.3217	0.2961	0.3209	0.3508	0.3770	0.4100	0.3637	0.3865	0.3874	0.3921
2	0.3243	0.3311	0.3789	0.3660	0.3488	0.3616	0.3712	0.4276	0.4668	0.4731
3	0.2654	0.2674	0.2852	0.2944	0.2745	0.2795	0.2980	0.3098	0.3097	0.3234
4	0.3350	0.3310	0.3318	0.3405	0.3530	0.3560	0.3556	0.3613	0.3613	0.3685
5	0.3779	0.3969	0.4062	0.4375	0.3927	0.3961	0.3947	0.4230	0.4392	0.4507
6	0.3331	0.3489	0.3746	0.3988	0.4031	0.4382	0.4561	0.4860	0.5383	0.5544
7	0.3115	0.3100	0.3326	0.3279	0.3278	0.3464	0.3585	0.3851	0.4154	0.4311
8	0.2818	0.2883	0.3041	0.3216	0.3312	0.3536	0.4262	0.3923	0.4195	0.4333
9	0.3601	0.3678	0.4004	0.4004	0.4161	0.3989	0.4041	0.4208	0.4312	0.4462
10	0.4293	0.4361	0.4580	0.4780	0.3329	0.3651	0.3835	0.4138	0.4358	0.4456
11	0.1590	0.1584	0.1661	0.1834	0.1894	0.2046	0.2165	0.2262	0.2326	0.2461
12	0.4169	0.4116	0.4547	0.4844	0.4444	0.4527	0.4731	0.5585	0.4878	0.5033
13	0.2891	0.2896	0.3131	0.3079	0.3030	0.2970	0.3001	0.3145	0.3188	0.3001
14	0.4244	0.5258	0.6361	0.6511	0.6088	0.5844	0.6000	0.6137	0.6532	0.6554
15	0.2848	0.2831	0.2938	0.2892	0.2801	0.3052	0.3107	0.3314	0.3072	0.3222
16	0.2433	0.2393	0.2558	0.2828	0.2432	0.2551	0.2609	0.2738	0.2916	0.2939
17	0.5093	0.5553	0.4390	0.6572	0.5927	0.6183	0.6290	0.6854	0.5257	0.2751
18	0.4147	0.4145	0.6221	0.4532	0.3242	0.3358	0.4628	0.5153	0.5193	0.5295
19	0.3729	0.3493	0.3713	0.4679	0.5241	0.5709	0.6173	0.5980	0.5305	0.5022
20	0.4028	0.3816	0.4168	0.4431	0.3185	0.3351	0.3533	0.3718	0.4048	0.4190
21	0.3466	0.3390	0.3671	0.3796	0.3756	0.3756	0.3637	0.3556	0.4042	0.3641
22	0.2626	0.2574	0.2559	0.2739	0.2644	0.2647	0.2745	0.3125	0.3215	0.3358
23	0.3740	0.1523	0.1110	0.3262	0.5031	0.5501	0.5694	0.6045	0.6410	0.6634
24	0.3363	0.3263	0.3445	0.3558	0.3302	0.3240	0.3364	0.3549	0.3868	0.4115
25	0.3925	0.3396	0.3496	0.3642	0.3634	0.3597	0.3761	0.4263	0.4474	0.4628
26	0.2789	0.2784	0.2722	0.2652	0.2617	0.2815	0.3191	0.5139	0.5764	0.6135
27	0.2745	0.2752	0.3049	0.3109	0.2347	0.2397	0.2505	0.2586	0.2560	0.2761
28	0.2681	0.2673	0.2907	0.3030	0.3062	0.3119	0.3232	0.3293	0.3463	0.3598
29	0.3977	0.3885	0.3868	0.4088	0.3581	0.3819	0.4012	0.3993	0.4011	0.4147
30	0.4875	0.5165	0.5962	0.7176	0.4970	0.5524	0.5823	0.5746	0.6003	0.6263
31	0.3884	0.3849	0.4232	0.4458	0.3342	0.3493	0.3810	0.4029	0.3242	0.3222
32	0.4145	0.4191	0.4377	0.4651	0.4338	0.4649	0.4806	0.5271	0.5554	0.4961
33	0.4052	0.4150	0.4253	0.4535	0.4046	0.4158	0.4426	0.4643	0.4806	0.5066
34	0.4098	0.3791	0.3955	0.4225	0.3876	0.3432	0.3378	0.3540	0.4077	0.4214
35	0.5125	0.5466	0.5131	0.5273	0.4594	0.4780	0.4816	0.4850	0.5012	0.4753
36	0.3894	0.3869	0.3988	0.4073	0.2919	0.2997	0.2986	0.3296	0.3364	0.3314
37	0.2771	0.2837	0.3404	0.3569	0.3177	0.3290	0.3106	0.3999	0.3920	0.4013

(*continued on next page*)

Table B1 (*continued*)

ID	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989
38	0.4462	0.4426	0.5086	0.5407	0.3663	0.3504	0.3509	0.3499	0.3671	0.3734
39	0.1655	0.1696	0.1791	0.2062	0.1626	0.1687	0.1748	0.1994	0.2076	0.2004
40	0.2766	0.2900	0.3144	0.3323	0.2695	0.2735	0.2785	0.2991	0.3059	0.3142
41	0.1261	0.1583	0.1542	0.1599	0.1322	0.1331	0.1414	0.1422	0.1285	0.1245
42	0.4206	0.4319	0.5193	0.5384	0.5014	0.5339	0.5054	0.5119	0.5241	0.5217
GM*	0.3324	0.3284	0.3477	0.3749	0.3401	0.3511	0.3641	0.3890	0.3960	0.3952

* Geometric means of the sample.

Table B2
Aggregate input levels for 42 Swedish pharmacies between 1980 and 1989.

ID	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989
1	0.4635	0.4407	0.4710	0.4427	0.4310	0.4598	0.4464	0.4420	0.4335	0.4194
2	0.3734	0.3728	0.3864	0.3979	0.3710	0.3855	0.5489	0.5288	0.5250	0.5087
3	0.2850	0.2911	0.2978	0.3038	0.2936	0.2924	0.3072	0.3022	0.3048	0.2993
4	0.4782	0.4672	0.4679	0.4675	0.4808	0.4743	0.4950	0.4655	0.4358	0.4257
5	0.7006	0.7030	0.7076	0.7225	0.7115	0.6996	0.6985	0.7111	0.7004	0.7022
6	0.5751	0.5862	0.5791	0.5468	0.6045	0.6051	0.6138	0.6082	0.6222	0.6257
7	0.4471	0.4325	0.3983	0.4031	0.3989	0.4033	0.4197	0.4090	0.3735	0.3947
8	0.5528	0.3821	0.3902	0.3838	0.3614	0.3680	0.3838	0.3836	0.3920	0.3817
9	0.6056	0.6165	0.6022	0.6082	0.6131	0.5997	0.6020	0.6001	0.5931	0.5866
10	0.6743	0.6702	0.6587	0.6614	0.4451	0.4249	0.4267	0.4355	0.4373	0.4157
11	0.2294	0.2325	0.2211	0.1965	0.2234	0.2278	0.2278	0.2356	0.2374	0.2123
12	0.5497	0.5443	0.3414	0.5583	0.5356	0.5100	0.5289	0.5496	0.5144	0.4924
13	0.4574	0.4675	0.4623	0.4636	0.4652	0.4710	0.4660	0.4554	0.4535	0.4455
14	0.5068	0.6143	0.6349	0.6240	0.5946	0.6181	0.6109	0.5933	0.5686	0.5809
15	0.3311	0.3264	0.3250	0.3334	0.3236	0.3305	0.3410	0.3519	0.3935	0.3780
16	0.3987	0.3901	0.3767	0.3852	0.3485	0.3619	0.3644	0.3680	0.3635	0.3466
17	0.7813	0.7989	0.7090	0.8345	0.8075	0.8052	0.8182	0.8250	0.7417	0.5972
18	0.6100	0.6016	0.7350	0.6013	0.5168	0.5049	0.5896	0.5999	0.5976	0.6013
19	0.5858	0.5695	0.5506	0.6203	0.5873	0.5763	0.5712	0.5483	0.4987	0.4511
20	0.5224	0.5185	0.5257	0.5247	0.3907	0.3857	0.3972	0.3841	0.3904	0.3861
21	0.4480	0.4410	0.4467	0.4581	0.4493	0.4551	0.4635	0.4316	0.4191	0.3898
22	0.4022	0.4073	0.4125	0.4183	0.4237	0.4326	0.4513	0.4586	0.4729	0.4435
23	0.4322	0.2042	0.1829	0.4138	0.5734	0.5538	0.5752	0.5443	0.5550	0.5577
24	0.4384	0.4390	0.4135	0.4211	0.4292	0.4115	0.4008	0.3938	0.4071	0.4115
25	0.5523	0.5284	0.4551	0.4660	0.4655	0.4625	0.4660	0.4628	0.4848	0.4443
26	0.3863	0.3809	0.3677	0.3525	0.3630	0.3490	0.4116	0.4980	0.4929	0.4884
27	0.3896	0.3775	0.3761	0.3621	0.3191	0.3162	0.3274	0.3089	0.3125	0.3069
28	0.3757	0.3770	0.3797	0.3906	0.4068	0.4048	0.4156	0.3931	0.3699	0.3461
29	0.5833	0.5854	0.5600	0.5500	0.4855	0.4762	0.4771	0.4779	0.4732	0.4509
30	0.6268	0.6558	0.6779	0.7788	0.6248	0.6061	0.6100	0.5695	0.5380	0.5492
31	0.5544	0.6492	0.6433	0.6544	0.5700	0.5655	0.5660	0.5410	0.5050	0.5009
32	0.4961	0.5097	0.4955	0.4912	0.4405	0.4431	0.4577	0.4423	0.4617	0.4141
33	0.5624	0.5370	0.5294	0.5313	0.4580	0.4672	0.4598	0.4273	0.4326	0.4175
34	0.5619	0.5204	0.4894	0.4821	0.4660	0.4343	0.4069	0.3833	0.3901	0.3763
35	0.6923	0.6322	0.6746	0.6780	0.6227	0.6377	0.6459	0.6516	0.6608	0.6783
36	0.5225	0.5156	0.5129	0.4973	0.4538	0.4629	0.4961	0.4931	0.4984	0.5014
37	0.4267	0.4136	0.4473	0.4751	0.4392	0.4396	0.5112	0.4884	0.4897	0.4668
38	0.5502	0.5799	0.6200	0.6195	0.5411	0.5411	0.5375	0.5250	0.5349	0.5316
39	0.1903	0.2004	0.1956	0.2029	0.1782	0.1899	0.1923	0.1841	0.1891	0.1804
40	0.4701	0.4710	0.4681	0.4671	0.4373	0.3538	0.3380	0.3334	0.3297	0.3038
41	0.2464	0.2596	0.2645	0.2704	0.2558	0.2593	0.2551	0.2589	0.2607	0.2530
42	0.6411	0.6267	0.6424	0.6466	0.6314	0.6393	0.6297	0.6356	0.6104	0.6135
GM*	0.4729	0.4619	0.4544	0.4710	0.4463	0.4439	0.4566	0.4507	0.4470	0.4325

* Geometric means of the sample.

Table B3
Standard reference technology total factor productivity index for 42 Swedish pharmacies between 1980 and 1989.

ID	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989
1	0.6941	0.6720	0.6813	0.7924	0.8747	0.8919	0.8148	0.8744	0.8937	0.9350
2	0.8687	0.8882	0.9807	0.9197	0.9402	0.9381	0.6763	0.8087	0.8890	0.9300
3	0.9315	0.9186	0.9577	0.9692	0.9348	0.9559	0.9700	1.0249	1.0162	1.0804
4	0.7004	0.7084	0.7092	0.7285	0.7343	0.7506	0.7183	0.7762	0.8291	0.8657
5	0.5393	0.5645	0.5741	0.6055	0.5519	0.5662	0.5650	0.5949	0.6271	0.6419
6	0.5791	0.5953	0.6469	0.7292	0.6669	0.7242	0.7430	0.7990	0.8652	0.8861
7	0.6968	0.7167	0.8352	0.8133	0.8218	0.8590	0.8540	0.9416	1.1121	1.0921
8	0.5098	0.7544	0.7792	0.8380	0.9163	0.9611	1.1103	1.0228	1.0701	1.1351
9	0.5946	0.5967	0.6649	0.6583	0.6787	0.6652	0.6713	0.7012	0.7271	0.7608
10	0.6366	0.6507	0.6954	0.7227	0.7479	0.8593	0.8986	0.9503	0.9966	1.0719
11	0.6934	0.6815	0.7511	0.9333	0.8480	0.8982	0.9505	0.9605	0.9796	1.1592

(*continued on next page*)

Table B3 (*continued*)

ID	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989
12	0.7585	0.7563	1.3318	0.8677	0.8298	0.8876	0.8944	1.0162	0.9483	1.0220
13	0.6319	0.6193	0.6773	0.6640	0.6513	0.6307	0.6440	0.6906	0.7031	0.6736
14	0.8373	0.8559	1.0019	1.0435	1.0238	0.9455	0.9823	1.0344	1.1488	1.1282
15	0.8604	0.8676	0.9041	0.8674	0.8657	0.9234	0.9113	0.9417	0.7806	0.8524
16	0.6101	0.6133	0.6790	0.7341	0.6979	0.7049	0.7160	0.7440	0.8023	0.8477
17	0.6519	0.6951	0.6192	0.7875	0.7341	0.7680	0.7688	0.8308	0.7088	0.4606
18	0.6799	0.6891	0.8465	0.7537	0.6274	0.6650	0.7849	0.8589	0.8689	0.8805
19	0.6366	0.6134	0.6744	0.7542	0.8924	0.9907	1.0807	1.0908	1.0637	1.1134
20	0.7711	0.7360	0.7929	0.8446	0.8152	0.8688	0.8896	0.9679	1.0369	1.0853
21	0.7736	0.7688	0.8217	0.8287	0.8359	0.8254	0.7847	0.8241	0.9645	0.9340
22	0.6529	0.6319	0.6203	0.6549	0.6240	0.6118	0.6081	0.6815	0.6798	0.7572
23	0.8654	0.7457	0.6069	0.7881	0.8775	0.9934	0.9899	1.1106	1.1550	1.1895
24	0.7671	0.7433	0.8333	0.8448	0.7693	0.7873	0.8391	0.9013	0.9500	1.0001
25	0.7107	0.6427	0.7682	0.7815	0.7808	0.7778	0.8071	0.9211	0.9228	1.0417
26	0.7219	0.7309	0.7403	0.7523	0.7210	0.8066	0.7753	1.0319	1.1693	1.2562
27	0.7045	0.7291	0.8107	0.8586	0.7354	0.7581	0.7650	0.8372	0.8192	0.8996
28	0.7138	0.7091	0.7656	0.7759	0.7528	0.7705	0.7776	0.8378	0.9361	1.0395
29	0.6819	0.6637	0.6908	0.7432	0.7376	0.8020	0.8408	0.8356	0.8477	0.9197
30	0.7779	0.7875	0.8795	0.9215	0.7954	0.9114	0.9546	1.0090	1.1159	1.1404
31	0.7006	0.5928	0.6579	0.6812	0.5864	0.6178	0.6731	0.7447	0.6419	0.6433
32	0.8356	0.8222	0.8833	0.9468	0.9846	1.0491	1.0501	1.1917	1.2029	1.1982
33	0.7203	0.7728	0.8033	0.8536	0.8835	0.8900	0.9625	1.0866	1.1110	1.2135
34	0.7293	0.7285	0.8081	0.8764	0.8319	0.7902	0.8301	0.9237	1.0452	1.1198
35	0.7402	0.8646	0.7607	0.7778	0.7378	0.7496	0.7457	0.7443	0.7585	0.7006
36	0.7454	0.7504	0.7775	0.8191	0.6431	0.6474	0.6018	0.6684	0.6751	0.6609
37	0.6493	0.6861	0.7611	0.7513	0.7233	0.7484	0.6076	0.8188	0.8005	0.8597
38	0.8110	0.7633	0.8204	0.8728	0.6770	0.6476	0.6529	0.6665	0.6862	0.7024
39	0.8695	0.8464	0.9159	1.0159	0.9123	0.8883	0.9087	1.0832	1.0981	1.1111
40	0.5885	0.6156	0.6717	0.7115	0.6162	0.7731	0.8240	0.8970	0.9279	1.0340
41	0.5119	0.6098	0.5828	0.5912	0.5168	0.5134	0.5543	0.5492	0.4928	0.4921
42	0.6561	0.6892	0.8083	0.8327	0.7941	0.8351	0.8026	0.8054	0.8586	0.8504
GM*	0.7028	0.7110	0.7653	0.7961	0.7620	0.7910	0.7975	0.8632	0.8857	0.9138

[*] Geometric means of the sample

Table B4
Standard reference technology total factor productivity change index for 42 Swedish pharmacies between 1980 and 1989.

ID	80-81	81-82	82-83	83-84	84-85	85-86	86-87	87-88	88-89
1	0.9681	1.0139	1.1630	1.1038	1.0196	0.9136	1.0731	1.0220	1.0463
2	1.0225	1.1042	0.9377	1.0223	0.9978	0.7209	1.1958	1.0994	1.0461
3	0.9861	1.0426	1.0120	0.9645	1.0226	1.0148	1.0566	0.9916	1.0632
4	1.0114	1.0012	1.0271	1.0080	1.0222	0.9570	1.0806	1.0681	1.0441
5	1.0466	1.0169	1.0547	0.9115	1.0259	0.9980	1.0527	1.0543	1.0235
6	1.0279	1.0867	1.1273	0.9145	1.0859	1.0259	1.0754	1.0829	1.0242
7	1.0286	1.1652	0.9739	1.0104	1.0452	0.9942	1.1025	1.1812	0.9820
8	1.4799	1.0329	1.0755	1.0934	1.0489	1.1553	0.9212	1.0462	1.0607
9	1.0036	1.1143	0.9901	1.0309	0.9801	1.0091	1.0447	1.0369	1.0463
10	1.0221	1.0687	1.0393	1.0350	1.1489	1.0458	1.0575	1.0487	1.0755
11	0.9829	1.1020	1.2427	0.9086	1.0592	1.0582	1.0104	1.0199	1.1833
12	0.9971	1.7610	0.6515	0.9564	1.0697	1.0076	1.1362	0.9332	1.0777
13	0.9800	1.0936	0.9804	0.9808	0.9684	1.0212	1.0724	1.0180	0.9581
14	1.0221	1.1706	1.0415	0.9812	0.9234	1.0390	1.0530	1.1107	0.9820
15	1.0083	1.0421	0.9594	0.9980	1.0667	0.9869	1.0333	0.8290	1.0920
16	1.0051	1.1071	1.0812	0.9507	1.0100	1.0158	1.0391	1.0783	1.0567
17	1.0663	0.8908	1.2719	0.9321	1.0462	1.0010	1.0807	0.8531	0.6499
18	1.0136	1.2284	0.8904	0.8324	1.0599	1.1803	1.0943	1.0116	1.0134
19	0.9635	1.0995	1.1183	1.1832	1.1102	1.0909	1.0093	0.9752	1.0468
20	0.9545	1.0773	1.0652	0.9652	1.0658	1.0238	1.0880	1.0713	1.0467
21	0.9938	1.0688	1.0086	1.0087	0.9874	0.9507	1.0502	1.1704	0.9684
22	0.9679	0.9817	1.0558	0.9528	0.9804	0.9940	1.1206	0.9975	1.1138
23	0.8617	0.8139	1.2986	1.1134	1.1321	0.9964	1.1220	1.0399	1.0299
24	0.9690	1.1211	1.0139	0.9106	1.0234	1.0658	1.0740	1.0541	1.0527
25	0.9043	1.1952	1.0174	0.9991	0.9962	1.0377	1.1412	1.0019	1.1288
26	1.0124	1.0128	1.0163	0.9583	1.1188	0.9612	1.3309	1.1332	1.0743
27	1.0349	1.1119	1.0590	0.8566	1.0308	1.0091	1.0944	0.9785	1.0981
28	0.9935	1.0796	1.0134	0.9702	1.0235	1.0093	1.0773	1.1174	1.1105
29	0.9734	1.0408	1.0758	0.9924	1.0873	1.0484	0.9939	1.0145	1.0849
30	1.0124	1.1167	1.0478	0.8632	1.1457	1.0475	1.0570	1.1059	1.0219
31	0.8462	1.1098	1.0355	0.8608	1.0536	1.0895	1.1064	0.8620	1.0022
32	0.9840	1.0744	1.0719	1.0399	1.0655	1.0009	1.1348	1.0094	0.9961
33	1.0728	1.0395	1.0625	1.0350	1.0074	1.0815	1.1289	1.0224	1.0923
34	0.9990	1.1093	1.0845	0.9492	0.9500	1.0505	1.1127	1.1316	1.0713
35	1.1680	0.8798	1.0225	0.9486	1.0160	0.9948	0.9982	1.0191	0.9237
36	1.0067	1.0360	1.0536	0.7851	1.0067	0.9296	1.1106	1.0100	0.9790

(*continued on next page*)

Table B4 (*continued*)

ID	80-81	81-82	82-83	83-84	84-85	85-86	86-87	87-88	88-89
37	1.0566	1.1094	0.9871	0.9627	1.0347	0.8119	1.3476	0.9776	1.0740
38	0.9412	1.0748	1.0639	0.7756	0.9566	1.0082	1.0208	1.0297	1.0236
39	0.9735	1.0820	1.1092	0.8981	0.9736	1.0230	1.1921	1.0138	1.0118
40	1.0461	1.0910	1.0593	0.8660	1.2546	1.0659	1.0886	1.0344	1.1143
41	1.1914	0.9557	1.0144	0.8742	0.9934	1.0798	0.9908	0.8973	0.9984
42	1.0505	1.1728	1.0302	0.9537	1.0516	0.9611	1.0034	1.0661	0.9904
GM*	1.0116	1.0764	1.0402	0.9572	1.0379	1.0082	1.0824	1.0261	1.0317

* Geometric means of the sample

Table B5
Output changes for 42 Swedish pharmacies between 1980 and 1989.

ID	80-81	81-82	82-83	83-84	84-85	85-86	86-87	87-88	88-89	80-89
1	0.9203	1.0837	1.0934	1.0745	1.0877	0.8870	1.0627	1.0022	1.0123	1.2188
2	1.0209	1.1444	0.9658	0.9532	1.0367	1.0264	1.1520	1.0916	1.0136	1.4589
3	1.0074	1.0667	1.0321	0.9323	1.0185	1.0659	1.0397	0.9998	1.0442	1.2184
4	0.9880	1.0027	1.0263	1.0367	1.0085	0.9987	1.0162	1.0000	1.0198	1.1001
5	1.0503	1.0236	1.0769	0.8977	1.0086	0.9965	1.0717	1.0384	1.0262	1.1928
6	1.0477	1.0736	1.0646	1.0110	1.0870	1.0407	1.0656	1.1077	1.0300	1.6647
7	0.9952	1.0730	0.9857	0.9998	1.0567	1.0348	1.0744	1.0786	1.0377	1.3837
8	1.0229	1.0548	1.0576	1.0298	1.0679	1.2050	0.9207	1.0692	1.0329	1.5375
9	1.0215	1.0884	1.0000	1.0392	0.9588	1.0129	1.0414	1.0247	1.0348	1.2393
10	1.0158	1.0502	1.0436	0.6964	1.0968	1.0503	1.0792	1.0530	1.0225	1.0380
11	0.9963	1.0482	1.1042	1.0330	1.0803	1.0582	1.0448	1.0279	1.0581	1.5474
12	0.9873	1.1045	1.0654	0.9176	1.0186	1.0450	1.1805	0.8735	1.0317	1.2070
13	1.0017	1.0812	0.9833	0.9841	0.9805	1.0102	1.0482	1.0136	0.9413	1.0382
14	1.2389	1.2097	1.0237	0.9350	0.9600	1.0267	1.0227	1.0644	1.0034	1.5444
15	0.9940	1.0377	0.9842	0.9686	1.0895	1.0182	1.0665	0.9268	1.0490	1.1312
16	0.9835	1.0691	1.1055	0.8600	1.0490	1.0228	1.0495	1.0650	1.0077	1.2079
17	1.0903	0.7906	1.4970	0.9019	1.0432	1.0172	1.0897	0.7670	0.5232	0.5400
18	0.9996	1.5009	0.7284	0.7154	1.0356	1.3783	1.1133	1.0078	1.0197	1.2768
19	0.9368	1.0629	1.2600	1.1201	1.0894	1.0812	0.9688	0.8870	0.9467	1.3467
20	0.9474	1.0923	1.0631	0.7188	1.0522	1.0543	1.0522	1.0887	1.0351	1.0401
21	0.9782	1.0826	1.0343	0.9893	1.0001	0.9684	0.9778	1.1366	0.9007	1.0505
22	0.9802	0.9942	1.0706	0.9653	1.0009	1.0369	1.1386	1.0288	1.0446	1.2790
23	0.4072	0.7291	2.9377	1.5426	1.0934	1.0350	1.0616	1.0605	1.0350	1.7739
24	0.9701	1.0560	1.0326	0.9280	0.9812	1.0382	1.0552	1.0897	1.0640	1.2235
25	0.8653	1.0294	1.0418	0.9979	0.9898	1.0456	1.1332	1.0495	1.0344	1.1790
26	0.9984	0.9777	0.9743	0.9868	1.0758	1.1335	1.6104	1.1215	1.0644	2.2001
27	1.0028	1.1078	1.0198	0.7547	1.0216	1.0448	1.0327	0.9898	1.0786	1.0061
28	0.9971	1.0874	1.0423	1.0106	1.0183	1.0363	1.0189	1.0516	1.0390	1.3417
29	0.9770	0.9956	1.0567	0.8760	1.0665	1.0505	0.9954	1.0045	1.0338	1.0427
30	1.0593	1.1545	1.2037	0.6926	1.1115	1.0541	0.9867	1.0448	1.0434	1.2847
31	0.9910	1.0996	1.0534	0.7498	1.0452	1.0905	1.0576	0.8046	0.9940	0.8296
32	1.0109	1.0445	1.0626	0.9326	1.0718	1.0338	1.0967	1.0537	0.8933	1.1969
33	1.0243	1.0248	1.0663	0.8921	1.0278	1.0644	1.0490	1.0351	1.0541	1.2503
34	0.9251	1.0432	1.0685	0.9173	0.8854	0.9842	1.0480	1.1517	1.0335	1.0282
35	1.0665	0.9388	1.0276	0.8713	1.0404	1.0076	1.0070	1.0334	0.9482	0.9274
36	0.9935	1.0306	1.0215	0.7165	1.0268	0.9963	1.1038	1.0207	0.9850	0.8509
37	1.0241	1.1998	1.0485	0.8900	1.0358	0.9440	1.2874	0.9803	1.0237	1.4484
38	0.9921	1.1492	1.0630	0.6775	0.9566	1.0014	0.9971	1.0491	1.0173	0.8370
39	1.0249	1.0564	1.1509	0.7885	1.0376	1.0360	1.1410	1.0412	0.9652	1.2111
40	1.0482	1.0844	1.0569	0.8108	1.0151	1.0181	1.0740	1.0229	1.0269	1.1356
41	1.2553	0.9737	1.0369	0.8269	1.0070	1.0623	1.0056	0.9035	0.9690	0.9871
42	1.0268	1.2022	1.0368	0.9313	1.0648	0.9466	1.0129	1.0237	0.9955	1.2402
GM*	0.9880	1.0589	1.0782	0.9071	1.0323	1.0372	1.0683	1.0178	0.9982	1.1890

* Geometric means of the sample.

Table B6
Input changes for 42 Swedish pharmacies between 1980 and 1989.

ID	8081	8182	8283	8384	8485	8586	8687	8788	8889	8089
1	0.9506	1.0688	0.9401	0.9734	1.0668	0.9709	0.9903	0.9806	0.9675	0.9047
2	0.9984	1.0365	1.0300	0.9324	1.0390	1.4238	0.9634	0.9929	0.9690	1.3626
3	1.0216	1.0231	1.0198	0.9666	0.9960	1.0503	0.9840	1.0084	0.9821	1.0504
4	0.9769	1.0015	0.9991	1.0284	0.9865	1.0437	0.9404	0.9362	0.9767	0.8901
5	1.0035	1.0066	1.0210	0.9848	0.9832	0.9985	1.0180	0.9849	1.0026	1.0023
6	1.0193	0.9879	0.9443	1.1055	1.0010	1.0145	0.9909	1.0229	1.0057	1.0880
7	0.9675	0.9208	1.0121	0.9895	1.0110	1.0408	0.9745	0.9132	1.0567	0.8828
8	0.6912	1.0213	0.9834	0.9418	1.0181	1.0430	0.9994	1.0219	0.9738	0.6905
9	1.0179	0.9768	1.0099	1.0080	0.9782	1.0038	0.9969	0.9883	0.9890	0.9685
10	0.9939	0.9828	1.0041	0.6729	0.9547	1.0043	1.0206	1.0041	0.9507	0.6165

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Table B6 (*continued*)

ID	8081	8182	8283	8384	8485	8586	8687	8788	8889	8089
11	1.0136	0.9511	0.8885	1.1369	1.0199	1.0000	1.0340	1.0078	0.8942	0.9256
12	0.9902	0.6272	1.6353	0.9594	0.9522	1.0371	1.0390	0.9360	0.9573	0.8959
13	1.0221	0.9887	1.0030	1.0034	1.0125	0.9893	0.9774	0.9957	0.9825	0.9740
14	1.2121	1.0334	0.9828	0.9529	1.0396	0.9882	0.9712	0.9584	1.0217	1.1462
15	0.9858	0.9957	1.0258	0.9706	1.0214	1.0317	1.0322	1.1181	0.9606	1.1417
16	0.9785	0.9656	1.0225	0.9046	1.0386	1.0068	1.0100	0.9876	0.9537	0.8694
17	1.0225	0.8875	1.1770	0.9676	0.9971	1.0162	1.0083	0.8991	0.8051	0.7643
18	0.9862	1.2218	0.8181	0.8595	0.9771	1.1678	1.0174	0.9962	1.0062	0.9858
19	0.9723	0.9667	1.1266	0.9467	0.9813	0.9911	0.9599	0.9096	0.9044	0.7700
20	0.9925	1.0139	0.9980	0.7447	0.9873	1.0298	0.9671	1.0163	0.9889	0.7390
21	0.9843	1.0129	1.0255	0.9807	1.0128	1.0186	0.9311	0.9711	0.9301	0.8700
22	1.0127	1.0127	1.0141	1.0131	1.0209	1.0432	1.0161	1.0313	0.9378	1.1027
23	0.4725	0.8958	2.2622	1.3855	0.9659	1.0387	0.9462	1.0197	1.0049	1.2905
24	1.0012	0.9420	1.0184	1.0191	0.9588	0.9742	0.9824	1.0338	1.0107	0.9384
25	0.9568	0.8613	1.0240	0.9988	0.9936	1.0076	0.9930	1.0476	0.9164	0.8044
26	0.9861	0.9653	0.9587	1.0298	0.9616	1.1793	1.2100	0.9897	0.9908	1.2644
27	0.9689	0.9963	0.9630	0.8810	0.9910	1.0354	0.9436	1.0116	0.9822	0.7879
28	1.0036	1.0073	1.0285	1.0417	0.9949	1.0268	0.9458	0.9411	0.9356	0.9213
29	1.0036	0.9566	0.9822	0.8827	0.9808	1.0020	1.0016	0.9902	0.9528	0.7730
30	1.0463	1.0338	1.1488	0.8023	0.9701	1.0063	0.9335	0.9447	1.0210	0.8763
31	1.1711	0.9908	1.0173	0.8711	0.9920	1.0009	0.9558	0.9334	0.9918	0.9035
32	1.0274	0.9722	0.9913	0.8967	1.0059	1.0329	0.9664	1.0439	0.8968	0.8347
33	0.9548	0.9858	1.0036	0.8619	1.0202	0.9843	0.9292	1.0124	0.9650	0.7422
34	0.9260	0.9404	0.9853	0.9664	0.9321	0.9369	0.9419	1.0178	0.9646	0.6696
35	0.9131	1.0671	1.0050	0.9185	1.0241	1.0129	1.0088	1.0141	1.0265	0.9798
36	0.9869	0.9948	0.9695	0.9126	1.0200	1.0718	0.9939	1.0107	1.0062	0.9598
37	0.9692	1.0815	1.0622	0.9244	1.0010	1.1627	0.9554	1.0027	0.9532	1.0939
38	1.0540	1.0692	0.9992	0.8734	1.0000	0.9933	0.9768	1.0189	0.9939	0.9663
39	1.0528	0.9763	1.0376	0.8780	1.0657	1.0127	0.9572	1.0271	0.9539	0.9477
40	1.0020	0.9939	0.9977	0.9363	0.8091	0.9552	0.9866	0.9888	0.9216	0.6464
41	1.0537	1.0188	1.0223	0.9459	1.0137	0.9838	1.0149	1.0069	0.9705	1.0268
42	0.9775	1.0251	1.0065	0.9765	1.0126	0.9849	1.0095	0.9602	1.0051	0.9569
GM*	0.9766	0.9838	1.0365	0.9477	0.9946	1.0287	0.9870	0.9919	0.9675	0.9146

* Geometric means of the sample.

Table B7
Efficiency changes for 42 Swedish pharmacies between 1980 and 1989.

ID	80-81	81-82	82-83	83-84	84-85	85-86	86-87	8788	88-89	80-89
1	1.0022	0.8813	1.1521	1.0994	0.9610	0.9946	1.0205	1.0253	1	1.1187
2	1	1	1	1	1	0.9806	1.0198	1	1	1
3	1	0.9454	1.0577	1	1	1	1	1	0.8967	0.8967
4	1.0085	0.9131	1.0327	0.9972	0.9817	0.9483	0.9926	1.0541	1.0206	0.9427
5	1.0687	0.7166	1.1821	1.1876	0.9659	0.9279	0.9990	1.0348	0.9756	0.9717
6	1.0442	0.8887	1.2019	0.9176	1.0110	0.9984	0.9593	1.0088	0.9667	0.9664
7	1.1588	1.0051	0.9380	1.0833	0.9984	0.9720	0.9946	1.0897	1	1.2448
8	1.5817	0.7598	1.3162	1	1	1	1	1	1	1.5817
9	0.9708	0.9708	1.0064	1.2077	0.9910	0.9116	0.9705	0.9959	0.9971	0.9973
10	1.0168	0.8248	1.2097	0.8932	1.0986	0.9667	1.0317	1.0109	1.0408	1.0447
11	0.9657	0.9299	1.2462	0.8940	1.0305	1.0175	1.0025	1.0138	1.0496	1.1191
12	0.9949	1.0361	1	0.9714	0.9696	0.9567	1.0532	0.8924	1.0342	0.9028
13	1	1	1	0.9857	0.9428	1.0446	0.9734	1.0583	1	1
14	1.0226	1	1	1	1	1	1	1	1	1.0226
15	1	1	1	1	1	1	1	1	1	1
16	1.0317	0.9457	1.1174	1.0010	0.9482	0.9937	0.9594	1.0376	1.0233	1.0474
17	1	0.9347	1.0698	1	1	1	1	0.9989	0.9990	0.9979
18	0.9914	1.0087	1	0.8827	1.0663	1.0373	0.9851	1.0396	0.9131	0.9131
19	1.1692	1	0.9760	1.0246	1	1	1	1	1	1.1692
20	0.9634	1.0013	1.0210	1.0154	1	1	1	1	1	1
21	1	0.8986	1.0704	0.9956	0.9098	0.9030	1.0122	1.1707	0.9927	0.9255
22	0.9306	0.8545	1.1615	0.9298	0.9622	0.9710	1.0481	1.0203	1.0519	0.9025
23	1	0.8786	1.1382	1	1	1	1	1	1	1
24	1.0838	0.9347	1.0139	0.9430	1.0172	1.0803	0.9405	1.0831	0.9229	1.0007
25	0.9917	0.9336	1.0126	1.0666	0.9386	1.0654	1	1	1	1
26	1	1	1	1	1	0.9363	1.0680	1	1	1
27	1.0908	0.9264	1.0013	0.9000	1.0228	0.9368	1.0401	0.9869	1.0658	0.9547
28	1	0.8143	1.1384	0.9974	0.9691	0.9845	1.0040	1.0587	1.0665	1
29	0.9352	0.8031	1.1935	1.0476	0.9695	0.9879	0.9943	0.9678	1.1612	1.0050
30	1.0374	1	1	0.8570	1.0630	0.9925	1.0488	1.0545	1	1.0374
31	1	1	1	0.8428	0.9912	1.0172	1.0553	0.9703	1.0057	0.8751
32	1	1	1	1	1	1	1	1	1	1
33	1	1	1	1	1	1	1	1	1	1
34	1.0839	1	1	1	1	1	0.8548	1.0888	1.0496	1.0587
35	1	1	1	1	1	1	1	1	1	1

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Table B7 (*continued*)

ID	80-81	81-82	82-83	83-84	84-85	85-86	86-87	8788	88-89	80-89
36	1	1	1	0.9057	1.0517	1.0498	1	1	1	1
37	1.1145	0.9227	0.9912	1.0098	1.0251	0.7797	1.3060	0.9867	1.0819	1.1469
38	1	1	1	0.9813	1.0191	1	1	0.9721	0.9338	0.9077
39	1	1	1	1	1	1	1	1	1	1
40	0.8441	0.8561	1.0968	0.9782	1.3138	0.9892	0.9970	1.0184	1.0138	1.0371
41	1	0.9266	0.9930	0.9668	1.0103	1.1127	0.8243	0.9030	1.0625	0.7908
42	1	0.9063	1.1034	1	1	0.9509	0.9896	1.0609	0.9519	0.9504
GM*	1.0220	0.9353	1.0548	0.9875	1.0039	0.9868	1.0015	1.0133	1.0056	1.0065

* Geometric means of the sample

Table B8
Local technical changes for 42 Swedish pharmacies between 1980 and 1989.

ID	80-81	81-82	82-83	83-84	84-85	85-86	86-87	87-88	88-89	80-89
1	0.9222	0.9058	1.1576	1.2356	0.9952	0.9190	1.0487	1.0096	1.0146	1.1739
2	0.9740	0.9864	0.9333	1.1444	0.9740	0.7251	1.1686	1.0860	1.0144	0.9330
3	0.9393	0.9314	1.0073	1.0796	0.9981	1.0207	1.0326	0.9795	1.0309	1.0108
4	0.9634	0.8945	1.0223	1.1284	0.9978	0.9626	1.0560	1.0551	1.0125	1.0771
5	0.9970	0.9085	1.0497	1.0204	1.0014	1.0038	1.0288	1.0415	0.9925	1.0371
6	0.9791	0.9709	1.1220	1.0237	1.0600	1.0319	1.0510	1.0697	0.9931	1.3334
7	0.9798	1.0410	0.9693	1.1311	1.0203	1.0000	1.0774	1.1668	0.9522	1.3658
8	1.4096	0.9228	1.0704	1.2240	1.0238	1.1620	0.9003	1.0335	1.0285	1.9404
9	0.9560	0.9955	0.9855	1.1540	0.9567	1.0150	1.0210	1.0243	1.0146	1.1151
10	0.9735	0.9547	1.0344	1.1585	1.1214	1.0519	1.0334	1.0360	1.0429	1.4672
11	0.9363	0.9845	1.2368	1.0171	1.0339	1.0644	0.9875	1.0076	1.1474	1.4568
12	0.9497	1.5732	0.6484	1.0706	1.0441	1.0135	1.1103	0.9219	1.0450	1.1741
13	0.9335	0.9770	0.9758	1.0979	0.9453	1.0272	1.0480	1.0057	0.9290	0.9289
14	0.9736	1.0458	1.0366	1.0983	0.9014	1.0450	1.0291	1.0972	0.9523	1.1741
15	0.9604	0.9310	0.9549	1.1172	1.0412	0.9927	1.0098	0.8189	1.0589	0.8634
16	0.9574	0.9891	1.0761	1.0642	0.9859	1.0218	1.0155	1.0652	1.0246	1.2108
17	1.0157	0.7958	1.2659	1.0434	1.0212	1.0069	1.0562	0.8428	0.6301	0.6158
18	0.9655	1.0975	0.8862	0.9318	1.0346	1.1872	1.0695	0.9993	0.9827	1.1287
19	0.9178	0.9823	1.1131	1.3244	1.0836	1.0972	0.9864	0.9633	1.0150	1.5242
20	0.9092	0.9624	1.0602	1.0805	1.0403	1.0298	1.0633	1.0583	1.0150	1.2266
21	0.9466	0.9549	1.0038	1.1292	0.9638	0.9562	1.0263	1.1562	0.9390	1.0522
22	0.9220	0.8770	1.0508	1.0666	0.9570	0.9998	1.0952	0.9854	1.0801	1.0107
23	0.8208	0.7272	1.2925	1.2463	1.1050	1.0023	1.0965	1.0273	0.9986	1.1979
24	0.9230	1.0016	1.0091	1.0194	0.9990	1.0720	1.0496	1.0413	1.0208	1.1362
25	0.8614	1.0678	1.0126	1.1184	0.9724	1.0438	1.1153	0.9897	1.0945	1.2772
26	0.9644	0.9048	1.0115	1.0727	1.0921	0.9668	1.3007	1.1194	1.0418	1.5164
27	0.9858	0.9934	1.0540	0.9589	1.0062	1.0150	1.0695	0.9666	1.0648	1.1128
28	0.9464	0.9645	1.0087	1.0860	0.9991	1.0152	1.0528	1.1039	1.0768	1.2692
29	0.9272	0.9299	1.0708	1.1109	1.0613	1.0546	0.9713	1.0022	1.0520	1.1755
30	0.9644	0.9977	1.0428	0.9663	1.1184	1.0536	1.0330	1.0925	0.9909	1.2775
31	0.8060	0.9915	1.0306	0.9636	1.0284	1.0959	1.0813	0.8515	0.9718	0.8003
32	0.9373	0.9599	1.0668	1.1641	1.0400	1.0068	1.1091	0.9971	0.9659	1.2496
33	1.0219	0.9287	1.0575	1.1586	0.9833	1.0878	1.1033	1.0100	1.0592	1.4680
34	0.9515	0.9910	1.0794	1.0625	0.9273	1.0566	1.0874	1.1179	1.0388	1.3381
35	1.1126	0.7860	1.0177	1.0619	0.9917	1.0006	0.9755	1.0067	0.8957	0.8248
36	0.9590	0.9256	1.0486	0.8789	0.9826	0.9350	1.0854	0.9977	0.9493	0.7726
37	1.0065	0.9911	0.9825	1.0777	1.0100	0.8166	1.3170	0.9658	1.0414	1.1539
38	0.8966	0.9602	1.0589	0.8682	0.9338	1.0141	0.9976	1.0172	0.9925	0.7548
39	0.9273	0.9667	1.1040	1.0053	0.9503	1.0290	1.1650	1.0015	0.9811	1.1137
40	0.9965	0.9747	1.0543	0.9694	1.2246	1.0721	1.0639	1.0219	1.0805	1.5311
41	1.1348	0.8538	1.0096	0.9786	0.9696	1.0861	0.9683	0.8864	0.9682	0.8377
42	1.0006	1.0478	1.0253	1.0676	1.0264	0.9667	0.9806	1.0532	0.9604	1.1295
GM*	0.9636	0.9616	1.0353	1.0716	1.0131	1.0141	1.0578	1.0137	1.0004	1.1330

* Geometric means of the sample.

Annex. C

```
# This code estimates the Standard Total Factor Productivity Index (STFPI).
# The STFPI is a multiplicatively complete index whose change can be also decomposed in:
# Efficiency Change, Global Technical Change and Local Technical Change
# The code is applied to the database of 42 Swedish pharmacies described in
# Färe et al. (1992) and Althin (2001).
# This Methodology was introduced by Juan Aparicio and Daniel Santín in the European
# Journal of Operational Research
# Loading 'Benchmarking' for calculating the required distances library(Benchmarking) library(optimbase) library(readxl) library(openxlsx) library(uniformly)
# To replicate the same results that appear in Aparicio and Santín's EJOR paper
# set this seed number set.seed(8012009)
```

```
######### STEP1 – IMPORT DATA - #############
# Data can be downloaded from mendeley (pharmacy.xlsx)
# https://data.mendeley.com/datasets/xznv43bvjp/1
# Importing data from excel
# This database has the 42 Swedish pharmacies production activity
# between 1980 and 1989 that appeared in the seminal paper by Färe et al. (1992)
# Write in your own script the right path to the file in your laptop. d80 <- read_excel("C:/Daniel/EM/pharmacy.xlsx", sheet = "R80") d81 <- read_excel("C:/Daniel/EM/pharmacy.xlsx", sheet = "R81") d82 <- read_excel("C:/Daniel/EM/pharmacy.xlsx", sheet = "R82") d83 <- read_excel("C:/Daniel/EM/pharmacy.xlsx", sheet = "R83") d84 <- read_excel("C:/Daniel/EM/pharmacy.xlsx", sheet = "R84") d85 <- read_excel("C:/Daniel/EM/pharmacy.xlsx", sheet = "R85") d86 <- read_excel("C:/Daniel/EM/pharmacy.xlsx", sheet = "R86") d87 <- read_excel("C:/Daniel/EM/pharmacy.xlsx", sheet = "R87") d88 <- read_excel("C:/Daniel/EM/pharmacy.xlsx", sheet = "R88") d89 <- read_excel("C:/Daniel/EM/pharmacy.xlsx", sheet = "R89")
######### STEP 2 – NORMALIZATION - ##############
# Define inputs and outputs and normalize information by the maximum.
# Output and inputs. This information is detailed in Althin (2001, Table 3)
# 4 inputs and 4 outputs
# Inputs: ’x1-pharmacists’, ’x2-technical staff’, ’x3-equipment services’,
# ’x4-building services’
# Outputs: ’y1-drug deliveries to hospitals’, ’y2-drugs for outpatient care’.
# ’y3-Medical appliances for the handicapped’ and ’y4-over the counter goods’. x80=matrix(c(d80$x1, d80$x2, d80$x3, d80$x4), nrow=42, ncol=4) y80=matrix(c(d80$y1, d80$y2, d80$y3, d80$y4), nrow=42,ncol=4) x81=matrix(c(d81$x1, d81$x2, d81$x3, d81$x4), nrow=42,ncol=4) y81=matrix(c(d81$y1, d81$y2, d81$y3, d81$y4), nrow=42,ncol=4) x82=matrix(c(d82$x1, d82$x2, d82$x3, d82$x4), nrow=42,ncol=4) y82=matrix(c(d82$y1, d82$y2, d82$y3, d82$y4), nrow=42,ncol=4) x83=matrix(c(d83$x1, d83$x2, d83$x3, d83$x4), nrow=42,ncol=4) y83=matrix(c(d83$y1, d83$y2, d83$y3, d83$y4), nrow=42,ncol=4) x84=matrix(c(d84$x1, d84$x2, d84$x3, d84$x4), nrow=42,ncol=4) y84=matrix(c(d84$y1, d84$y2, d84$y3, d84$y4), nrow=42,ncol=4) x85=matrix(c(d85$x1, d85$x2, d85$x3, d85$x4), nrow=42,ncol=4) y85=matrix(c(d85$y1, d85$y2, d85$y3, d85$y4), nrow=42,ncol=4) x86=matrix(c(d86$x1, d86$x2, d86$x3, d86$x4), nrow=42,ncol=4) y86=matrix(c(d86$y1, d86$y2, d86$y3, d86$y4), nrow=42,ncol=4) x87=matrix(c(d87$x1, d87$x2, d87$x3, d87$x4), nrow=42,ncol=4) y87=matrix(c(d87$y1, d87$y2, d87$y3, d87$y4), nrow=42,ncol=4) x88=matrix(c(d88$x1, d88$x2, d88$x3, d88$x4), nrow=42,ncol=4) y88=matrix(c(d88$y1, d88$y2, d88$y3, d88$y4), nrow=42,ncol=4) x89=matrix(c(d89$x1, d89$x2, d89$x3, d89$x4), nrow=42,ncol=4) y89=matrix(c(d89$y1, d89$y2, d89$y3, d89$y4), nrow=42,ncol=4)
# Reading IDs, DMUs are sorted in the same way in all periods.
ID=matrix(c(d80$ID), nrow=42,ncol=1)
J=length(ID)
# Normalize the variables for the analysis
# For using the standard reference technology we have to look for every output
# the maximum observed output value over all periods.
# Building a matrix with all observed values for every output. y1=matrix(c(d80$y1, d81$y1, d82$y1, d83$y1, d84$y1, d85$y1, d86$y1, d87$y1, d88$y1, d89$y1), nrow=42,ncol=10) y2=matrix(c(d80$y2, d81$y2, d82$y2, d83$y2, d84$y2, d85$y2, d86$y2, d87$y2, d88$y2, d89$y2), nrow=42,ncol=10) y3=matrix(c(d80$y3, d81$y3, d82$y3, d83$y3, d84$y3, d85$y3, d86$y3, d87$y3, d88$y3, d89$y3), nrow=42,ncol=10) y4=matrix(c(d80$y4, d81$y4, d82$y4, d83$y4, d84$y4, d85$y4, d86$y4, d87$y4, d88$y4, d89$y4), nrow=42,ncol=10)
# Seeking the maximum output value for all outputs. y1max=max(y1) y2max=max(y2) y3max=max(y3) y4max=max(y4)
# Normalizing the outputs with respect to the observed maximum values in any period d80$y1m=d80$y1/y1max d81$y1m=d81$y1/y1max d82$y1m=d82$y1/y1max d83$y1m=d83$y1/y1max d84$y1m=d84$y1/y1max d85$y1m=d85$y1/y1max d86$y1m=d86$y1/y1max d87$y1m=d87$y1/y1max d88$y1m=d88$y1/y1max d89$y1m=d89$y1/y1max d80$y2m=d80$y2/y2max d81$y2m=d81$y2/y2max d82$y2m=d82$y2/y2max d83$y2m=d83$y2/y2max d84$y2m=d84$y2/y2max d85$y2m=d85$y2/y2max d86$y2m=d86$y2/y2max d87$y2m=d87$y2/y2max d88$y2m=d88$y2/y2max d89$y2m=d89$y2/y2max d80$y3m=d80$y3/y3max d81$y3m=d81$y3/y3max d82$y3m=d82$y3/y3max d83$y3m=d83$y3/y3max d84$y3m=d84$y3/y3max d85$y3m=d85$y3/y3max d86$y3m=d86$y3/y3max d87$y3m=d87$y3/y3max d88$y3m=d88$y3/y3max d89$y3m=d89$y3/y3max d80$y4m=d80$y4/y4max d81$y4m=d81$y4/y4max d82$y4m=d82$y4/y4max d83$y4m=d83$y4/y4max d84$y4m=d84$y4/y4max d85$y4m=d85$y4/y4max d86$y4m=d86$y4/y4max d87$y4m=d87$y4/y4max d88$y4m=d88$y4/y4max d89$y4m=d89$y4/y4max
#For using the standard reference technology, we have to look for every input
#the maximum input value over all periods.
#Building a matrix with all observed input values for every input. x1=matrix(c(d80$x1, d81$x1, d82$x1, d83$x1, d84$x1, d85$x1, d86$x1, d87$x1, d88$x1, d89$x1), nrow=42,ncol=10) x2=matrix(c(d80$x2, d81$x2, d82$x2, d83$x2, d84$x2, d85$x2, d86$x2, d87$x2, d88$x2, d89$x2), nrow=42,ncol=10) x3=matrix(c(d80$x3, d81$x3, d82$x3, d83$x3, d84$x3, d85$x3, d86$x3, d87$x3, d88$x3, d89$x3), nrow=42,ncol=10) x4=matrix(c(d80$x4, d81$x4, d82$x4, d83$x4, d84$x4, d85$x4, d86$x4, d87$x4, d88$x4, d89$x4), nrow=42,ncol=10) x1max=max(x1) x2max=max(x2) x3max=max(x3) x4max=max(x4)
#Transforming input according to observed minimum input in all periods d80$x1m=d80$x1/x1max d81$x1m=d81$x1/x1max d82$x1m=d82$x1/x1max d83$x1m=d83$x1/x1max d84$x1m=d84$x1/x1max d85$x1m=d85$x1/x1max d86$x1m=d86$x1/x1max d87$x1m=d87$x1/x1max d88$x1m=d88$x1/x1max d89$x1m=d89$x1/x1max d80$x2m=d80$x2/x2max d81$x2m=d81$x2/x2max d82$x2m=d82$x2/x2max d83$x2m=d83$x2/x2max d84$x2m=d84$x2/x2max d85$x2m=d85$x2/x2max d86$x2m=d86$x2/x2max d87$x2m=d87$x2/x2max d88$x2m=d88$x2/x2max d89$x2m=d89$x2/x2max d80$x3m=d80$x3/x3max d81$x3m=d81$x3/x3max d82$x3m=d82$x3/x3max d83$x3m=d83$x3/x3max d84$x3m=d84$x3/x3max d85$x3m=d85$x3/x3max d86$x3m=d86$x3/x3max d87$x3m=d87$x3/x3max d88$x3m=d88$x3/x3max d89$x3m=d89$x3/x3max d80$x4m=d80$x4/x4max d81$x4m=d81$x4/x4max d82$x4m=d82$x4/x4max d83$x4m=d83$x4/x4max d84$x4m=d84$x4/x4max d85$x4m=d85$x4/x4max d86$x4m=d86$x4/x4max d87$x4m=d87$x4/x4max d88$x4m=d88$x4/x4max d89$x4m=d89$x4/x4max
# Now, we have the database with outputs and inputs normalized by maximum outputs values
# This means that for each output and input the maximum observed output value is 1
```

```
# and remaining DMUs have less than one and greater than zero values.
# Remember that DEA is unit invariant so analyzing these data we will obtain
# the same results than in Althin (2001).
# The 'm' indicates that data are 'modified' after the normalization. x80m=matrix(c(d80$x1m, d80$x2m, d80$x3m, d80$x4m), nrow=42,
ncol=4) y80m=matrix(c(d80$y1m, d80$y2m, d80$y3m, d80$y4m), nrow=42,ncol=4) x81m=matrix(c(d81$x1m, d81$x2m, d81$x3m, d81$x4m),
nrow=42,ncol=4) y81m=matrix(c(d81$y1m, d81$y2m, d81$y3m, d81$y4m), nrow=42,ncol=4) x82m=matrix(c(d82$x1m, d82$x2m, d82$x3m,
d82$x4m), nrow=42,ncol=4) y82m=matrix(c(d82$y1m, d82$y2m, d82$y3m, d82$y4m), nrow=42,ncol=4) x83m=matrix(c(d83$x1m, d83$x2m,
d83$x3m, d83$x4m), nrow=42,ncol=4) y83m=matrix(c(d83$y1m, d83$y2m, d83$y3m, d83$y4m), nrow=42,ncol=4) x84m=matrix(c(d84$x1m,
d84$x2m, d84$x3m, d84$x4m), nrow=42,ncol=4) y84m=matrix(c(d84$y1m, d84$y2m, d84$y3m, d84$y4m), nrow=42,ncol=4) x85m=matrix(c
(d85$x1m, d85$x2m, d85$x3m, d85$x4m), nrow=42,ncol=4) y85m=matrix(c(d85$y1m, d85$y2m, d85$y3m, d85$y4m), nrow=42,ncol=4)
x86m=matrix(c(d86$x1m, d86$x2m, d86$x3m, d86$x4m), nrow=42,ncol=4) y86m=matrix(c(d86$y1m, d86$y2m, d86$y3m, d86$y4m),
nrow=42,ncol=4) x87m=matrix(c(d87$x1m, d87$x2m, d87$x3m, d87$x4m), nrow=42,ncol=4) y87m=matrix(c(d87$y1m, d87$y2m, d87$y3m,
d87$y4m), nrow=42,ncol=4) x88m=matrix(c(d88$x1m, d88$x2m, d88$x3m, d88$x4m), nrow=42,ncol=4) y88m=matrix(c(d88$y1m, d88$y2m,
d88$y3m, d88$y4m), nrow=42,ncol=4) x89m=matrix(c(d89$x1m, d89$x2m, d89$x3m, d89$x4m), nrow=42,ncol=4) y89m=matrix(c(d89$y1m,
d89$y2m, d89$y3m, d89$y4m), nrow=42,ncol=4)
######### STEP 3 – THE MALMQUIST INDEX -##############
# Calculate now the necessary distances for the 'Efficiency Change' component
# The calculus coincides with the traditional adjacent Malmquist Index (MI)
# and base period MI decompositions
# First, we need to calculate DEAs for each period referred to its own period CRS technology
# We choose output oriented analysis
# Note that the notation in Aparicio and Santín's paper uses Shephard's measures while
# the library 'Benchmarking' uses Farrell's measures. For this reason we run
# the 'input orientation' for obtaining directly the 'output orientation' interpretation.
# This delivers the Färe et al.'s (1992) Efficiency Change results. dea80=dea(x80m, y80m, RTS="crs", ORIENTATION="in", XREF=NULL,
YREF=NULL) dea81=dea(x81m, y81m, RTS="crs", ORIENTATION="in", XREF=NULL, YREF=NULL) dea82=dea(x82m, y82m, RTS="crs", ORIENTATION="in", XREF=NULL, YREF=NULL) dea83=dea(x83m, y83m, RTS="crs", ORIENTATION="in", XREF=NULL, YREF=NULL) dea84=dea
(x84m, y84m, RTS="crs", ORIENTATION="in", XREF=NULL, YREF=NULL) dea85=dea(x85m, y85m, RTS="crs", ORIENTATION="in", XREF=NULL,
YREF=NULL) dea86=dea(x86m, y86m, RTS="crs", ORIENTATION="in", XREF=NULL, YREF=NULL) dea87=dea(x87m, y87m, RTS="crs", ORIENTATION="in", XREF=NULL, YREF=NULL) dea88=dea(x88m, y88m, RTS="crs", ORIENTATION="in", XREF=NULL, YREF=NULL) dea89=dea
(x89m, y89m, RTS="crs", ORIENTATION="in", XREF=NULL, YREF=NULL)
# Traditional efficiency change. These values are exactly equal than in
# Färe et al. (1992) and Althin (2001)
# EFFICIENCY CHANGE effc80=prod(dea80$eff)ˆ(1/J) effc81=prod(dea81$eff)ˆ(1/J) effc82=prod(dea82$eff)ˆ(1/J) effc83=prod(dea83$eff)ˆ
(1/J) effc84=prod(dea84$eff)ˆ(1/J) effc85=prod(dea85$eff)ˆ(1/J) effc86=prod(dea86$eff)ˆ(1/J) effc87=prod(dea87$eff)ˆ(1/J) effc88=prod(dea88
$eff)ˆ(1/J) effc89=prod(dea89$eff)ˆ(1/J) effc8081=effc81/effc80 effc8182=effc82/effc81 effc8283=effc83/effc82 effc8384=effc84/effc83
effc8485=effc85/effc84 effc8586=effc86/effc85 effc8687=effc87/effc86 effc8788=effc88/effc87 effc8889=effc89/effc88 effc8089=effc89/effc80
ec8081=dea81$eff/dea80$eff ec8182=dea82$eff/dea81$eff ec8283=dea83$eff/dea82$eff ec8384=dea84$eff/dea83$eff ec8485=dea85$eff/
dea84$eff ec8586=dea86$eff/dea85$eff ec8687=dea87$eff/dea86$eff ec8788=dea88$eff/dea87$eff ec8889=dea89$eff/dea88$eff ec8089=dea89
$eff/dea80$eff
########### STEP 4 – THE STANDARD - ############
# The standard reference technology productivity index is calculated by
# plug-in transformed inputs and outputs into the standard technology using
# Eq. (11) with the parameters in (15)
# m = Number of inputs; n = Number of outputs; m=4 n=4
# The constant elasticity of transformation function in the numerator is a (n-1)-Sphere y80num=sqrt(rowSums(y80mˆ2))*(1/(sqrt(n)))
y81num=sqrt(rowSums(y81mˆ2))*(1/(sqrt(n))) y82num=sqrt(rowSums(y82mˆ2))*(1/(sqrt(n))) y83num=sqrt(rowSums(y83mˆ2))*(1/(sqrt(n)))
y84num=sqrt(rowSums(y84mˆ2))*(1/(sqrt(n))) y85num=sqrt(rowSums(y85mˆ2))*(1/(sqrt(n))) y86num=sqrt(rowSums(y86mˆ2))*(1/(sqrt(n)))
y87num=sqrt(rowSums(y87mˆ2))*(1/(sqrt(n))) y88num=sqrt(rowSums(y88mˆ2))*(1/(sqrt(n))) y89num=sqrt(rowSums(y89mˆ2))*(1/(sqrt(n)))
# The constant elasticity of substitution function in the denominator
# is the inverse of a (m-1)-sphere x80denom=sqrt(rowSums(x80mˆ2))*(1/(sqrt(m))) x81denom=sqrt(rowSums(x81mˆ2))*(1/(sqrt(m)))
x82denom=sqrt(rowSums(x82mˆ2))*(1/(sqrt(m))) x83denom=sqrt(rowSums(x83mˆ2))*(1/(sqrt(m))) x84denom=sqrt(rowSums(x84mˆ2))*(1/(sqrt
(m))) x85denom=sqrt(rowSums(x85mˆ2))*(1/(sqrt(m))) x86denom=sqrt(rowSums(x86mˆ2))*(1/(sqrt(m))) x87denom=sqrt(rowSums(x87mˆ2))*
(1/(sqrt(m))) x88denom=sqrt(rowSums(x88mˆ2))*(1/(sqrt(m))) x89denom=sqrt(rowSums(x89mˆ2))*(1/(sqrt(m)))
# Productivity levels plevel80=y80num/x80denom plevel81=y81num/x81denom plevel82=y82num/x82denom plevel83=y83num/x83denom
plevel84=y84num/x84denom plevel85=y85num/x85denom plevel86=y86num/x86denom plevel87=y87num/x87denom plevel88=y88num/
x88denom plevel89=y89num/x89denom
############## STEP 5 -THE STANDARD TFP CHANGE INDEX - #############
# The Standard Reference Technology TFP Change Index
SI8180=plevel81/plevel80
SI8281=plevel82/plevel81
SI8382=plevel83/plevel82
SI8483=plevel84/plevel83
SI8584=plevel85/plevel84
SI8685=plevel86/plevel85
SI8786=plevel87/plevel86
```

```
SI8887=plevel88/plevel87
SI8988=plevel89/plevel88
SI8980=plevel89/plevel80
# Geometric means for each year
AVSI8180=prod(SI8180)^(1/J)
AVSI8281=prod(SI8281)^(1/J)
AVSI8382=prod(SI8382)^(1/J)
AVSI8483=prod(SI8483)^(1/J)
AVSI8584=prod(SI8584)^(1/J)
AVSI8685=prod(SI8685)^(1/J)
AVSI8786=prod(SI8786)^(1/J)
AVSI8887=prod(SI8887)^(1/J)
AVSI8988=prod(SI8988)^(1/J)
AVSI8980=prod(SI8980)^(1/J)
# OUTPUT CHANGE
OC8180=y81num/y80num
OC8281=y82num/y81num
OC8382=y83num/y82num
OC8483=y84num/y83num
OC8584=y85num/y84num
OC8685=y86num/y85num
OC8786=y87num/y86num
OC8887=y88num/y87num
OC8988=y89num/y88num
OC8980=y89num/y80num
# Geometric means for output change
AVOC8180=prod(OC8180)^(1/J)
AVOC8281=prod(OC8281)^(1/J)
AVOC8382=prod(OC8382)^(1/J)
AVOC8483=prod(OC8483)^(1/J)
AVOC8584=prod(OC8584)^(1/J)
AVOC8685=prod(OC8685)^(1/J)
AVOC8786=prod(OC8786)^(1/J)
AVOC8887=prod(OC8887)^(1/J)
AVOC8988=prod(OC8988)^(1/J)
AVOC8980=prod(OC8980)^(1/J)
# INPUT CHANGE
IC8180=x81denom/x80denom
IC8281=x82denom/x81denom
IC8382=x83denom/x82denom
IC8483=x84denom/x83denom
IC8584=x85denom/x84denom
IC8685=x86denom/x85denom
IC8786=x87denom/x86denom
IC8887=x88denom/x87denom
IC8988=x89denom/x88denom
IC8980=x89denom/x80denom
# Geometric means for input change
AVIC8180=prod(IC8180)^(1/J)
AVIC8281=prod(IC8281)^(1/J)
AVIC8382=prod(IC8382)^(1/J)
AVIC8483=prod(IC8483)^(1/J)
AVIC8584=prod(IC8584)^(1/J)
AVIC8685=prod(IC8685)^(1/J)
AVIC8786=prod(IC8786)^(1/J)
AVIC8887=prod(IC8887)^(1/J)
AVIC8988=prod(IC8988)^(1/J)
AVIC8980=prod(IC8980)^(1/J)
############### STEP 6 - THE SRTFPCI AND ITS DECOMPOSITION - ##############
# Numerical analysis to calculate the Global Technical Change (GTC)
# Generate a big K number of synthetic DMUs inside the unit-hypercube of dimension H.
# H=m+n:m = number of inputs; n = number of outputs. H = unit-hypercube size
# K = Number of synthetic DMUs inside the unit-hypercube H for calculating GTC
# The library ‘uniformly’ allows to generate the unit hypercube H in one step.
# Another possibility is generating independent uniform variables U(0,1) for each input
# and output and then combining all in a vector with m inputs and n outputs.
```

```
H=m+n
K=1000000 simH<-runif_in_cube(K, H)
Hh<-abs(simH)
Xh<-Hh[,1:4]
Yh<-Hh[,5:8]
# Now, project the K synthetic DMUs inside the hypercube against
# the two empirical technologies of every pair of periods assuming CRS
# (You can project more if you wish, it depends on the strength of your computer and your time)
# This information will be used for calculating
# average global technical change (GTC) and the local technical change (LTC).
# Global Technical Change (GTC) hcube80=dea(Xh, Yh, RTS="crs", ORIENTATION="in", XREF=x80m, YREF=y80m) hcube81=dea(Xh, Yh, RTS="crs", ORIENTATION="in", XREF=x81m, YREF=y81m) hcube82=dea(Xh, Yh, RTS="crs", ORIENTATION="in", XREF=x82m, YREF=y82m) hcube83=dea(Xh, Yh, RTS="crs", ORIENTATION="in", XREF=x83m, YREF=y83m) hcube84=dea(Xh, Yh, RTS="crs", ORIENTATION="in", XREF=x84m, YREF=y84m) hcube85=dea(Xh, Yh, RTS="crs", ORIENTATION="in", XREF=x85m, YREF=y85m) hcube86=dea(Xh, Yh, RTS="crs", ORIENTATION="in", XREF=x86m, YREF=y86m) hcube87=dea(Xh, Yh, RTS="crs", ORIENTATION="in", XREF=x87m, YREF=y87m) hcube88=dea(Xh, Yh, RTS="crs", ORIENTATION="in", XREF=x88m, YREF=y88m) hcube89=dea(Xh, Yh, RTS="crs", ORIENTATION="in", XREF=x89m, YREF=y89m)
Global80=10^(mean(log10(hcube80$eff),na.rm=TRUE))
Global81=10^(mean(log10(hcube81$eff),na.rm=TRUE))
Global82=10^(mean(log10(hcube82$eff),na.rm=TRUE))
Global83=10^(mean(log10(hcube83$eff),na.rm=TRUE))
Global84=10^(mean(log10(hcube84$eff),na.rm=TRUE))
Global85=10^(mean(log10(hcube85$eff),na.rm=TRUE))
Global86=10^(mean(log10(hcube86$eff),na.rm=TRUE))
Global87=10^(mean(log10(hcube87$eff),na.rm=TRUE))
Global88=10^(mean(log10(hcube88$eff),na.rm=TRUE))
Global89=10^(mean(log10(hcube89$eff),na.rm=TRUE))
# The GTC is calculated every two periods,
GTFPC8081=Global80/Global81
GTFPC8182=Global81/Global82
GTFPC8283=Global82/Global83
GTFPC8384=Global83/Global84
GTFPC8485=Global84/Global85
GTFPC8586=Global85/Global86
GTFPC8687=Global86/Global87
GTFPC8788=Global87/Global88
GTFPC8889=Global88/Global89
GTFPC8089=Global80/Global89
# The local technical change can be easily obtained as a residual:
LTC8081=SI8180/ (GTFPC8081*effc8081)
LTC8182=SI8281/ (GTFPC8182*effc8182)
LTC8283=SI8382/ (GTFPC8283*effc8283)
LTC8384=SI8483/ (GTFPC8384*effc8384)
LTC8485=SI8584/ (GTFPC8485*effc8485)
LTC8586=SI8685/ (GTFPC8586*effc8586)
LTC8687=SI8786/ (GTFPC8687*effc8687)
LTC8788=SI8887/ (GTFPC8788*effc8788)
LTC8889=SI8988/ (GTFPC8889*effc8889)
LTC8089=SI8980/ (GTFPC8089*effc8089)
AVLTC8081=prod(LTC8081)^(1/J)
AVLTC8182=prod(LTC8182)^(1/J)
AVLTC8283=prod(LTC8283)^(1/J)
AVLTC8384=prod(LTC8384)^(1/J)
AVLTC8485=prod(LTC8485)^(1/J)
AVLTC8586=prod(LTC8586)^(1/J)
AVLTC8687=prod(LTC8687)^(1/J)
AVLTC8788=prod(LTC8788)^(1/J)
AVLTC8889=prod(LTC8889)^(1/J)
AVLTC8089=prod(LTC8089)^(1/J)
############## STEP 7 – EXPORTING THE RESULTS - #############
### Dataframes for exporting the results
STFPI=data.frame(SI8180,SI8281,SI8382,SI8483,SI8584,SI8685,SI8786,SI8887,SI8988,SI8980)
AGGOUT=data.frame(y80num,y81num,y82num,y83num,y84num,y85num,y86num,y87num,y88num,y89num)
AGGIN=data.frame(x80denom,x81denom,x82denom,x83denom,x84denom,x85denom,x86denom,x87denom, x88denom,x89denom)
PRODLEVEL=data.frame(plevel80,plevel81,plevel82,plevel83,plevel84,plevel85,plevel86, plevel87,plevel88,plevel89)
# Output Change and Input Change
```

```
OCH=data.frame(OC8180,OC8281,OC8382,OC8483,OC8584,OC8685,OC8786,OC8887,OC8988,OC8980)
ICH=data.frame(IC8180,IC8281,IC8382,IC8483,IC8584,IC8685,IC8786,IC8887,IC8988,IC8980)
AVOC=data.frame(AVOC8180,AVOC8281,AVOC8382,AVOC8483,AVOC8584,AVOC8685,AVOC8786,
AVOC8887,AVOC8988,AVOC8980)
AVIC=data.frame(AVIC8180,AVIC8281,AVIC8382,AVIC8483,AVIC8584,AVIC8685,AVIC8786,
AVIC8887,AVIC8988,AVIC8980)
# GTC and LTC
GTC=data.frame(GTFPC8081,GTFPC8182,GTFPC8283,GTFPC8384,GTFPC8485,GTFPC8586,GTFPC8687,
GTFPC8788,GTFPC8889,GTFPC8089)
LTC=data.frame(LTC8081,LTC8182,LTC8283,LTC8384,LTC8485,LTC8586,LTC8687,LTC8788,
LTC8889,LTC8089)
AVLTC=data.frame(AVLTC8081,AVLTC8182,AVLTC8283,AVLTC8384,AVLTC8485,AVLTC8586,
AVLTC8687,AVLTC8788,AVLTC8889,AVLTC8089)
EC=data.frame(ec8081,ec8182,ec8283,ec8384,ec8485,ec8586, ec8687,ec8788,ec8889,ec8089)
#Finally, we export all results to excel to build Tables in Annex B.
SRTTFPI <- list(’Sheet1′ = STFPI, ’Sheet2′ = AGGOUT, ’Sheet3′ = AGGIN,
’Sheet4′ = PRODLEVEL)
OICHANGES <- list(’Sheet1′ = OCH, ’Sheet2′ = ICH, ’Sheet3′ = AVOC, ’Sheet4′ = AVIC)
TCDESC <- list(’Sheet1′ = GTC, ’Sheet2′ = LTC, ’Sheet3′ = AVLTC, ’Sheet4′ = EC)
```

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