Analytical calculation of the flow superposition effect on the power consumption in oscillatory baffled reactors

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Abstract

The aim of this communication is to clarify some aspects related to the power consumption in continuous Oscillatory Baffled Reactors (OBRs). The first aspect studied is the effect of the flow superimposition on the power consumptions associated to the net flow and the oscillatory flow. The quasi-steady model is used to obtain an expression for the oscillatory flow effect on the net flow power consumption, and vice versa. The expression obtained for the oscillatory flow effect is in good agreement with the one available in the literature, whose deduction is still unknown. The second aspect is the effect of the energy recovery in the power consumption. An analytical expression, function of the pressure drop-velocity phase lag, is derived, showing that the power consumption difference between a system with and without energy recovery can be significant.

Keywords: oscillatory baffled reactors, oscillatory flow, flow interaction, power density

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Nomenclature

A	cross sectional area of the tube (m^2), $\pi D^2/4$
D	tube inner diameter (m)
f	oscillation frequency (Hz)
F_n	factor for the influence of the oscillatory flow on the net pump power consumption
F_{osc}	factor for the influence of the net flow on the oscillator power consumption
F_{NER}	factor for the influence of no energy recovery on the oscillator power consumption
K_p	constant for the pressure drop losses along the test section ($Pa/(m^2/s^2)$)
Δp	instantaneous pressure drop (Pa)
Δp_{max}	pressure drop amplitude, considered as a perfect sine wave (Pa)
q_{max}	flow rate amplitude, considered as a perfect sine wave (m^3/s)
t	time (s)
T	period (s)
u_n	mean velocity of the net flow (m/s)
u_{osc}	mean instantaneous velocity of the oscillatory flow (m/s)
u	mean instantaneous overall velocity of the flow (m/s)
$\overline{W}$	averaged power consumption (W)
x_0	oscillation amplitude, center to peak (m)
Z	OBR length (m)

Dimensionless groups

Re_n	net Reynolds number, $\rho U_n D/\mu$
Re_{osc}	oscillatory Reynolds number, $\rho(2\pi f x_0)D/\mu$
Ψ	velocity ratio, Re_{osc}/Re_n

Greek symbols

δ	pressure drop-velocity phase lag (rad)
μ	dynamic viscosity (kg/(m·s))
ρ	fluid density (kg/m^3)
θ	phase (rad)
θ_0	phase at which the overall velocity is zero (rad)
ω	angular frequency (rad/s)

Subscripts

ER	considering energy recovery
NER	considering no energy recovery
n	related to the net flow pump
osc	related to the oscillator
0	without considering the effect of the flow superposition

1. Introduction

Oscillatory baffled reactors (OBRs) are a form of plug flow reactor, ideal for performing long reactions in continuous mode, as the mixing is independent of the net flow rate [1]. The superposition of an oscillating motion on a low velocity, net flow through a baffled tube creates a series of well-mixed volumes, which are responsible for the decoupling of plug flow from net flow velocity. A full review of the applications of oscillatory flow technology is presented in [2].

Several aspects of OBRs have been thoroughly analysed in the open literature, like mixing [3], heat transfer enhancement [4] and scaling-up [5]. However, the number of investigations dealing with the power consumption mechanisms in OBRs is comparatively scant, and only a few works based on CFD have addressed this problem in the last years [4, 6, 7]. In most of the cases, the approach for analysing the power consumption is based on neglecting the effect of the net flow rate. This is supported by the high oscillatory-to-net velocity ratios that are required in OBRs for achieving an adequate level of plug flow [1]. As a result of this approach, the most used empirical models for the analysis of power consumption in OBRs, namely the quasi-steady model [8] and the eddy enhancement model [9], take only into account the power consumption for the generation of the oscillatory motion (oscillator power consumption).

The crossed influence of net flow and oscillatory flow components on power consumption is, however, intuitive, and both contributions must be simultaneously considered for a proper modelling analysis in OBRs. A precursor work following this strategy was performed in 1984 by Noh and Baird [10], who analysed the net component of pressure drop in a cocurrent reciprocating plate extraction column. Applying the quasi-steady model, they derived a numerical expression for the averaged pressure drop in a flow with superimposed net and oscillatory components. The results showed an overestimation of the quasi-steady model for high net flow velocities, and an underestimation for low net flow rates when there is no oscillation. The authors proved a better performance of the model for the higher frequencies, which was explained by the more uniform orifice coefficient values found for the resulting higher Reynolds number regime. However, the expression proposed has been seldom adopted by other authors, apparently due to its relative complexity.

In 1995, Mackley and Stonestreet [11] introduced a factor to quantify the augmentation of the pumping power that sustains the net flow rate (net flow pumping power), when an oscillatory flow is superimposed (Equation 1).

$$F_n = \left(1 + \left(\frac{4\Psi}{\pi}\right)^3\right)^{\frac{1}{3}} \quad (1)$$

This factor is referenced in [11] as a personal communication with Prof. M.H.I. Baird, who authored a significant number of works in the field of pulsating columns [12, 13, 14]. It is striking that, in spite of being the most utilised approximation for the assessment of the interaction between net and oscillatory flows [7, 15, 16], the experimental or theoretical evidence of this expression is not available -to the best of our knowledge- in the open literature.

A complementary factor, able to evaluate the increased oscillator power consumption as a result of a net flow rate through the tube has not been devised so far. This aspect has been justified by the fact that, in OBRs, the oscillatory flow is much higher than the net flow (i.e. high velocity ratio). However, it would be interesting to quantify that statement.

Alternatively, recent studies [7] have proposed to nondimensionalize the power consumption by using the maximum velocity (net plus oscillatory) as the characteristic velocity. This total power is of high interest to correlate the mixing quality and the power density, and quantify the performance of the OBRs under given flow conditions. However, this approach is not able to distinguish the net flow pumping power from the oscillator power, which is an important aspect in order to design both systems.

Another aspect of interest, which has been barely treated in the modelling of power consumption in continuous OBRs, is the ability of the oscillation mechanism for absorbing and releasing mechanical energy. Jealous and Johnson [8] pointed out that this aspect could be relevant as, in a certain period of the oscillation cycle, the power demand is negative and therefore susceptible to storage. This takes place when the pressure drop and velocity signals have opposite signs. Hafez and Baird [13] explained that the ability of an oscillation system to store mechanical energy requires the inclusion of a flywheel or other highly inertial components. Baird and Stonestreet [9] discussed the possible energy recovery effects at the end of strokes of the oscillation piston of a continuous OBR, since velocity becomes null and a different sign between pressure and velocity could occur. The most extended expression for power dissipation in OBRs [17] accepts that the system is able to recover energy, even though the classical assemblies for the generation of oscillation in continuous OBRs are not designed to this aim.

The aim of this communication is to clarify, from a theoretical basis, some open questions exposed above related to the power demand in OBRs. Firstly, the quasi-steady model and a formulation based on the integral form of the momentum equation along an OBR is developed, in order to devise analytical expressions for the net flow pumping power, the oscillator power consumption and the consequent implications of the interactions between both components of the flow in continuous OBRs. Secondly, a mathematical expression is obtained for the evaluation of the oscillator power consumption in a pure oscillatory flow system without energy storage. It should be noticed that, while of interest, the calculation of the absolute power is out of the scope of this communication.

2. Oscillatory and net flows interaction

2.1. Physical model

Elements related to power consumption. A simplified schematic of the elements of a general continuous OBR is shown in Figure 1. A pump (1) sustains the net flow rate across the test section, and a double acting cylinder (2) generates the oscillatory flow, which is superimposed upon the net flow. The details of the oscillatory flow device are irrelevant in the context of the following deductions, e.g., it could equally be a diaphragm or a syringe pump. For sake of simplicity, it will be referred as an oscillator.

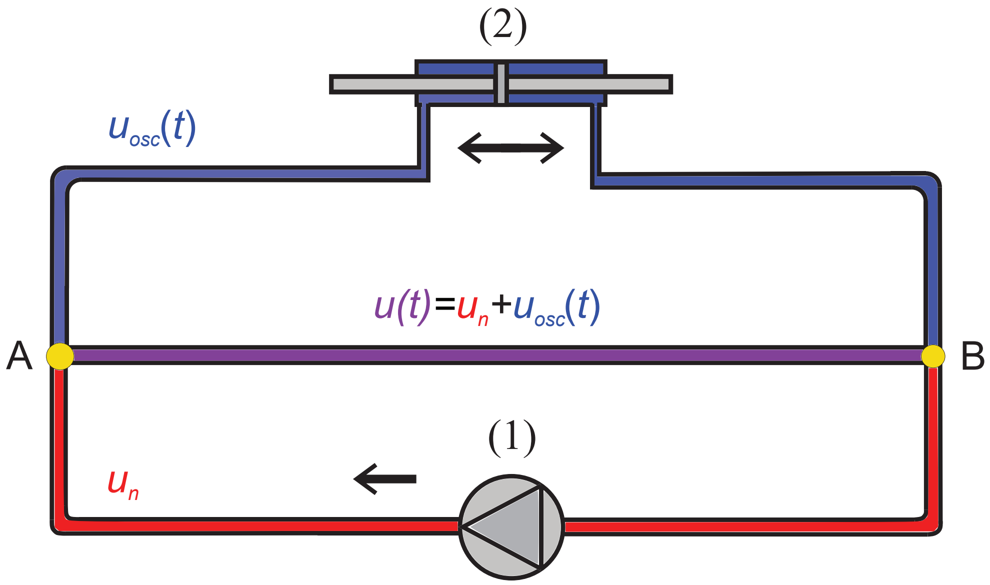

Figure 1: Hydraulic schematic of a continuous Oscillatory Baffled Reactor: (1) net flow pump, (2) oscillatory flow system.

Along the section of the circuit painted in red there is only a constant net flow, as imposed by the pump (1). In the blue section there is only a pure oscillatory flow, which follows, typically, a sinusoidal curve, according to the displacement of the oscillator. In the violet section, representing the test section A-B, both flows are superimposed.

Model assumptions and simplifications. These are the assumptions and simplifications of the model:

- The oscillatory flow follows a perfect sinusoidal wave. This is a reasonable approximation because the devices which generate the oscillatory flow are designed in order to follow a sinusoidal movement.

- The flow behaviour is quasi-steady, i.e., the instantaneous frictional pressure drop is identical to the one which would exist in a steady flow whose mean velocity equals the instantaneous velocity. Apart from fully turbulent flows [7], this behaviour has also been reported in laminar flows at very low oscillation intensities [18].

- The instantaneous pressure drop is a quadratic function of the flow velocity during the whole oscillation cycle. It is evident that the flow would be laminar during some fractions of the cycle when the mean flow velocity is low, and the instantaneous friction factor would be a function of the instantaneous Reynolds number. However, it is considered that the chaotic behaviour affects the whole cycle and the relation between the instantaneous pressure drop and velocity is not dependent on the Reynolds number.

With regard to the quasi-steady and turbulent assumptions, we present here a mathematical derivation that might clarify our assumption. The power density, $W_{osc}/(AZ)$, of the oscillator assuming the previous hypothesis: 1) pure sinusoidal velocity; 2) quasi-steady flow; and 3) fully turbulent flow, is:

$$\epsilon = \frac{4K_p}{3\pi Z}(x_0\omega)^3 \quad (2)$$

where K_p is a constant which is the relationship between the instantaneous pressure drop and the flow velocity. This constant depends on the fluid properties and the geometry of the baffles, but it has been considered as independent of the Reynolds number. If this expression is compared with the power density provided by the quasi-steady and the eddy enhancement models, the constant K_p for these models is, respectively:

$$K_{p,qs} = \frac{n_b\ \rho}{2\ C_0^2} \frac{(1 - S^2)}{S^2} \tag{3}$$

$$K_{p,ee} = \frac{9\pi\ n_b\ \rho\ l_m}{8\ x_0\ S} \tag{4}$$

As a conclusion, both models can be seen as quasi steady and fully turbulent models. The eddy-enhancement model has been validated at moderately low oscillatory Reynolds numbers for single-orifice baffles with sudden constrictions [9]: $Re_{osc} = 40 - 85$ at $St = 0.95$, and $Re_{osc} = 130 - 350$ at $St = 0.15$. Ergo our model should be valid, at least, for the same operational regimes. This statement should be taken very cautiously, because in those ranges the flow in that geometry has been identified -at least- as chaotic, whereas this hydraulic status can change completely for a different geometry. As an example, the quasi steady model provided results lacking physical meaning [7]in single-orifice baffled tubes with smooth constrictions for $Re_{osc} \approx 100$.
The third assumption implies that the pressure drop in the test section cannot be obtained by superposition, i.e., by adding the pressure drop due to the oscillatory flow and the net flow separately. Consequently, the respective power consumptions of the net flow pump (1) and the oscillator (2) are affected by the superposition of both flows.

Problem formulation. The mean flow velocity, $u(t)$, between the points A and B is the superposition of the net and the oscillatory flows:

$$u(t) = u_n + u_{osc}(t) = u_n + x_0\ \omega\ \cos(\omega t) \tag{5}$$

The existence of a negative velocity (flow reversal) depends on the ratio of the maximum oscillatory flow velocity and the net flow velocity, defined as the velocity ratio:

$$\Psi = \frac{x_0 \ \omega}{u_n} \tag{6}$$

The typical evolution of the net and the oscillatory flows is shown in Figure 2. As can be seen in Figure 2 (a), for the case with $\Psi < 1$, there is no flow reversal. The interest of this case is merely theoretical, because in COBRs a velocity ratio of $\Psi > 1$ is required (Figure 2 (b)) in order to achieve flow reversal and generate cyclic vortex dispersion during both halves of the cycle, increasing the radial mixing.

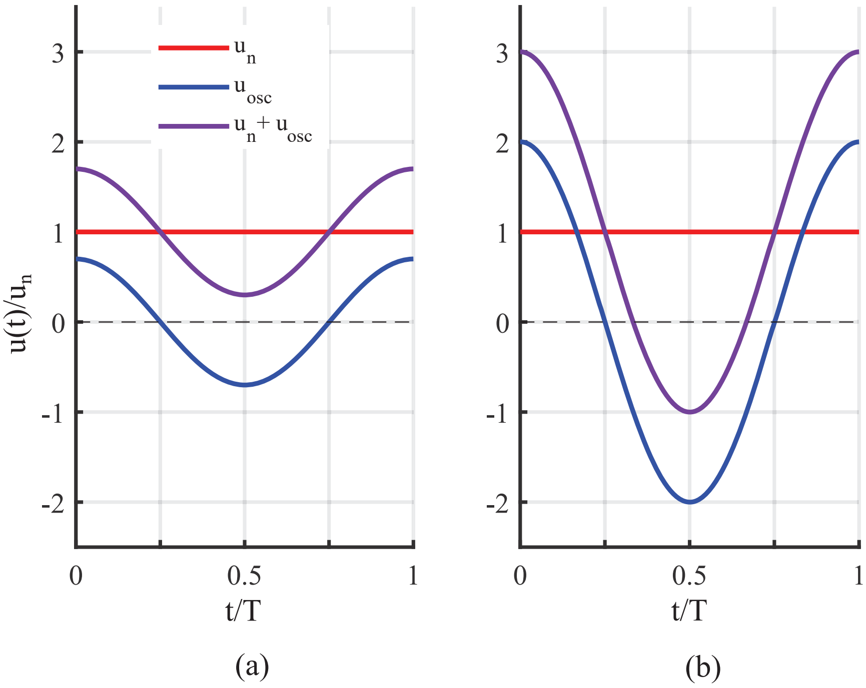

Figure 2: Flow in the different sections of the circuit, for a case with a) $\Psi = 0.7$; (b) $\Psi = 2$

Following the third assumption, the pressure drop in the test section, A-B, experienced by the net flow pump and the oscillator can be expressed as:

$$\Delta p_{AB}(t) = K_p \ (u_n + x_0 \ \omega \ \cos(\omega \ t)) \ |u_n + x_0 \ \omega \ \cos(\omega \ t)| \tag{7}$$

The constant K_P takes into consideration all the elements between the points A and B, where the pressure drop is a result of the overall flow velocity, i.e. superposition of the net and the oscillatory flows. The absolute value ensures that the pressure drop direction is the same as the overall flow direction at each instant.This pressure drop has to be overcome by both the net flow pump and the oscillator. Therefore, the averaged net flow pumping power is given by:

$$\bar{W}_n = \frac{1}{T}\int_0^T A\ u_n\ \Delta p_{AB}(t)\ dt \tag{8}$$

and the averaged oscillator power consumption is:

$$\bar{W}_{osc} = \frac{1}{T}\int_0^T A\ u_{osc}(t)\ \Delta p_{AB}(t)\ dt \tag{9}$$

2.2. Effect of the oscillatory flow on the net flow pumping power consumption

In order to solve the integral (Equation 8), the expression of the pressure drop in the section A-B (Equation 7) is introduced and the change of variable $\theta = \omega t$ is done:

$$\bar{W}_n = \frac{1}{2\pi}\int_0^{2\pi} A\ u_n\ K_p\ \left(u_n + x_0\ \omega\ \cos(\theta)\right)\ \left|u_n + x_0\ \omega\ \cos(\theta)\right|\ d\theta \tag{10}$$

Due to the presence of the absolute value in the integral, the way to solve it is different if the velocity is negative or not during a fraction of the cycle. For a case with flow reversal, $\Psi > 1$, the minimum flow velocity is negative during a fraction of the cycle. The part of the cycle when there is flow reversal can be delimited calculating the phase of the cycle when the velocity is zero. From Equation 5:

$$\theta_0 = \arccos\left(-\frac{u_n}{x_0\ \omega}\right) = \arccos\left(-\frac{1}{\Psi}\right) \tag{11}$$

Thus, the integral to be developed is:

$$\frac{\bar{W}_n}{A} = \frac{K_p}{2\pi}\left[-\int_{\theta_0}^{2\pi-\theta_0} u_n\ \left(u_n + x_0\ \omega\ \cos(\theta)\right)^2 d\theta + \int_{2\pi-\theta_0}^{2\pi+\theta_0} u_n\ \left(u_n + x_0\ \omega\ \cos(\theta)\right)^2 d\theta\right] \tag{12}$$

As can be observed, the lower and upper limits of the integration are $\theta = \theta_0$ and $\theta = 2\pi + \theta_0$, respectively. This way the number of sections to consider in the integration is reduced.

For a case without flow reversal, $\Psi < 1$, the integral can be solved directly:

$$\frac{\bar{W}_n}{A} = \frac{K_p}{2\pi} \int_0^{2\pi} u_n \ \left(u_n + x_0 \ \omega \ \cos(\theta)\right)^2 d\theta \tag{13}$$

The resolution of both expressions allows us to obtain the oscillatory flow effect on the net flow, F_n, as the relation between the net flow pumping power considering the effect of the oscillatory flow, $\bar{W}_n$, and the pumping power without considering it, $\bar{W}_{n,0}$. Thus:

$$F_n = \frac{\bar{W}_n}{\bar{W}_{n,0}} = \begin{cases} A + B \ \Psi + C \ \Psi^2 & \Psi \geq 1 \\ 1 + \frac{\Psi^2}{2} & \Psi < 1 \end{cases} \tag{14}$$
$$A = \frac{2\theta_0}{\pi} - 1; \ B = \frac{4\sin(\theta_0)}{\pi}; \ C = \frac{2\theta_0 - \pi + \sin(2\theta_0)}{2\pi}$$

In Figure 3 the oscillatory flow effect factor is plotted as a function of the velocity ratio, and compared with the expression proposed by Baird and used by Mackley and Stonestreet [11].

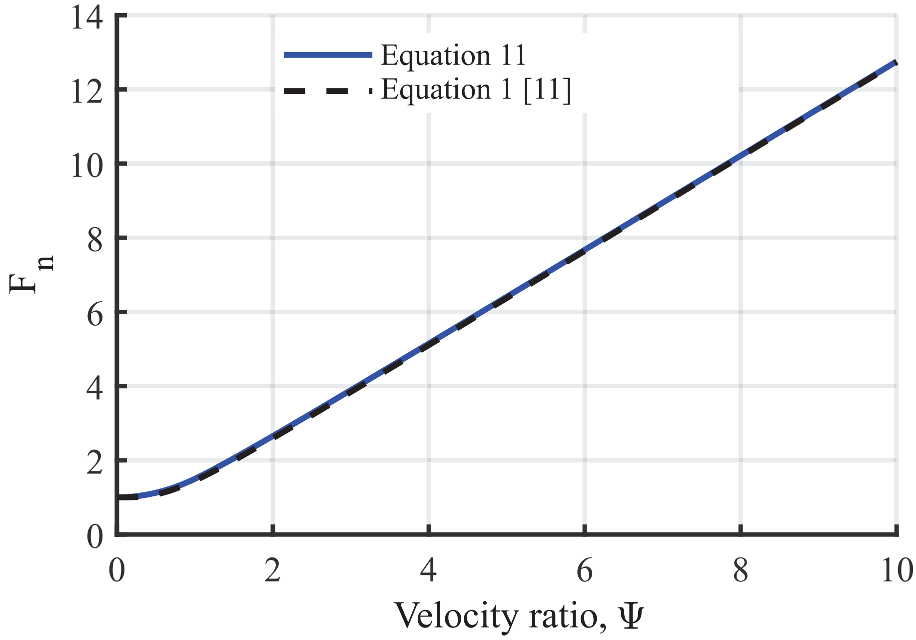

Figure 3: Oscillatory flow effect factor on the net power consumption according to Equation 14

As can be seen, both expressions provide seemingly identical results. In order to observe in detail the difference between both expressions, Figure 4 shows, as a percentage, the deviation of the expression proposed by Baird from the one calculated in this section (Equation 14).

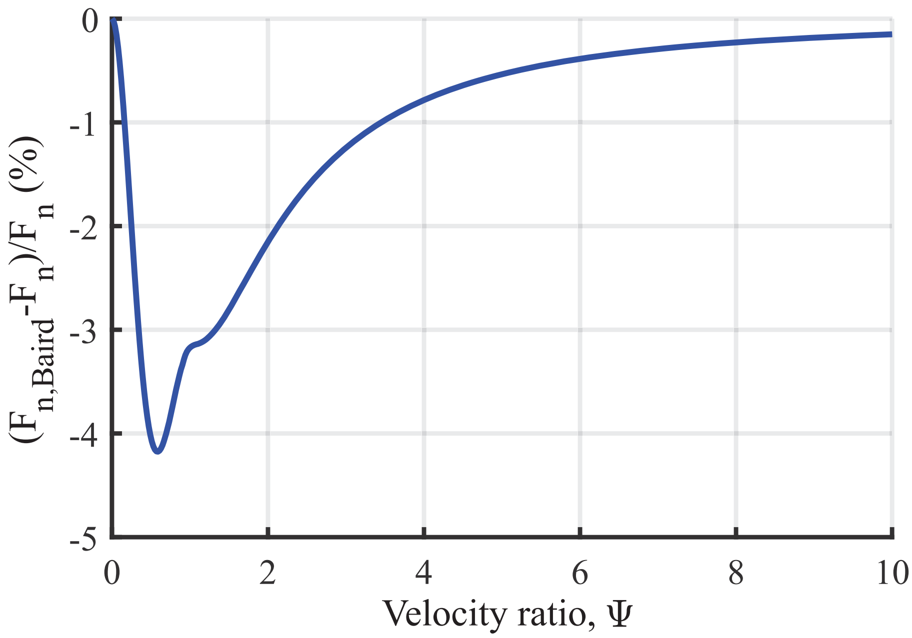

Figure 4: Deviation of the factor used by Mackley and Stonestreet [11] from the factor obtained from Equation 14

The maximum deviation is around a 4 %, and it decreases significantly for higher velocity ratios. For a velocity ratio $\Psi > 2$, typical in an OBR, the deviation between both expressions is lower than a 2 %. Based on these observations, it can be concluded that the expression proposed by Baird, in spite of not being exact, is precise enough for design purposes.

2.3. Effect of the net flow on the oscillator power consumption

In order to solve the integral (Equation 9), the expression of the pressure drop in the section A-B (Equation 7) is introduced and the change of variable $\theta = \omega t$ is done:

$$\bar{W}_{osc} = \frac{1}{2\pi}\int_0^{2\pi} A\ (x_0\ \omega\ \cos(\theta))\ K_p\ (u_n + x_0\ \omega\ \cos(\theta))\ |u_n + x_0\ \omega\ \cos(\theta)|\ d\theta \tag{15}$$

For the cases with $\Psi < 1$ and $\Psi \geq 1$ we proceed in an analogous way to the previous section.

The factor that accounts for the effect of the net flow rate on the oscillator power consumption, F_{osc}, is calculated as the relation between the oscillator power consumption in the presence of a net flow rate, $\bar{W}_{osc}$, and without it, $\bar{W}_{osc,0}$:

$$F_{osc} = \frac{\bar{W}_{osc}}{\bar{W}_{osc,0}} = \begin{cases} \frac{3}{8}\left(\frac{A}{\Psi^2} + \frac{2B}{\Psi} + C\right) & \Psi \geq 1 \\ \frac{3\pi}{4\Psi} & \Psi < 1 \end{cases} \tag{16}$$
$$A = 4\ \sin(\theta_0);\ B = 2\theta_0 - \pi + \sin(2\theta_0);\ C = 4\sin(\theta_0) - \frac{4}{3}\sin^3(\theta_0)$$

This factor is represented in Figure 5 as a function of the velocity ratio. As can be observed, the factor tends to infinity when the velocity ratio is near zero, when the oscillator power consumption would be zero. The factor decreases sharply when the velocity ratio is increased. In practice, OBRs work with a velocity ratio $\Psi > 2$, being $2 < \Psi < 4$ the optimum range proposed by Stonestreet and Van der Veeken [1]. For this range, it would be obtained an increase in the oscillator power consumption of around 36 % for $\Psi = 2$ and ~9 % for $\Psi = 4$. Therefore, it can be stated that the net flow effect on the oscillator power consumption can be significant in the typical working range of an OBR.

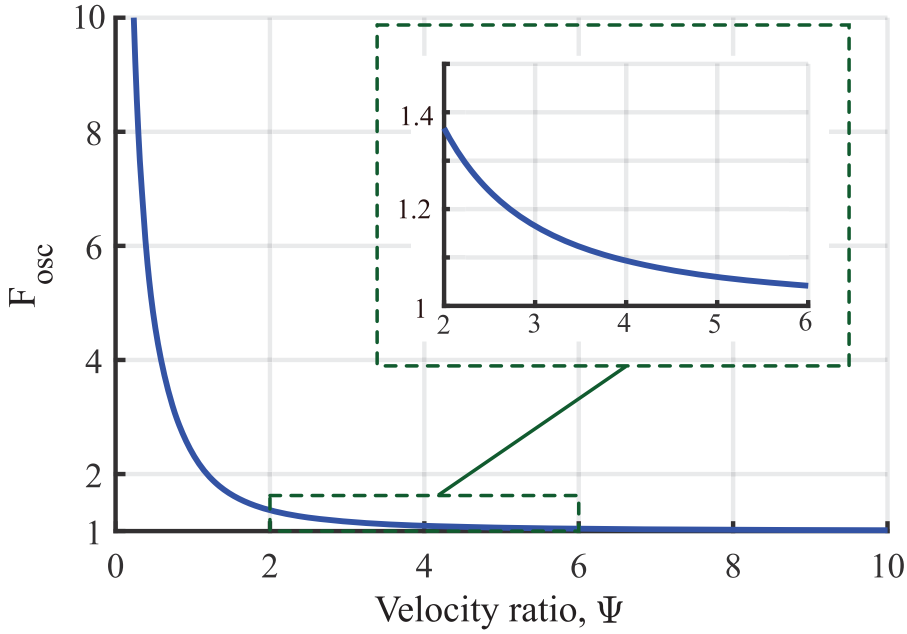

Figure 5: Net flow effect factor on the oscillatory flow power consumption

253 The implementation of Equation 16 is not simple, so a simplified expression is proposed in the form of Equation 1:

$$F_{osc} = \left(1 + \left(\frac{3\pi}{4\Psi}\right)^3\right)^{\frac{1}{3}} \tag{17}$$

This expression has been obtained from a statistical fitting, obtaining the coefficient and the two exponents which provide the best fitting to the values predicted by Equation 16 in the range $0 < \Psi < 20$. The coefficient and the two exponents have been rounded to achieve a more simplified expression. The maximum deviation of this expression from the exact solution (Equation 16) is lower than a 4 % in the range $0 < \Psi < 20$. Thus, as Equation 1, it can be applied with enough accuracy for design purposes.

We can infer that Equation 1 was obtained in a similar way: solving the integral for the averaged net flow pumping power (Equation 8), either analytically or numerically. Then, a suitable mathematical expression was fitted to the values predicted by that solution.

Application of the flow interaction factors. In this paragraph, a simple scheme for applying the flow interaction factors is presented. The first step is to decide the nominal conditions for the design of the OBR, i.e., net Reynolds number, oscillatory Reynolds number and Strouhal number. Then:

- The net flow pumping power consumption should be calculated from the net flow rate and the pressure drop related to that net flow. This information is not commonly available in the open literature on OBRs, because the studies are commonly focused on the consumption of the oscillator. As an example, the net Fanning friction factor can be found for single-orifice and tri-orifice baffles in [19] and [18], respectively.

- The previous power is multiplied by the factor in Equation 1, function of the velocity ratio, Re_{osc}/Re_n. This power should be used for the selection of the net flow pump engine.

- The oscillator power consumption is calculated by means of experimental or numerical tests, or alternatively using one of the two models available [9].

- The previous power is multiplied by the factor in Equation 17, function of the velocity ratio, Re_{osc}/Re_n. This power should be used for the selection of the oscillator engine.

3. Effect of the energy recovery on the power consumption

The ability of the system to store/recover energy has been discussed in few number of investigations. The flywheel effect, i.e., the ability of the oscillator to store kinetic energy, was firstly mentioned by Jealous and Johnson [8], but the first attempt to assess if a system fulfils that model was made by Hafez and Baird [13]. The authors proposed that the flywheel effect could be checked if the power consumption deducted from the pressure drop-velocity curves matches the value measured by a wattmeter connected to the motor of the oscillator.

If there is energy recovery, i.e., if the energy is recovered by the system during the fraction of the cycle when the pressure drop and the velocity have opposite directions, the power consumption, W_{ER}, is:

$$\bar{W}_{ER} = \frac{1}{T}\int_0^T q_{max}\ \sin(\omega\ t)\ \Delta p_{max}\ \sin(\omega\ t + \delta)dt \quad (18)$$

where q_{max} and Δp_{max} are the amplitudes of the instantaneous flow rate and pressure drop signals, respectively. The only assumption in this procedure is that both variables follow a sine function. This assumption is based on some experimental results in single-orifice baffles [9] and the fact that, if the pressure drop is not sinusoidal, it can be described by a fundamental sine wave obtained by statistical fitting [6, 18]. It is important to point out that no assumptions have been made regarding the flow behaviour (quasi-steadiness or turbulence) or the type of baffle geometry.

The change of variable: $\theta = \omega t$ is made:

$$\bar{W}_{ER} = \frac{1}{T}\int_0^{2\pi} q_{max}\ \sin(\theta)\ \Delta p_{max}\ \sin(\theta + \delta)d\theta \quad (19)$$

306 The result of the above integral over an oscillation cycle gives the expression
307 obtained by Mackay et al. [17]:

$$\bar{W}_{ER} = \frac{q_{max}\ \Delta p_{max}\ \cos(\delta)}{2} \tag{20}$$

308 However, for the case with no energy recovery, the part of the cycle with
309 opposite directions of the pressure drop and the flow velocity should not be
310 taken into account for the calculation of the averaged power consumption.
311 In Figure 6 this part of the cycle is delimited between dashed lines.

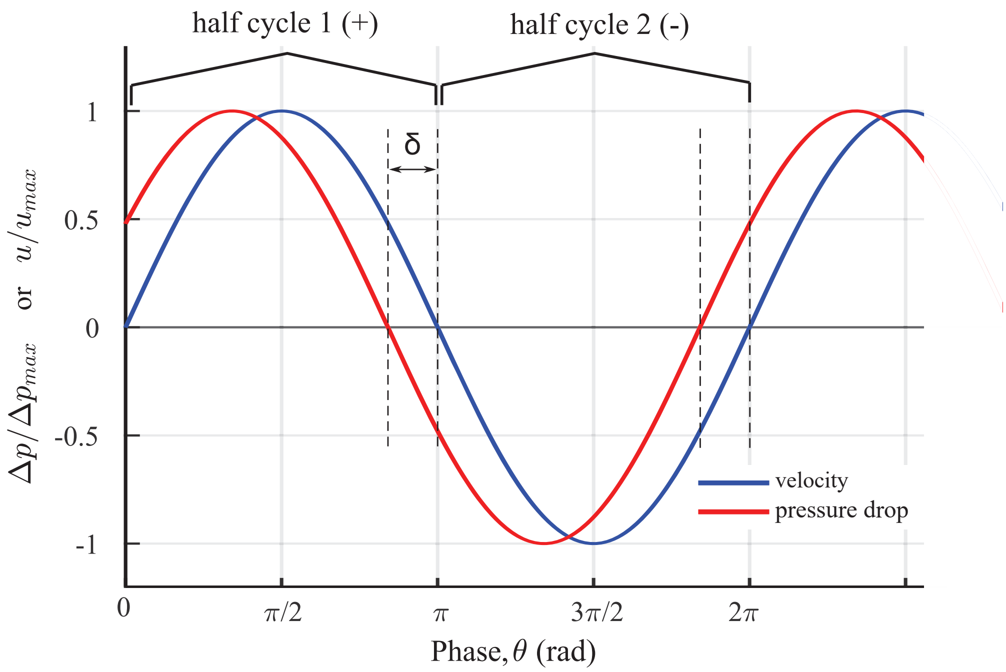

Figure 6: Velocity and pressure drop in a pure oscillatory flow

312 Due to the symmetry of the functions between $\theta = [0, \pi]$ and $\theta = [\pi, 2\pi]$ it is
313 possible to integrate over only one of the ranges. After removing the part of

the cycle when the pressure drop and the velocity have opposite directions, corresponding to $\theta = [\pi - \delta, \pi]$, the integral to solve is:

$$\bar{W}_{NER} = \frac{2}{2\pi} \int_0^{\pi-\delta} q_{max} \ \sin(\theta) \ \Delta p_{max} \ \sin(\theta + \delta) d\theta \tag{21}$$

The amplitudes of the pressure drop and the flow rate are constants, so they can be extracted from the integral:

$$\bar{W}_{NER} = \frac{q_{max} \ \Delta p_{max}}{\pi} \int_0^{\pi-\delta} \sin(\theta) \ \sin(\theta + \delta) d\theta \tag{22}$$

Applying the product-to-sum trigonometric identity, the expression left is:

$$\bar{W}_{NER} = \frac{q_{max} \ \Delta p_{max}}{2\pi} \int_0^{\pi-\delta} - \cos(2\theta + \delta) + \cos(\delta) d\theta \tag{23}$$

$$\bar{W}_{NER} = \frac{q_{max} \ \Delta p_{max}}{2\pi} \left[-\frac{1}{2} \sin(2\theta + \delta) + \cos(\delta) \ \theta \right]_0^{\pi-\delta} \tag{24}$$

$$\bar{W}_{NER} = \frac{q_{max} \ \Delta p_{max} \ [\cos(\delta)(\pi - \delta) + \sin(\delta)]}{2\pi} \tag{25}$$

This is the power consumption required by the oscillator in a circuit with only oscillatory flow and without energy recovery. The relation between the power consumption in a system without energy recovery (Equation 25) and with it (Equation 20) is given by:

$$F_{NER} = \frac{\bar{W}_{NER}}{\bar{W}_{ER}} = \frac{\cos(\delta)(\pi - \delta) + \sin(\delta)}{\pi \ \cos(\delta)} = \frac{\pi - \delta + tan(\delta)}{\pi} \tag{26}$$

The power increase (as a percentage) is plotted in Figure 7 as a function of the pressure drop-velocity phase lag. For a low phase lag, $\delta < 0.5$, there is almost no increase in the power consumption, lower than a 1.5 %, but the effect is significantly higher when the phase lag is increased above that value. The power consumption increase tends to infinity when the phase lag reaches the value $\pi/2$. This situation corresponds to a null power consumption if there is energy recovery in the system.

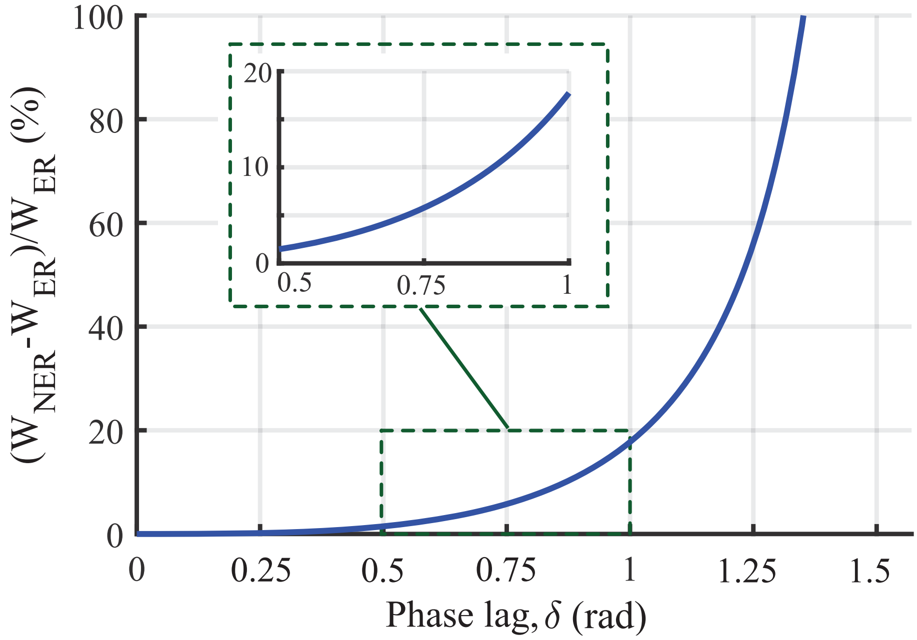

Figure 7: Pressure drop-velocity phase lag effect on the power consumption of a system without energy recovery

It should be noticed that, in order to apply this factor, the phase lag should be known. The phase lag for a given condition has to be obtained by experimental testing or numerical simulation (assessment of the pressure drop and the velocity signals), because there are no predictive models in the open literature, and the available experimental data is scant. For the sake of proving the relevance of these results in practice, the only experimental results available in the open literature [9] are used as a mere reference: the maximum phase lag reported, 1 rad, would imply a power consumption increase of around 18 % as compared with the expression proposed by Mackay et al. (Equation 20).

To sum up, in the first place, it should be established if the system is able to store/recover energy. If the system is capable of recovering energy, the expression proposed by Mackay et al. [17] (Equation 20) for the power density can be used to obtain the power consumption. If there is no possibility for energy recovery, the factor proposed in this section (Equation 26) should be applied, multiplying the power consumption of a system with energy recovery (Equation 20).

4. Conclusions

- Assuming that the flow is quasi-steady and fully turbulent, a factor which considers the nonlinear effect of the oscillatory flow on the net pump power consumption has been proposed. This factor is a function of the velocity ratio. The factor proposed by Baird: $F_n = (1 + (4\ \Psi/\pi)^3)^{1/3}$ is in agreement with the one proposed in this communication (with deviations lower than 2 % for $\Psi > 2$), while no justification has been found for Baird's factor. Due to its simplicity we recommend the use of the factor proposed by Baird.

- Assuming that the flow is queasi-steady and fully turbulent, a factor which considers the nonlinear effect of the net flow on the oscillator power consumption has been proposed. The superposition of the net flow can lead to an increase of around 36 % for $\Psi = 2$.

- Assuming that both the velocity and the pressure drop follow a perfect sine wave, an expression has been proposed to quantify the increase in the power consumption if the system is not able to recover energy. For example, an 18 % increase in the oscillator power consumption can be observed for a phase lag $\delta = 1$ rad.

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