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Proposal for a Fuzzy Model to Assess Cost Overrun in Healthcare Due to Delays in Treatment

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Abstract: Apart from the effects of treating those infected with COVID-19, the pandemic has also affected treatment for other diseases, which has been either interrupted or canceled. The aim of this paper is to provide a financial model for obtaining the cost overrun resulting from the worsening of illnesses and deaths for each of the causes considered. To achieve this, first deaths have been classified into causes of death and for each of these causes, an estimation has been made of the worsening condition of patients due to delay in treatment. Through these data, a fuzzy relation between deaths and the worsening condition of patients can be obtained. Next, the expertise process has been used to estimate cost overrun in relation to patients' pathologies. The experts' opinions have been aggregated using ordered weighted average (OWA). Lastly, using fuzzy logic again, a correction coefficient has been determined, which optimizes the future implementation of the proposed model without the need for a new estimation of inputs. The paper concludes with a numerical example for a better comprehension of the proposed theoretical model. Ultimately, it provides the scientific community in general and in particular managers of public administration entities with a novel tool for improving the efficiency of the healthcare system.

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1. Introduction

On 31st January 2020, the first COVID-19 case was reported in Spain, followed by an exponential increase in the number of people infected. On 14th March 2020, a state of emergency was declared in Spain, calling for a national lockdown to try and stop the pandemic from spreading. Healthcare saturation due to this virus has led to a reorganization of healthcare procedures as well as a relocation of physical and material resources. As a result, tests, diagnoses, and medical visits have not only been delayed in Spain but in many other parts of the world as well. A great number of surgeries have been cancelled or delayed due to the pandemic [1], and Klocker et al. [2] highlight serious problems and even psychological effects experienced by patients. Atreya et al. [3] affirm a 44% decrease in hospital admissions of patients without COVID-19. On many occasions it has been the citizens themselves who have decided of their own freewill not to use the health services because they are afraid of becoming infected or of saturating the system [4].

As a result, the impact of the pandemic has been such that it transcends COVID-19 infection and affects several other pathologies such as cardiovascular, neurological, or oncological diseases. Rodriguez-Leon et al. [5] point out a decrease in healthcare in interventional cardiology, and Solomon et al. [6] confirm a decrease of up to 48% in weekly hospitalization rates for acute myocardial infarction in the COVID-19 period. Some authors, such as Masroor [7] or Metzler et al. [8], indicate a worsening of coronary diseases because of treatment delays.

Zadnik et al. [9] highlight that during the pandemic there has been a significant decrease in patients' first referral to oncology specialists, and delays in imaging studies, which has caused a late cancer diagnosis for several patients. Maringe et al. [10] estimated a substantial increase in the number of deaths from cancer-related illnesses as a result of a delay in diagnosis.

In fact, COVID-19 has simply exacerbated and put the spotlight on a problem that exists in a majority of countries to a greater or lesser extent: the worsening of illnesses because of delays in treatment, and the human as well as the economic cost that this implies. This study is specifically aimed at providing an assessment model that can estimate this economic cost.

However, in the development of the financial model required, the latent inaccuracy and uncertainty in any kind of valuation must be considered. Despite this, the incorporation of uncertainty in models of financial decisions improves forecasting as well as descriptive power [11]. For this reason, it is appropriate to use techniques to improve the design of financial models by introducing the subjectivity and uncertainty inherent in them.

To overcome these limitations, a branch of mathematics called fuzzy mathematics has been developed, whose application has been extended to several fields, one of which is economics. Fuzzy logic entails a proper treatment of subjectivity and uncertainty, and it also provides a series of instruments for analysts to be able to transmit the information [12].

The use of fuzzy logic in financial fields is not new. Faircloth and Ricchiute [13] use it to quantify truthfulness in financial reports. Gil Lafuente [14,15] develops extensive applications in Financial Analysis that have been useful for later studies such as the one by Couturier [16] among others. Siegel, Korvin, and Omer [17] use fuzzy logic to model the detection of irregular customers, which is especially useful in the banking and financial sector. Terceño et al. [18] incorporate fuzzy methodologies to study life assurances. Martinez et al. [19] develop a simulation fuzzy model for financial planning, and Dourra and Siy [20] implement fuzzy engineering tools in the financial field, especially in the technical analysis field. Tung et al. [21] develop a neural fuzzy system as an alternative to traditional statistical models to forecast bankruptcies in the banking sector. Tang and Chi [22] use the fuzzy model to predict commercial credit risks in the context of international commerce. Jin and Doloi [23] use a fuzzy inference system (FIS) to facilitate the risk allocation decision-making (RADM) process. Hachicah et al. [24] develop a modeling technique using fuzzy sets optimized through differential evolution that improves the explanation of the dynamics of emerging and international financial markets. Mandic et al. [25] and Reig et al. [26] use fuzzy AHP and TOPSIS methods to analyze banking parameters and to establish rankings.

The contribution of this paper to the scientific community lies in the fact that it is a subject of great interest which is approached innovatively. Moreover, the determination of cost overrun, which could cause a delay in the treatment of specific pathologies in any healthcare system, is a subject of relevance as well as timeliness. Although the subject itself justifies our interest in carrying out this study, the completely novel approach used is without precedents in the specialized bibliography. In order to quantify the aforementioned cost overrun, a financial fuzzy model is proposed in which the implementation of several fuzzy logic tools is absolutely innovative.

2. Proposed Methodology

2.1. Relation between Deaths and Patients Whose Pathology Has Become Worse Due to a Delay in Treatment

This first stage aims to provide a model which can relate patients' deaths caused by worsening pathologies due to delays in treatments to the total number of patients whose pathologies are worse because of these delays. In this sense, we can highlight previous

studies in the field of medicine, where symptoms are linked to diseases [27], and also in the field of business, where ratios are linked to the causes leading to problems observed in business [28,29].

This methodology is based on determining a set of periods, $T = \{T_k\}$, $k = 1, 2, \dots, t$, where it is possible to identify the number of deaths caused by the worsening condition of patients' previous pathologies due to delays in treatments. The causes of death are denoted by $C = \{C_i\}$, $i = 1, 2, \dots, n$. For each C_i , which denotes the cause of death due to a worsening of a previous pathology, the extent of how negative attention could affect the worsening of the pathology is measured on a scale of six elements: (1) very mildly, (2) mildly, (3) neutrally, (4) somewhat seriously, (5) seriously, and (6) very seriously. In this sense, cardinality must be low enough to not require excessive accuracy in the information to be expressed and high enough to achieve discrimination of valuations in a limited number of degrees [30]. According to Miller [31], a subject is able to reasonably deal with and remember a maximum of around 7 or 9 terms, so for reasons of prudence only six scales have been selected. There were N_{ki} deaths for each period (k) and each cause of death (i). Out of these, for $n_i^{(1)}$ the worsening of the initial pathology is considered to influence the patient's death very mildly, $n_i^{(2)}$ mildly, ..., and $n_i^{(6)}$ very seriously. The importance of the cause of death i in each year is measured through the expression:

$$p_{ki} = \sum_{j=1}^6 \frac{j-1}{5} n_{ki}^{(j)}, \quad k = 1, \dots, t, \quad i = 1, \dots, n \tag{1}$$

This process is repeated for each cause of death i in each of the t analysed periods, obtaining matrix $P = [p_{ki}]_{t \times n}$, which shows the importance of the cause i in each period k . The membership function of the cause of death i for the period k , q_{ki} , is obtained by dividing the importance of each kind of death generated in the period k by the maximum for this cause for all the periods.

$$q_{ki} = \mu(C_{ki}) = p_{ki} \max_k^{-1}(p_{ki}), \quad k = 1, \dots, t, \quad i = 1, \dots, n \tag{2}$$

From this, it is possible to obtain matrix $C = [q_{ki}]_{t \times n}$ which shows the intensity of the cause of death in the period k .

Next, an estimation is made of the number of patients whose pathology has worsened because of a delay in treatment throughout the analysis period. A delay in treatment can be due to late detection of the illness because of postponed medical tests or the cancellation or delay in certain medical procedures considered not to be urgent for patients whose pathologies have already been diagnosed. Since not all patients whose condition worsens die, it would be interesting to interview health professionals to find out the number of patients whose pathologies worsen and to what extent for the reference period. They could then be grouped according to the worsening of their initial situation using linguistic labels: very high, high, medium, etc.

The membership function for each of these patients (l_{kh}) is obtained by dividing the number of patients of each period k and kind h (l_{kh}) by the maximum number of patients of this kind (h) for all the periods.

$$l_{kh} = \mu_L(L_{kh}) = n_{l_{kh}} \max_k^{-1}(n_{l_{kh}}), \quad k = 1, \dots, t, \quad h = 1, \dots, l \tag{3}$$

The matrix $L = [l_{kh}]_{t \times l}$ is defined as the matrix that shows the nominal level of each kind of patient whose pathology worsens due to a delay in medical care or bad care for each period, for the t periods, and l kind of patients.

Once matrices L and C have been obtained, the next step is to obtain matrix $R = [r_{ih}]_{n \times p}$, with $r_{ih} \in [0,1]$, $i = 1, \dots, n$ and $h = 1, \dots, l$, which represents the fuzzy relation between the kind of patient whose pathology has become worse, L_i , and the cause of death C_j . Each of the elements shows the degree to which each patient whose pathology has worsened L_h implies a death C_i . The following expression must be considered

$$L = C \circ R \tag{4}$$

According to Sanchez [27], the largest relation which is a solution to this equation is,

$$R = C^{-1} \alpha L = [c_{ik}] \alpha [1_{kh}] \tag{5}$$

Being $C = [c_{ki}]^{-1} = [c_{ik}]$ y $[r_{ih}] = \Lambda_k [q_{ik} \alpha 1_{kh}]$, where

$$q_{ik} \alpha 1_{kh} = \begin{cases} 1 & \text{if } q_{ik} \leq 1_{kh} \\ q_{ik} & \text{if } q_{ik} > 1_{kh} \end{cases} \tag{6}$$

Matrix R shows the relation between deaths and patients whose pathology has worsened. For this reason, it will be used to predict the number and level of worsening of patients' pathologies for future periods through the data about deaths from these periods.

2.2. Additional Cost Due to the Worsening of Patients' Pathology

Given that Matrix R predicts the number of patients whose pathologies have worsened, in this section, cost overrun per patient for the healthcare system will be approximated for each level of worsening. In this way, an equivalence table is created to allocate a previously established assessment to each level of worsening pathology. In order to establish the defined standards in advance, the following sequential process will be proposed for each level.

2.2.1. Application of the Expertise Process for the Quantification of Healthcare Cost Quantification

For the first estimation of cost overrun due to worsening pathology, the opinion of several experts in the subject will be considered. According to Robbins [32], the number of participants required for decision-making problems varies between 5 and 7. This number is much lower than required in traditional surveys because those consulted are experts in the subject. It is necessary to have highly qualified professionals (by way of example, these would be managers of different healthcare areas, financial and medical managers from hospitals, insurance companies, etc.) duly advised by those responsible for the medical departments related to the pathologies analyzed. As well as this, they should be provided with any information considered relevant to the study and should be informed at all times of its purpose. For each level of worsening of the pathology, the experts must make an estimation of the cost of the treatments required to try and help a patient recover which is additional to that of the treatments that would have been required if this worsening had not occurred. However, to improve the treatment of the subjectivity and uncertainty inherent in the information provided by the experts, we propose the use of the expertise process introduced by Kaufmann [33]. In this way, firstly, according to their knowledge and the information available, a total of H experts will be asked to make an assessment of the increase in the average cost per patient arising from the treatment of the analyzed pathology according to the level of worsening of their condition. As the experts will provide approximations, the information will be collected through triangular fuzzy numbers (TFN) $\tilde{Q}_i = (q_i^1, q_i^2, q_i^3)$, $i = 1, \dots, H$.

The H expert opinions will be weighted according to the degree of confidence in each expert, since they do not all necessarily have the same qualifications or experience, and confidence in the healthcare system. For the latter, a group of L experts will be asked their

opinion about the healthcare system’s ability to reach its goals at the least possible cost. Greater confidence in the healthcare system’s efficiency means a greater weight will be assigned to the opinions by the H experts who have provided a lower cost, and vice versa.

The use of ordered weighted averages (OWA) will permit the aggregation and summary of the information provided by experts. This operator and its extensions have been used in a wide range of applications [34–42].

Definition 1. An ordered weighted average (OWA) is defined as a mapping of dimension n , $F : \mathbb{R}^n \rightarrow \mathbb{R}$ that has an associated weighting vector W of dimension n , $W^T = [w_1, w_2, \dots, w_n]$, such that $w_j \in [0, 1]$ and $\sum_{j=1}^n w_j = 1$, with

$$f(a_1, a_2, \dots, a_n) = \sum_{j=1}^n w_j \cdot b_j \tag{7}$$

where b_j is the j th largest of a_i .

The essence of OWA [43–45] is the rearrangement of the elements or arguments, causing aggregation in a_j not associated with a weighting w_j but with the order of placement instead. An interesting extension of the OWA operator is the Basic Defuzzification Distribution OWA operator (BADD-OWA) [46,47].

Definition 2. A BADD-OWA is an OWA in which the weights w_j are obtained as,

$$w_j = b_j^\beta / \sum_{j=1}^n b_j^\beta \tag{8}$$

where $\beta \in \mathbb{R}$, b_j is the j th largest of a_i .

2.2.2. Weighting According to the Level of Confidence in the Healthcare System

The healthcare system’s ability to efficiently respond to a worsening pathology will be considered. The more efficient the system is, the lower the cost will be. Based on this premise, the confidence level as a weighting factor is introduced. For this purpose, five levels of confidence will be proposed, $S_j, j = 1, \dots, 5$, so that S_1 represents the minimum degree of confidence in the healthcare system, and S_5 is the maximum degree of confidence in it. In the degree of confidence S_1 , the aggregate cost of the healthcare system will be obtained by assigning a greater weight to the highest costs provided by the H experts, so the aggregate will be higher. Conversely, in the degree of confidence S_5 , the maximum degree of confidence, the aggregate cost of the healthcare system will be obtained by assigning a maximum weight to the lowest costs provided by the experts, so the result will be lower.

To establish a level of confidence in the healthcare system, it would be necessary to consider the opinion of L experts. They may or may not be the same experts, or only some of the previous H experts, and the number of experts may differ.

Determining the level of confidence in the healthcare system

A group of L experts will assess the healthcare system’s membership to each of the defined stages, using linguistic labels {totally disagree, strongly disagree, disagree, neutral, true, and very true}. Each linguistic label will be defined with the following membership functions $\mu_k, k = 1, \dots, 6 : \{0.0, 0.2, 0.4, 0.6, 0.8, 1.0\}$, which permit the index of each stage j to be obtained as an average of the products of these membership functions by the number of experts who assess each stage with each one of the k degrees.

In short, this attempts to overcome the problem

$$I_j = \frac{1}{L} \sum_{k=1}^6 \mu_k a_{jk}, j = 1, 2, \dots, 5 \tag{9}$$

where L is the number of experts who assess the membership function to each of the levels of confidence in the healthcare system and a_{jk} is the number of experts that assess the j stage of the healthcare system with the linguistic label k. The total stage index I_T is obtained as:

$$I_T = \sum_{j=1}^5 j \cdot I_j \cdot \left(\sum_{j=1}^5 I_j \right)^{-1} \tag{10}$$

For its part, Expression (11) defines to what degree the healthcare system belongs to each level of the previously defined confidence levels.

$$\mu_i(I_T) = \begin{cases} I_t - i + 1 & 1 \leq i - 1 < I_T < i \\ i + 1 - I_t & i < I_T < i + 1 \leq 5, i = 1, \dots, 5 \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

Due to the very definition of the previously established levels, the result will show membership of confidence in the efficiency of the system to two of these levels simultaneously with its corresponding membership functions. Lastly, the sum of all membership functions must be equal to one.

Delimitation cost of each stage

Through BAAD-OWA the costs given by the H experts can be aggregated, the weights being different for each of the TFN extremes, and a $\gamma \in [2, 1, 0, -1, -2]$ can be applied according to the level S_1, S_2 , etc. it belongs to. In this way, the experts predicting higher costs are weighted higher if confidence in the healthcare system belongs to level S_1 ($\gamma = 2$), and they are weighted lower if their confidence in the healthcare system belongs to a level of complete confidence in S_5 ($\gamma = -2$). The weights $\omega_{s_j}^r$ for each of the r extremes of the TFN are

$$\omega_{s_j}^r = q_i^{r,j-3} \left(\sum_{l=1}^m q_l^{r,j-3} \right)^{-1}, r = 1, 2, 3, j = 1, \dots, 5 \tag{12}$$

where j shows the level of confidence in the healthcare system and l is the lth expert with the greatest value communicated. Based on these weightings, the TFN \tilde{Q}_{s_j} is obtained for each of the levels S_1, \dots, S_5

$$Q_{s_j}^r = \sum_{l=1}^m q_l^{r,j-2} \left(\sum_{l=1}^m q_l^{r,j-3} \right)^{-1} \tag{13}$$

In this way, based on the information provided by the experts, it will be possible to estimate several costs.

Value of healthcare cost generated by taking the level of confidence in the healthcare system as a weighting factor

For this purpose, the estimated costs for each stage must be multiplied by the membership function of each one. These costs have been obtained according to Expression (13) and the membership function to each stage is determined by Expression (11). The result is the TFN \tilde{Q}_C whose r extremes are defined by:

$$Q_C^r = \sum_{j=1}^5 \mu_j(I_T) \cdot Q_{S_j}^r, \quad r = 1, 2, 3 \tag{14}$$

2.2.3. Weighting Based on the Importance Assigned to Each Expert and Final Result

The TFN obtained according to Expression (14) weights the first valuation provided by the H experts according to confidence in the healthcare System (11) without considering the relevance of the degree of confidence of each H expert that has previously taken part in the procedure.

Through the BADD-UPOWA, the confidence in each of the H selected experts can be introduced. The aggregation of the two weighting factors considered is reflected in Expression (15), whose result is an approximation of the final estimated costs

$$\tilde{Q}_F = \beta \cdot \tilde{Q}_C + (1 - \beta) \sum_{i=1}^H v_i \cdot \tilde{Q}_i \tag{15}$$

Where $\beta \in [0, 1]$ shows the importance allocated to confidence in the healthcare system as a weighing factor and $v_i \in [0, 1], \sum_i v_i = 1$ is the probability allocated to an expert i regarding the degree of confidence that they deserve. As can be seen through the β , it is possible to decide which of the two weighting factors (confidence level in the healthcare system or confidence level in the experts) will have a higher specific weight in the final valuation. The use of BADD-UPOWA in the group of new opinions allows the same previously considered weighing factors to be introduced into the analysis.

Even though the healthcare cost calculated in this way becomes a triangular fuzzy number, (TFN), in order to approximate through a certain value, the TFN can be defuzzified using any of the existing methods.

This process will be repeated for each level of worsening. The healthcare cost obtained, derived from the worsening of pathologies, will be considered constant over time, and as a result, it will be necessary to revise and update it.

2.3. Estimation of the Number of Patients with Worsening Pathologies in a Future Period t + k (for Level of Worsening)

Firstly, we carry out the estimation of row matrix $L^* = \{l_{t+k,h}^*\}, h = 1, \dots, l$ applying Expression (4) where matrix R (order $n \times 1$) has been obtained previously and matrix C^* (order $1 \times n$) has been obtained from the deaths registered during the new period of analysis t + k for each kind of patient whose pathology is worsening (i)

$$L^* = C^* \circ R \tag{16}$$

As matrix L^* gives values between 0 and 1, it will be necessary to determine the number of patients for each level of worsening. To do so, the row matrix “number of patients with worsening pathologies” has been estimated $C_L^* = \{c_{t+k,h}^*\}$. Matrix L^* shows the degree of patients’ membership to each of the levels of a worsening medical condition. Multiplying the abovementioned degree of membership by the maximum number of patients in each group, $\max_k(p_{ki})$, the number of patients for the period t + k for each level of worsening has been obtained. Each element of the matrix C_L^* is obtained as:

$$c_{lh}^* = l_{t+k,h}^* \cdot \max_k(p_{kh}), \quad h = 1, \dots, l \tag{17}$$

2.4. Quantification in Period $t + k$ of Healthcare Cost Overrun Arising from the Worsening of Pathologies

The estimation of health care cost overrun during the period $t + k$ due to delays in healthcare attention will be done by multiplying the number of patients i in the above-mentioned period, calculated according to Section 2.3, by Q_{fh} , which includes the value of the healthcare cost for each kind of patient h estimated according to Section 2.2. In this way, the resulting cost for the period $t + k$ is reflected in matrix L_L^* , which is obtained by multiplying matrix C_L^* by Q_{fh} , which behaves as a converter into monetary units for each kind of patient.

$$L_L^* = C_L^* \cdot Q_{fh} \tag{18}$$

However, once the proposed model has been applied to a future period, we must be aware of the fact that the model’s inputs are taken from information defined in past periods. Knowledge about the previous healthcare crisis, the implementation of new public policies, investments or disinvestments in the healthcare system, changes in healthcare management, etc. can make this information obsolete. New research in this field as well as the opinion of new experts would be required to update all this information. Even though it is recommended to do this regularly, given the difficulty in compiling all the accurate information, it would be interesting to extend these periods to the maximum. In this respect, we propose the following and final stage, which aims to provide an easier and cheaper way to automatically update the final data each time the proposed financial model is applied.

2.5. Adjusting the Calculated Healthcare Cost to the Situation in the Period $t + k$

Firstly, $2n + 1$ possible scenarios $A_i, i = 1, \dots, 2n + 1$ will be defined, n being the number of positive scenarios (or negative) and an additional scenario considered neutral. These scenarios will show changes in the healthcare system for the period in which the last input update (base period) took place, allocating a specific correction coefficient $m_i, i = 1, \dots, 2n + 1$ to each scenario. For the level of worsening of the situation in the base period, coefficients greater than one will be used. In this way, a corrected cost greater than the first one will be reflected, whereas in the improvement case, correction coefficients lower than the unit will be applied. The correction coefficients will be considered constants until a new comprehensive update of the model is made, when all the variables required for the model to function adequately will be estimated again.

Considering the scenarios established previously for the period in which the model is implemented (period $t + k$), a new group of experts will be required to assess the improvement or the worsening of the healthcare system with respect to the base period.

Using a similar methodology to the one presented in Section 2.2 for determining the level of confidence in the healthcare system, the experts will now compare the present period with the base period and they will be placed in one of the $2n + 1$ scenarios defined. Once again, the experts will use a six-element scale: totally disagree, strongly disagree, disagree, neutral, true, and very true. As the experts’ stance with respect to each scenario is also uncertain, the total index will be obtained according to the following expression:

$$I = \frac{1}{5} \sum_{i=1}^{2n+1} i \sum_{j=1}^6 (j-1) a_{ij} / \sum_{i=1}^{2n+1} \sum_{j=1}^6 (j-1) a_{ij} \tag{19}$$

The membership function of each stage i , $\mu_i(I)$ is obtained by applying Expression (11) to the result obtained in (19).

To consider the current situation of the healthcare system, the cost of the period $t + k$ will be obtained by multiplying the previously calculated cost by the correction factor m_{t+k} :

$$m_{t+k} = \sum_{i=1}^{2n+1} [1 + z(i - n - 1)] \tag{20}$$

where z measures the increase in the correction factor, measured as one unit because of a change in stage.

Lastly, for the financial valuation over time, the result obtained will be multiplied by a suitable price index for the period $t + k$ (P_{t+k}) including inflation from the base period. As a result, the final cost LL^* for the period $t + k$ is:

$$LL^* = L_c^* m_{t+k} P_{t+k} \tag{21}$$

3. Example of the Application of the Proposed Methodology

To illustrate the proposed financial model, the following example is proposed.

3.1. Obtaining the Relation between Deaths and the Number of Patients Whose Situation Has Worsened

In a healthcare region and for a five-year-period, the causes of death due to the worsening of a specific pathology have been analyzed and grouped (A, B, C, D, E). Each of them has been graded on a six-value scale (very slight, slight, neutral, slightly strong, strong, and very strong) according to how shortcomings in initial attention could affect the previous pathology. In this sense, for the first year and cause of death A (Table 1), there is a neutral influence for 3 deaths, slightly strong influence for 6, strong for 9, and very strong for 13.

Table 1. Impact of shortcomings in healthcare attention on each cause of death (A, B, C, D, E) and each year.

Year	A	B	C	D	E
1	{0; 0; 3; 6; 9; 13}	{0; 0; 2; 4; 5; 8}	{0; 0; 3; 8; 9; 14}	{0; 0; 2; 5; 8; 9}	{0; 0; 1; 2; 4; 6}
2	{0; 1; 5; 7; 10; 14}	{0; 1; 3; 4; 6; 9}	{1; 3; 6; 8; 16; 21}	{0; 3; 4; 8; 10; 13}	{0; 1; 2; 3; 6; 9}
3	{1; 2; 6; 10; 15; 19}	{0; 1; 3; 4; 9; 11}	{1; 2; 8; 10; 20; 28}	{1; 2; 5; 7; 10; 15}	{0; 2; 3; 4; 7; 10}
4	{0; 0; 4; 8; 11; 12}	{0; 1; 2; 4; 6; 7}	{0; 1; 5; 9; 13; 17}	{0; 1; 4; 6; 9; 11}	{0; 1; 1; 3; 5; 7}
5	{0; 1; 3; 4; 6; 8}	{0; 0; 1; 2; 5; 6}	{0; 0; 4; 6; 10; 13}	{0; 1; 2; 3; 5; 7}	{0; 0; 2; 2; 4; 4}

Following the proposed methodology, firstly, the values p_{ki} (1) can be obtained, and then, as shown in Table 2, matrix C shows the incidence of insufficient medical attention for each cause of death (A, B, C, D, E).

Table 2. Matrix C.

Year	A	B	C	D	E
1	0.63	0.69	0.51	0.65	0.55
2	0.71	0.80	0.78	0.95	0.85
3	1.00	1.00	1.00	1.00	1.00
4	0.68	0.69	0.65	0.80	0.68
5	0.42	0.53	0.49	0.47	0.47

In Table 3, patients have been classified for each of the five years analyzed according to the worsening of their initial situation, with the linguistic labels: a little worse, moderately worse, much worse and completely worse.

Table 3. Number of patients for each worsening level.

Year	Level of Worsening				Total
	A Little	Moderately	Much	Completely	
1	3010	2780	800	520	7110
2	2300	3260	1780	840	8180
3	1520	2540	2840	1350	8250
4	2580	2950	1450	950	7930
5	3600	1850	750	450	6650
Total					38,120

From Table 3, it is possible to obtain matrix L (3). As you can see, Table 4 shows the importance of each level of worsening per year. Therefore, it is possible to observe, for instance, how the “low” level of worsening was important in year 5 but very slight in year 3.

Table 4. Matrix L.

Year	Level of Worsening			
	A Little	Moderately	Much	Completely
1	0.78	0.98	0.33	0.45
2	0.52	1.00	0.63	0.63
3	0.34	0.77	1.00	1.00
4	0.60	0.93	0.53	0.73
5	1.00	0.70	0.33	0.41

The relation between deaths and patients whose pathology has worsened is shown in matrix R (Table 5). This matrix is obtained from (5) and relates the number of deaths due to insufficient healthcare attention to the set of patients whose situation has worsened.

Table 5. Matrix R.

	A Little Worse	Moderately Worse	Much Worse	Completely Worse
A	0.34	0.77	0.33	0.41
B	0.34	0.77	0.33	0.41
C	0.34	0.77	0.33	0.41
D	0.34	0.77	0.33	0.41
E	0.34	0.77	0.33	0.41

3.2. Cost Overrun for the Healthcare System Caused by the Worsening of Patients

A group of six experts define the cost overrun for the healthcare system per patient for each level of worsening. Each expert provides a triplet of values (Table 6) indicating the minimum cost, the most possible cost, and the maximum cost per patient that they believe this level of worsening causes in the healthcare system. In this sense, expert 1 considers that for the least worsening scenario, the most possible cost per patient is EUR 2000, although it can vary between EUR 1000 and EUR 3000.

In the same way, as can be seen in the last column of Table 6, a probability is assigned according to the level of confidence in each expert. For instance, expert 1 has been assigned a probability of 0.20, and as a result, his opinion is considered the most important.

Table 6. Cost overrun for each level of worsening.

Expert	A Little Worse	Moderately Worse	Much Worse	Completely Worse	Probability
1	(1000, 2000; 3000)	(3000, 4000; 5000)	(10,000, 11,000; 12,000)	(20,000, 23,000; 24,000)	0.20
2	(2000, 3000; 4000)	(4000, 5000; 6000)	(14,000, 15,000; 17,000)	(22,000, 24,000; 27,000)	0.18
3	(2500, 3000; 3500)	(4500, 6500; 7000)	(13,000, 14,500; 16,000)	(24,000, 26,000; 28,000)	0.18
4	(1500, 2000; 3000)	(3000, 4500; 6000)	(11,000, 14,000; 15,000)	(18,000, 20,000; 23,000)	0.18
5	(1000, 1500; 2000)	(2000, 3000; 4000)	(9000, 11,000; 13,000)	(19,000, 23,000; 24,000)	0.15
6	(1000, 1500; 2500)	(2000, 4000; 5000)	(9500, 10,500; 12,000)	(20,000, 22,000; 25,000)	0.13

To measure the degree of confidence in the healthcare system, five stages have been considered: stage 1 being the one which generates the lowest confidence and stage 5 the one which generates the highest confidence. Once again, 5 experts are required to assess each stage according to the following scale: 1 (totally disagree), 2 (strongly disagree), 3 (disagree), 4 (neutral), 5 (true), and 6 (very true). Table 7 shows the results obtained and as can be observed 3 experts have classified stage S_1 as totally disagree, 2 as strongly disagree, etc. The membership functions to stage 1 to 5 are 0.08, 0.80, 0.48, and 0.04 respectively, obtained according to (11) and lastly, the total index is 2.343.

Table 7. Confidence level in the healthcare system.

Stage	Experts	μ
S_1	(3; 2; 0; 0; 0; 0)	0.080
S_2	(0; 0; 0; 1; 3; 1)	0.800
S_3	(0; 0; 3; 2; 0; 0)	0.480
S_4	(4; 1; 0; 0; 0; 0)	0.040
S_5	(5; 0; 0; 0; 0; 0)	0.000
Total index		2.343

The application of the proposed methodology allows cost overrun to be obtained for each level of worsening (Table 8) according to the BADD-OWA (14) incorporating the probability assigned to each expert. In this sense, both weighting factors are considered equally relevant, and that is why the confidence level assigned to the healthcare system $\beta = 0.5$ is equal to the confidence in the experts. Mean value defuzzification allows the values in the last column of Table 8 to be obtained.

Table 8. Summary of cost overrun per patient by level of worsening of initial pathologies.

Level of Worsening	BADD-OWA	Probability Assigned to Experts	Cost Overrun ($\beta = 0.5$)	Defuzzified
a little	(1646.03; 2284.62; 3091.27)	(1525.00; 2212.50; 3050.00)	(1585.52; 2248.56; 3070.63)	2.28832
moderately	(3268.34; 4670.37; 5609.52)	(3162.50; 4550.00; 5550.00)	(3215.42; 4610.19; 5579.76)	4.50389
much	(11,283.03; 12,846.80; 14,343.19)	(11,187.50; 12,775.00; 14,250.00)	(11,235.26; 12,810.90; 14,296.60)	12.78842
completely	(20,625.55; 23,095.24; 25,248.63)	(20,550.00; 23,050.00; 25,175.00)	(20,587.78; 23,072.62; 25,211.81)	22.98621

3.3. Cost Overrun Estimation for Year 6

We now carry out the implementation of the model in year 6 in order to be able to financially assess the healthcare cost overrun in that year arising from shortcomings in the treatment of the pathology.

Table 9 shows the incidence of shortcomings in healthcare attention for each cause of death (A, B, C, D, E) for year 6. To do this, the same scale is used (very slight, slightly neutral, a little strong, strong, and very strong)

Table 9. Incidence of worsening of initial pathology for each cause of death in year 6.

	A	B	C	D	E
Year 6	{0; 0; 2; 4; 5; 7}	{0; 0; 1; 3; 4; 5}	{0; 0; 4; 4; 9; 11}	{0; 0; 1; 3; 4; 5}	{0; 0; 1; 2; 2; 3}

As shown in Table 10, through the implementation of the proposed model, matrix L^* of year 6 can be obtained, by multiplying matrix C^* , as defined for the abovementioned period, by matrix R (16). As a result, the maximum cost per patient is obtained for each kind of worsening (a little, moderately, much, and completely worse). The product of the corresponding element of matrix L^* by the cost per patient and by the maximum number of patients provides the final cost. Lastly, the sum of the final cost for each kind of worsening allows the cost overrun for the healthcare system to be obtained for year 6 in the healthcare region analyzed (EUR 34,448,201).

Table 10. Final cost overrun.

Matrix C^* year 6	(0.36; 0.47; 0.41; 0.35; 0.32)
Matrix L^*	(0.34; 0.47; 0.33; 0.41)
Maximum cost overrun per patient (€)	(2288.32; 4503.89; 12,788.42; 22,986.21)
Maximum number of patients per degree	(3600; 3260; 2840; 1350)
Cost overrun for the set of patients (€)	(2,803,673; 6,940,900; 11,871,103; 12,832,525)
Previous final cost (€)	34,448,201

Nevertheless, following the proposed methodology, the result obtained initially needs to be modeled according to the situation of worsening or improvement with regard to the base year; that is to say, the year in which the group of experts made the initial estimations (see Table 6). For this purpose, we have appealed to a new group of 5 experts who will state their view about the membership of each of the previously defined possible scenarios of the healthcare system at this time (Table 11). In this way, for instance, you can see how the five experts seem to completely disagree on the fact that the healthcare system is in scenarios A_1 , A_2 , and A_3 . However, with regard to A_4 , while 4 experts still disagree, there is one who totally disagrees. This table also shows the membership function of each of the defined scenarios.

Table 11. Valuation of the situation with regard to the base year (year 5).

Stage	Valuation	μ
A1	(5; 0; 0; 0; 0)	0.00
A2	(5; 0; 0; 0; 0)	0.00
A3	(5; 0; 0; 0; 0)	0.00
A4	(4; 1; 0; 0; 0)	0.04
A5	(0; 0; 1; 2; 2)	0.64
A6	(0; 0; 1; 0; 2)	0.80
A7	(4; 1; 0; 0; 0)	0.04
A8	(5; 0; 0; 0; 0)	0.00
A9	(5; 0; 0; 0; 0)	0.00
		5.55

The resulting multiplier coefficient is 1.041. Its application, considering the rate of inflation of 2% provides a final cost of 36,593,508.

4. Conclusions

Through the mathematical model developed in this paper, it is possible to obtain the healthcare cost resulting from the worsening of pathologies because of delays in treatment. In order to do this, an analysis and classification over time are made of deaths caused by a certain pathology and patients whose pathology has become worse due to a delay in treatment. Through the use of fuzzy mathematics, the information gathered can be treated and a matrix which links the relation between deaths and deteriorating patients can be obtained at any given moment.

In order to obtain the cost overrun per patient caused by the worsening of their pathologies, expert's opinions are asked for and they are weighted using BADD-OWAs. As weighting factors, both the degree of confidence given by the experts and the degree of confidence in the healthcare system are used.

To optimize the implementation of the proposed methodology in future periods, the designed model allows the healthcare cost to be adjusted to the circumstances of each period through the determination of a suitable correction factor.

To illustrate the proposed theoretical model, this paper ends with an implementation example. In this way, by knowing the number of deaths in a particular year for a certain pathology, the designed model facilitates the quantification of the cost for the healthcare system for that year due to the worsening of the analyzed pathology for all patients as a result of a delay in medical attention. In future studies, it would be of interest to apply the model to real data for a short-term horizon.

The motivation behind this study was the seriousness of the global COVID-19 pandemic and its collateral effects on different pathologies whose treatment has been postponed.

The implication of this study for researchers and public managers lies in the timeliness of the subject as well as in the innovative methodology proposed to financially approach cost overrun due to a delay in the treatment of illnesses and diseases. Leaving aside the important ethical implications and focusing exclusively on financial aspects, the cost overrun that delays in the treatment of pathologies generate brings to light the profitability of investing in healthcare resources. In this way, the application of the proposed model will provide public administrators with the economic arguments to justify these investments.

However, there are some problems that make the application of the proposed model difficult. As the quantification process requires the participation of experts, the cost of the application could turn out to be significant. Apart from this, there is the added difficulty of the availability of experts. That is why in order to try and reduce these problems, we proposed a final stage that aims to automatically update the final data each time the proposed model is applied.

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