Omega ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Omega

journal homepage: www.elsevier.com/locate/omega

Graph productivity change measure using the least distance to the pareto-efficient frontier in data envelopment analysis

Juan Aparicio [a,*], Eva M. Garcia-Nove [a], Magdalena Kapelko [b], Jesus T. Pastor [a]

[a] Center of Operations Research, University Miguel Hernandez, Elche, 03202 Alicante, Spain
[b] Institute of Applied Mathematics, Department of Logistics, Wroclaw University of Economics, Poland

ARTICLE INFO

Article history:
Received 19 August 2015
Accepted 27 October 2016

Keywords:
Data envelopment analysis
Productivity change
Closest targets
Least distances
Principle of Least Action
Graph measures

ABSTRACT

This paper proposes a new method to measure productivity change of decision making units in the full input-output space. The new approach is based on the calculation of the least distance to the Pareto-efficient frontier and hence provides the closest targets for evaluated decision making units to reach the strongly efficient frontier with least effort. Another advantage of the new methodology is that it always leads to feasible solutions. The productivity change in the new approach is operationalized as a Luenberger-type indicator in the Data Envelopment Analysis framework and it is decomposed into efficiency change and technical change. The paper empirically illustrates the new method using recent data on the Spanish quality wine sector.

1. Introduction

The measurement of productivity change over time using frontier methods continues to claim considerable attention in the literature that centers on the assessment of economic performance of decision making units (DMUs). The most popular approach to evaluate productivity change is the Malmquist productivity index introduced by Caves et al. [20] and popularized by Färe et al. [31] that made it empirically tractable in the Data Envelopment Analysis (DEA) framework allowing for decomposition of productivity change into efficiency and technical changes. Malmquist index is a ratio-based index that uses Shephard [51] distance functions to represent technology and, in its most popular forms, adopt either an input contraction or an output expansion perspective.

Meanwhile, more general indices were developed aimed to measure the productivity change in the full input-output space on the graph representation of the technology.[1] In many practical situations it is desirable to use measures that are non-oriented in which units are able to change both inputs and outputs. Chambers et al. [22] and Chambers and Pope [23] define the Luenberger productivity change indicator that is a difference-based index of directional distance functions that account for both input contractions and output improvements. Zofio and Lovell [56] define the hyperbolic Malmquist index that considers both input and output dimensions when evaluating productivity change over time. Portela and Thanassoulis [50] propose to estimate productivity change through observed values only and use the geometric distance function in the estimation of a Malmquist index that allows for simultaneous changes in inputs and outputs towards the efficient frontier, while Tone [55] defines a non-oriented Malmquist index based on the non-oriented slacks-based measure that includes both input and output slacks.

The determination of closest efficient targets and the calculation of the least distance to the efficient frontier is a recent theme that has drawn the attention of a large number of authors in the DEA literature.[2] So far, the approach based on the least distance to the frontier has been fundamentally applied to the field of technical efficiency measurement and benchmarking. Chronologically speaking, we can list the following references. Coelli [27] proposed a multi-stage methodology based on solving a sequence of radial models, seeking targets as similar as possible to the original DMU. The Joro et al. [38] approach mixes DEA and multiple objective linear programming searching for

* Corresponding author. fax: +34 966658715.
E-mail address: j.aparicio@umh.es (J. Aparicio).

[1] Referring to the assessment of efficiency in the full input-output space as a graph efficiency measurement is due to Färe et al. [30]. See also Juo et al. [41].

[2] For an evaluated DMU, its targets are the coordinates of the projection point on the estimated efficient frontier and represent levels of operation of inputs and outputs that would make the corresponding unit performs efficiently.

http://dx.doi.org/10.1016/j.omega.2016.10.005
0305-0483/

Please cite this article as: Aparicio J, et al. Graph productivity change measure using the least distance to the pareto-efficient frontier in data envelopment analysis. Omega (2016), http://dx.doi.org/10.1016/j.omega.2016.10.005

the closest projections on the frontier. Briec [14] and Briec and Lemaire [15] calculated the minimum distance to the weakly efficient frontier using Hölder norms. Frei and Harker [32] focused on determining projection points by minimizing the Euclidean distance to the efficient frontier. Cherchye and Van Puyenbroeck [21] defined the deviation between mixes in an oriented-space framework as the angle between the input vector of the assessed DMU and its projection and maximize the corresponding cosine, consequently finding the closest targets. Gonzalez and Alvarez [36] minimized the sum of input contractions required to reach the production frontier, which is equivalent to 'maximizing' the input-oriented Russell efficiency measure. Another interesting paper on closest targets is Portela et al. [49] that proposes a multistage procedure to determine the closest targets that identify all the facets of the efficient frontier. Later, Lozano and Villa [43] introduced a method that determines a sequence of targets to be achieved in successive leaps, which converge to the efficient frontier. Aparicio et al. [3] introduced a single-stage methodology based on MILP (Mixed Integer Linear Programming) for determining closest targets for any DEA measure in an easy way. More recently, other authors have focused their analysis on the Euclidean distance, as Amirteimoori and Kordrostami [1] and Aparicio and Pastor [5]. Jahanshahloo et al. [37] introduced the directional closest-target based measure of efficiency, integrating Hölder norms and directional distance functions in DEA. More recently, the literature on deriving the least distance to the frontier invokes the concept of the Principle of Least Action (PLA), which works with the notion of closeness/similarity [7]. Finally, another recent and related stream of the literature analyses the properties that the measures adapted to the PLA satisfy, as for example strong monotonicity. In this respect, we must mention Baek and Lee [10], Ando et al. [2], Aparicio and Pastor [4,6], Aparicio et al. [10] and [33,34].

Although it is usual to adapt traditional DEA technical efficiency measures for estimating productivity change over time (see, e.g., [56] and Kapelko et al.[42,43]), revising the literature about the determination of closest efficient targets, we highlight that, to the best of our knowledge, there is no paper that applies least distance for estimating productivity change and its components (technical change and efficiency change) on the graph representation of the technology.[3] An intriguing prospect is, therefore, to incorporate the idea of least distance and the Principle of Least Action into the productivity change measurement in the context of the full input-output space and Pareto-efficiency [40] and Mirdehghan and Fukuyama [51]. In this paper we aim to do so by developing a new approach to the assessment of productivity change that satisfies these characteristics. In particular, the new approach assures the determination of Pareto-efficient projections both for units located in the interior of the technology and outside the production possibility set and, additionally, it always leads to feasible results for the mixed period distances. Our approach is operationalized in the Data Envelopment Analysis framework exploiting the Luenberger-type productivity change measurement. For illustration purposes we apply the new approach to a dataset of the Spanish quality wine sector (the so called Designation of Origin of wines).

The remainder of the paper unfolds as follows. Section 2 deals with the review of existing approaches that consider both output and input dimensions simultaneously in measuring productivity change. Section 3 develops a new approach to the measurement of productivity change based on the estimation of the least distance to the Pareto-efficient frontier. The empirical application to the data on the Spanish quality wine sector is described in Section 4. Section 5 offers concluding comments.

2. Review of the literature

In this section, we briefly review existing approaches where the issue of measuring productivity change in the full input-output space has been analyzed. Let us begin with the introduction of some notation and definitions.

Consider n DMUs that use m inputs to produce s outputs. These are denoted by $\left(x_j, y_j\right)$, $j = 1, \ldots, n$. It is assumed that $x_j = \left(x_{1j}, \ldots, x_{mj}\right) \in R^m_{++}$, $j = 1, \ldots, n$, and $y_j = \left(y_{1j}, \ldots, y_{sj}\right) \in R^s_{++}$, $j = 1, \ldots, n$. The relative efficiency of each DMU_0 in the sample is assessed with reference to the production technology that is defined as follows:

$$T = \left\{(x, y)/\ x \text{ can produce } y\right\}. \tag{1}$$

T can be empirically constructed from n observations as follows:

$$T_{CRS} = \left\{(x, y) \in R^{m+s}_{+} /\ x \geq \sum_{j=1}^{n} \lambda_j x_j,\ \ y \leq \sum_{j=1}^{n} \lambda_j y_j,\ \ \lambda_j \geq 0,\ j = 1, \ldots, n\right\}. \tag{2}$$

The above formulation assumes Constant Returns to Scale (CRS). It uses λ_j as intensity weights, $\lambda_j \geq 0$. We will assume this hypothesis hereafter. Moreover, the strongly efficient frontier of T_{CRS} is defined as:

$$\partial^s(T_{CRS}) := \left\{(x, y) \in T_{CRS} : \hat{x} \leq x, \hat{y} \geq y,\ (\hat{x}, \hat{y}) \neq (x, y) \Rightarrow (\hat{x}, \hat{y}) \notin T_{CRS}\right\} \tag{3}$$

Regarding the notation for denoting observations in different periods of time, we will use hereafter when needed $\left(x^h_j, y^h_j\right)$, T^k_{CRS}, $\partial^s\left(T^k_{CRS}\right)$ with $h = t, t+1$ and $k = t, t+1$.

Now we turn to the first model that measures productivity change in the full input-output space and we introduce the efficiency measures that form the basis of the Luenberger indicator [22,23]. The Luenberger indicator is based on the directional technology distance function that for time t is defined as follows [24] [4]:

$$D^t\left(x^t, y^t; g^I, g^O\right) = \max\left\{\beta^t : \left(x^t - \beta^t g^I, y^t + \beta^t g^O\right) \in T_{CRS}\right\}, \tag{4}$$

where $g = (g^I, g^O)$ is a directional vector for inputs and outputs, respectively and β measures the degree of technical inefficiency. Directional distance function projects input and output vector from itself to the technology frontier in a pre-assigned direction given by the directional vector. Therefore, this measure does not reach the strongly efficient frontier $\partial^s(T_{CRS})$ and it does not calculate the least distance to this frontier. Directional distance function for time t can be calculated by the following DEA

[3] The only related paper within this line is Aparicio et al. [9]. However, this paper considers a specific case of input orientation and develops an approach for the measurement of input-specific productivity change. Hence, it was not developed for the general framework of the full input-output space, as the current paper does. Also, the paper by Aparicio et al. [9] develops a model that only works under the assumption of dealing with only one output.

[4] Luenberger [44,45] introduced the concept of benefit function as a representation of the amount that an individual is willing to trade, in terms of a specific reference commodity bundle g, for the opportunity to move from a consumption bundle to a utility threshold. Luenberger also defined a so-called shortage function [44]. 242, Definition 4.1), which basically measures the distance in the direction of a vector g of a production plan from the boundary of the production possibility set. In other words, the shortage function measures the amount by which a specific plan is short of reaching the frontier of the technology. In recent times, Chambers et al. [24] redefined the benefit function and the shortage function as efficiency measures, introducing to this end the so-called directional distance function.

Please cite this article as: Aparicio J, et al. Graph productivity change measure using the least distance to the pareto-efficient frontier in data envelopment analysis. Omega (2016), http://dx.doi.org/10.1016/j.omega.2016.10.005

optimization program:

$$D^t(x^t, y^t; g^I, g^O) = \max \beta^t$$
$$\text{s.t.}$$
$$\sum_{j=1}^{n} \lambda_j x_{ij}^t \le x_{i0}^t - \beta^t g_i^I, \qquad i = 1, \ldots, m$$
$$-\sum_{j=1}^{n} \lambda_j y_{rj}^t \le -y_{r0}^t - \beta^t g_r^O, \qquad r = 1, \ldots, s$$
$$\lambda_j \ge 0 \tag{5}$$

However, in order to compute the Luenberger indicator, the directional distance function for time $t+1$ needs to be estimated $\left(D^{t+1}(x^{t+1}, y^{t+1}; g^I, g^O)\right)$ together with the so called mixed-period distance functions that reflect the distance of a data point in time period t relative to the technology of period $t+1$ $\left(D^{t+1}(x^t, y^t; g^I, g^O)\right)$ as well as the distance of a data point in time period $t+1$ relative to the technology of period t $(D^t(x^{t+1}, y^{t+1}; g^I, g^O))$. It is well known that these mixed period directional distance functions can yield infeasible results [18]. Moreover, Briec and Kerstens [18] show that infeasibilities may also occur when estimating single period directional distance functions when the output direction vector is non-zero and the number of inputs is larger than or equal to two, or the directional input vector is not of full dimension whenever the output direction is null. In addition, Briec and Kerstens [19] notice that the computation of mixed period directional distance functions can lead to projections with a negative output, which in general have little meaning in standard economic production applications. In order to avoid such problem one needs to add an additional constraint into program (6): the output translated by the directional distance function into the direction of the directional vector must be positive (that is, $y_{r0} + \beta g_r^O \ge 0$). It is worth noticing that imposing this constraint may lead to additional infeasibilities. See Aparicio et al. [11] for revising other properties of the directional distance function.

A second approach is that of Zofio and Lovell [56], who define a Malmquist-type productivity change measure based on the hyperbolic distance function of Färe et al. [30]:

$$H^t(x^t, y^t) = \min\left\{\delta^t : (x^t\delta^t, y^t/\delta^t) \in T_{CRS}\right\} \tag{6}$$

This measure consists in a simultaneous equiproportionate expansion of outputs and contraction of inputs. As in the case of the directional distance function, this approach does not aim to signal Pareto-efficiency or apply the Principle of Least Action. The measure can be represented by the following nonlinear optimization problem:

$$H^t(x_0^t, y_0^t) = \min \delta^t$$
$$\text{s.t.}$$
$$\delta^t x_{i0}^t \ge \sum_{j=1}^{n} \lambda_j x_{ij}^t, \qquad i = 1, \ldots, m$$
$$y_{r0}^t/\delta^t \le \sum_{j=1}^{n} \lambda_j y_{rj}^t, \qquad r = 1, \ldots, s$$
$$\lambda_j \ge 0 \tag{7}$$

Of course as for the Luenberger indicator, to compute the hyperbolic Malmquist index, the distance function for $t+1$ as well as mixed period distance functions need to be solved in addition to (7). As it is shown in Pastor et al. [48] the nonlinear CRS hyperbolic program as represented by (7) can be linearized to the CRS input-oriented program. And an input-oriented program used to estimate mixed-period distance functions can lead to infeasibilities [39]. Hence, it can be concluded that the variation of program (7) for mixed period problems also suffers from the problem of infeasibilities.

The two remaining approaches to the assessment of productivity change in the full input-output space were developed with the Pareto-efficiency criterion in mind.

Tone [55] develops a non-oriented Malmquist index based on the non-oriented slacks-based measure of efficiency that can be computed as follows:

$$S^t(x_0^t, y_0^t) = \min\left(1 - \frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{x_{i0}^t}\right) \Big/ \left(1 + \frac{1}{s}\sum_{r=1}^{s}\frac{s_r^+}{y_{r0}^t}\right)$$
$$\text{s.t.}$$
$$x_{i0}^t = \sum_{j=1}^{n} \lambda_j x_{ij}^t + s_i^-, \qquad i = 1, \ldots, m$$
$$y_{r0}^t = \sum_{j=1}^{n} \lambda_j y_{rj}^t - s_r^+, \qquad r = 1, \ldots, s$$
$$\lambda_j \ge 0$$
$$s_i^- \ge 0$$
$$s_r^+ \ge 0 \tag{8}$$

where s^- indicates input slack (excess) and s^+ indicates output slack (shortfall).

Tone [55] shows that model (8) is equivalent to the following model:

$$S^t(x_0^t, y_0^t) = \min\left(\frac{1}{m}\sum_{i=1}^{m}\theta_i\right) \Big/ \left(\frac{1}{s}\sum_{r=1}^{s}\tau_r\right)$$
$$\text{s.t.}$$
$$\theta_i x_{i0}^t \ge \sum_{j=1}^{n} \lambda_j x_{ij}^t \qquad i = 1, \ldots, m$$
$$\tau_r y_{r0}^t \le \sum_{j=1}^{n} \lambda_j y_{rj}^t \qquad r = 1, \ldots, s$$
$$\lambda_j \ge 0$$
$$\theta_i \le 1$$
$$\tau_r \ge 1 \tag{9}$$

The above two models are meant to be solved for units within the production possibility set (that is, the units that lie below the frontier), which occurs when solving the single-period distance functions, that is for time t and time $t+1$. For units located outside the technology (that is, the units that lie beyond the frontier), the occurrence of which can be associated with cross period evaluations, Tone [55] proposes to solve another problem. In particular, assuming that we estimate the distance of a data point in time period t relative to the technology of period $t+1$, the following problem needs to be solved (so called super-slack based measure):

$$S^{t+1}(x_0^t, y_0^t) = \min_{\bar{x},\bar{y}}\left(\frac{1}{m}\sum_{i=1}^{m}\frac{\bar{x}_i}{x_{i0}^t}\right) \Big/ \left(\frac{1}{s}\sum_{r=1}^{s}\frac{\bar{y}_r}{y_{r0}^t}\right)$$
$$\text{s.t.}$$
$$\bar{x}_i \ge \sum_{j=1}^{n} \lambda_j x_{ij}^{t+1} \qquad i = 1, \ldots, m$$
$$\bar{y}_r \le \sum_{j=1}^{n} \lambda_j y_{rj}^{t+1} \qquad r = 1, \ldots, s$$
$$\lambda_j \ge 0$$
$$\bar{x} \ge x_0^t$$
$$\bar{y} \le y_0^t \tag{10}$$

Tone [55] shows that model (10) is equivalent to the following model:

$$S^{t+1}(x_0^t, y_0^t) = \min\left(\frac{1}{m}\sum_{i=1}^{m}\theta_i\right) \Big/ \left(\frac{1}{s}\sum_{r=1}^{s}\tau_r\right)$$
$$\text{s.t.}$$

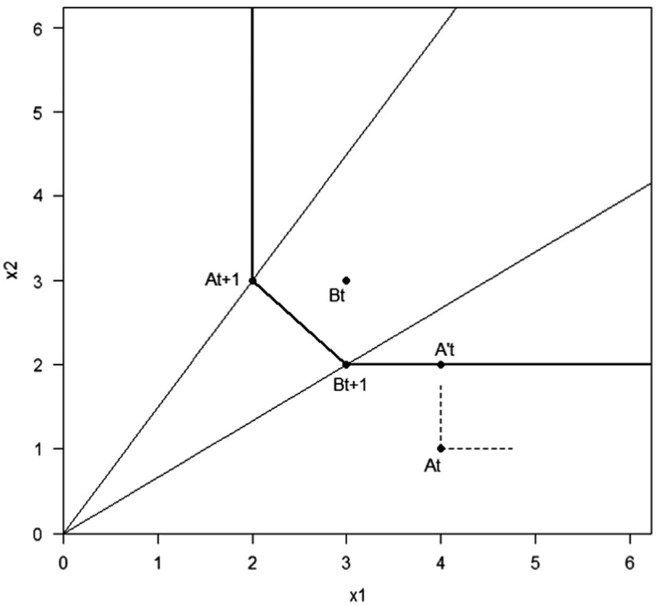

Fig. 1. Example for Tone's [55] model not reaching Pareto efficiency.

$$\begin{aligned} &\theta_i x_{i0}^t \geq \sum_{j=1}^{n} \lambda_j x_{ij}^{t+1} && i = 1, \ldots, m \\ &\tau_r y_{r0}^t \leq \sum_{j=1}^{n} \lambda_j y_{rj}^{t+1} && r = 1, \ldots, s \\ &\lambda_j \geq 0 \\ &\theta_i \geq 1 \\ &\tau_r \leq 1 \end{aligned} \tag{11}$$

Hence, Tone's [55] approach by incorporating a computation solution for units outside technology, avoids the problem of infeasibilities in the productivity change measure.[5] Moreover, models (8) and (9) resort to the strongly efficient frontier as any slacks-based measure does (see [46,54]). However, a problem appears when using models (10) and (11) for which the subset of the corresponding frontier used to calculate the distance between the assessed unit and the technology does not always match the strongly efficient frontier. Let us demonstrate it by a simple example in Fig. 1.

In Fig. 1 we consider two units A and B, which consume two inputs to produce one output. Units At+1=(2,3,1) and Bt+1=(3,2,1) determine the strongly efficient frontier of the production possibility set in time *t+1*, which consists of the convex combinations of the points on the ray that passes through At+1 and the ray that passes through Bt+1. The figure also shows unit A observed in the period *t*, At =(4,1,1). If we want to evaluate unit A observed in period *t* (At) with respect to the technology in *t+1*, we have to compute a mixed period distance function. In addition, in this case, unit At is outside the production possibility set in *t+1*, that is we need to solve models (10) or (11). The application of models (10) and (11) obliges to increase inputs and decrease output ($\theta_i \geq 1, \tau_r \leq 1$) for At. In particular, the projection point is A't=(4,2,1). So, for this example, the resulted projection through (10) or (11) does not reach the strongly efficient frontier. Hence, Tone's [55] model does not always lead to a Pareto-efficient solution for units outside of the technology. In this respect, an additional issue that deserves to be studied is how often this situation happens. However, it is outside the scope of this paper.

Portela and Thanassoulis [50] propose the usage of the geometric distance function in the estimation of productivity change in the full input-output space, which is defined as below:

$$GDF^t(x^t, y^t) = \left(\prod_i \theta_i\right)^{\frac{1}{m}} / \left(\prod_r \tau_r\right)^{\frac{1}{s}} \tag{12}$$

The geometric distance function is defined as the ratio between the geometric mean of θ_i (that is the ratio between a target input and an observed input i), and the geometric mean of τ_r (that is the ratio between a target output and an observed output r). In general, geometric distance function is meant to be used after targets have been computed by any known procedure. In particular, Portela and Thanassoulis [50] show the following model to determine the targets for single period computations[6]:

$$\begin{aligned} EFF = \ &\min \frac{\theta}{\tau} \\ s.t. \\ &\sum_{j \in E^t} \lambda_j x_{ij}^t \leq \theta x_{i0}^t, && i = 1, \ldots, m \\ &\sum_{j \in E^t} \lambda_j y_{rj}^t \geq \tau y_{r0}^t, && r = 1, \ldots, s \\ &\sum_{j \in E^t} \lambda_j = 1, \\ &\lambda_j \geq 0, && j \in E^t \\ &0 \leq \theta \leq 1, \\ &\tau \geq 1 \end{aligned} \tag{13}$$

E^k denotes the set of Pareto-efficient DMUs observed in period k (k=t, $t+1$). For mixed period distance functions, when observation lies above the frontier, the model (13) changes slightly and becomes:

$$\begin{aligned} EFF = \ &\min \frac{\theta}{\tau} \\ s.t. \\ &\sum_{j \in E^{t+1}} \lambda_j x_{ij}^{t+1} \leq \theta x_{i0}^t, && i = 1, \ldots, m \\ &\sum_{j \in E^{t+1}} \lambda_j y_{rj}^{t+1} \geq \tau y_{r0}^t, && r = 1, \ldots, s \\ &\sum_{j \in E^{t+1}} \lambda_j = 1, \\ &\lambda_j \geq 0, && j \in E^{t+1} \\ &\theta \geq 1, \\ &0 \leq \tau \leq 1 \end{aligned} \tag{14}$$

Hence, Portela and Thanassoulis´ [50] approach, by incorporating models with solutions for units located both inside and outside technology, does not suffer from the infeasibility problem for similar reasons as Tone [55]. However, this framework does not always lead to Pareto-efficient targets. In particular, it is not true that "We assure that Pareto-efficient targets result from the linear combinations of the λ by restricting the reference set to Pareto-efficient units (units in the set E)" ([50], p. 40). To support our claim, let us consider a numerical example of four units A, B, C and D, which consume two inputs to produce two outputs: A=

[5] Note that if we only use models (8) and (9) for evaluating any observation, regardless of whether it is located inside or outside the reference technology, then we can have problems related to the infeasibility of the models. Let us assume that we have observed one unit in period t that consumes one input to produce one output, under Constant Returns to Scale, At=(1,1), and that we have observed the same unit in period t+1, At+1=(0.5,2). In this case, if we use, for example, model (8) to evaluate At+1 with respect to the technology in t, then the infeasibility problem would occur, since model (8) projects the evaluated units following a monotone scheme (decreasing inputs and increasing outputs). Something similar happens with Portela and Thanassoulis' approach.

[6] Portela and Thanassoulis [50] assume Variable Returns to Scale in order to estimate productivity change over time. However, this hypothesis contrasts to the usual assumption of Constant Returns to Scale followed by most researchers (see, for example, [42]).

(1,3,2,1), B=(3,1,2,1), C=(2,2,2,4) and D=(2,2,2,2), and assume Variable Returns to Scale (VRS) as Portela and Thanassoulis [50] did. The calculations using the additive model of Charnes et al. [26] for this dataset show that the Pareto efficient set E={A,B,C}. Now we want to evaluate unit D through model (13). On the one hand, from the input constraints and the convexity restriction in this model, we have that $\lambda_A+3\lambda_B+2\lambda_C \le \theta 2$, $3\lambda_A+\lambda_B+2\lambda_C \le \theta 2$ and $\lambda_A+\lambda_B+\lambda_C=1$. Substituting λ_C by $1-\lambda_A-\lambda_B$ in the two first inequalities, we get $-\lambda_A+\lambda_B \le \theta 2-2$ and $\lambda_A-\lambda_B \le \theta 2-2$. Summing up the two last inequalities yields $0 \le \theta 4-4$, which is equivalent to $\theta \ge 1$. However, following model (13), $0 \le \theta \le 1$. Therefore, $\theta^* = 1$ at the optimum when unit D is assessed. On the other hand, the constraint associated with the first output is $2\lambda_A+2\lambda_B+2\lambda_C \ge \tau 2$, which is equivalent to $\tau \le 1$ since $\lambda_A+\lambda_B+\lambda_C=1$. Then, using the model constraint $\tau \ge 1$, we have that $\tau^*=1$ at the optimum. In this way, the optimal value of model (13) when unit D is evaluated is equal to 1. In particular, an optimal solution of this model is $(\theta^*, \tau^*, \lambda_A^*, \lambda_B^*, \lambda_C^*)$ with $\theta^*=1$, $\tau^*=1$, $\lambda_A=\lambda_B=\lambda_C=\frac{1}{3}$. However, following this solution, the determined projection point would be D'=D=(2,2,2,2), which is dominated by unit C in the sense of Pareto. Hence, Portela and Thanassoulis' [50] approach does not always lead to Pareto-efficient projections, even when the evaluated unit belongs to the reference technology.

Another interesting feature of Tone's and Portela and Thanassoulis' approaches is the direction that the evaluated units follow when they are projected by the corresponding models. Both approaches resort to monotone procedures for adjusting inputs and outputs in order to reach the frontier of the production possibility set. The implication of this is twofold. On the one hand, it means that when the assessed DMU belongs to the reference technology, the models, (8) or (9) for Tone's approach and (13) for Portela and Thanassoulis' method, project units looking for target points on the frontier with less or equal quantity of each input and more or equal quantity of each output. On the other hand, if the assessed unit does not belong to the reference technology, something that can only occur with mixed period evaluations, then the corresponding models, (10) or (11) for Tone's approach and (14) for Portela and Thanassoulis' method, project units looking for target points on the frontier with more or equal quantity of each input and less or equal quantity of each output, i.e. the opposite direction to what was followed in the first scenario. However, as we next show, any approach based on monotone procedures for projecting units onto the frontier satisfies at most one property between Pareto-efficiency and feasibility. In order to show that, let us first point out that if using a monotone procedure is the focus and the evaluated unit belongs to the reference technology, then there will always be at least one Pareto-efficient target point as a candidate to be the final projection, i.e. a Pareto-efficient point that dominates the assessed unit. Consequently, the problem we want to show can only occur when the evaluated unit is outside the reference technology.

In order to show that monotone procedures of projection do not fit well with Pareto-efficiency when the evaluation corresponds to a unit that does not belong to the production possibility set, let us consider the numerical example associated with Fig. 2. In this way, let us assume that the reference technology T_{CRS} is estimated from the observation of exclusively one unit, A=(3,2,1), which consumes two inputs to produce one output under Constant Returns to Scale and that we want to evaluate a unit that is outside the technology through a monotone scheme. In particular, we are referring to unit B=(4,1,1) (see Fig. 2). Following a monotone scheme would imply determining a target point with expression $(4+s_1^-, 1+s_2^-; 1-s_1^+)$, satisfying s_1^-, s_2^-, $s_1^+ \ge 0$ and belonging to $\partial^s(T_{CRS})$. However, in this simple example $\partial^s(T_{CRS}) = \{(x_1,x_2,y): \ (x_1,x_2,y)=\gamma(3,2,1),\ \gamma \ge 0\}$. So, we now want to prove that $(4+s_1^-, 1+s_2^-; 1-s_1^+) \notin \partial^s(T_{CRS})$ for any

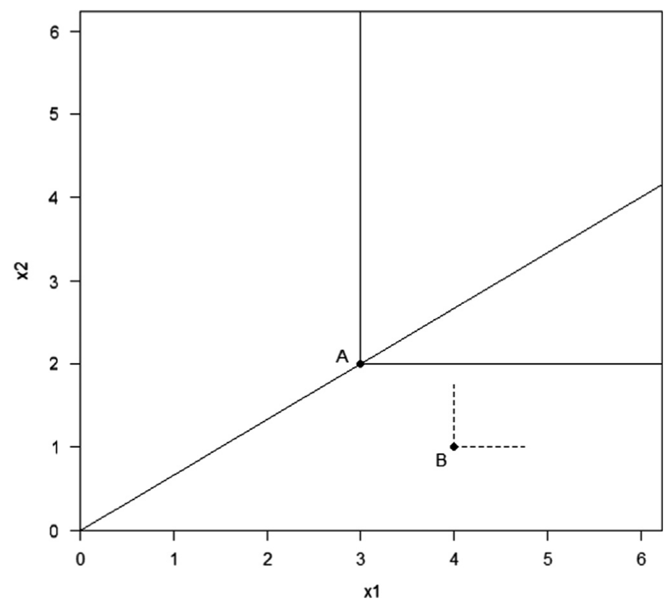

Fig. 2. Example showing that Pareto-efficiency is not compatible with monotone procedures of projection.

considered combination of s_1^-, s_2^-, $s_1^+ \ge 0$. Let $(4+s_1^-, 1+s_2^-; 1-s_1^+) = \gamma(3,2,1)$. From the output component, we have that $\gamma = 1-s_1^+$. Substituting in $4+s_1^- = 3\gamma$, we get $4+s_1^- = 3-3s_1^+$, which is equivalent to $s_1^- + 3s_1^+ = -1$. However, the left hand side of the last equality is non-negative by s_1^-, $s_1^+ \ge 0$ and the right hand side is strictly negative. This is the contradiction we were seeking. Therefore, this example illustrates the fact that if we resort to monotone procedures, then we cannot assure Pareto-efficiency.

Before finalizing this section, it is worth mentioning that we focused our attention on three properties: Pareto-efficiency, feasibility and monotonicity[7] with respect to the adjustment of inputs and outputs. We are aware that this was arbitrary and it was due to the fact that the new approach that will be introduced in the next section satisfies, in particular, two of these properties: Pareto-efficiency and feasibility. Note that, following our discussion above, the satisfaction of both properties is not compatible with monotone procedures of projection. In other words, it is not possible to define a measure that satisfies the three properties at the same time. Consequently, and giving priority to Pareto-efficiency, the new approach will need to abandon monotone schemes of projection in order to yield target points on the Pareto-efficient frontier both for units located in the interior of the technology and outside the production possibility set. Nevertheless, if the fulfillment of other properties were the focus, we are aware that other approaches would be preferred by practitioners. For example, if the possibility of determining a statistical measure of goodness of fit is important, then the approaches based on the directional distance function and the hyperbolic measure would be the best options (see [25] and [29]). Additionally, if resorting to a more flexible graph measure, which permits different changes in inputs and outputs, is the objective, then using the Malmquist index based on the slacks-based measure would the best choice [54]. Finally, if the practitioners and researchers would wish to utilize a productivity change measure based upon the combination of partial productivities, then the suitable alternative would be the

[7] In this paper, we distinguish between monotonicity with respect to the projection of the evaluated unit onto the frontier of the technology and the property of (weak and strong) monotonicity of the values of the technical efficiency measure (see Definitions 1 and 2 in Section 3.3).

Geometric Distance Function by Portela and Thanassoulis [50]. Anyway, and focusing our attention back on the aforementioned three properties, i.e. Pareto-efficiency, feasibility and monotonicity with respect to the projection of the assessed unit, if practitioners would prefer feasibility and monotonicity before Pareto-efficiency, then the best option between Tone's approach and Portela and Thanassoulis' method seems to be the former, since, as we have shown, both approaches guarantee feasibility and monotonicity but Tone's approach additionally assures Pareto-efficiency for contemporaneous evaluations, while Portela and Thanassoulis' method can fail even in this context.

After revising the literature, we can summarize that none of the existing approaches are based on measures that yield the closest Pareto-efficient targets, despite this line of research has attracted the attention of a large number of researchers in recent times. In this way, our main aim in this paper is to endow DEA practitioners and theoretical researchers with a methodology for estimating productivity change directly associated with the determination of the least distance to the Pareto efficient frontier, assuring both Pareto-efficiency and feasibility.

3. A new Luenberger indicator using the least distance to the Pareto-efficient frontier

Once we have shown in Section 2 that none of the existing graph approaches for estimating productivity change have as an objective to apply the Principle of Least Action, we introduce in this section a new method based on the determination of closest Pareto-efficient targets to measure and decompose productivity change over time in the full input-output space.

3.1. A graph measure based on the least distance to the strongly efficient frontier

From a computational point of view, trying to minimize a certain distance to the Pareto-efficient frontier of a reference technology from a unit located inside the production possibility set is not an easy task, since it is equivalent to minimizing the distance to the complement of a polyhedral set, which is not a convex set [13]. This was one of the reasons why Aparicio et al. [3] characterized the set of Pareto-efficient points in the technology dominating DMU_0 by means of a set of linear constraints and a set of continuous and binary variables. This characterization allowed them to determine Pareto-efficient closest targets by means of a Mixed Integer Linear Program in a single step, in contrast to many other alternatives that resorted to the determination of all the Pareto-efficient faces of the technology, a NP-hard problem (see [8]).

We invoke in this section the main theorem in Aparicio et al. [3] in order to define our model. However, we will need to slightly modify this result to suit our context. Specifically, we will characterize the set of Pareto efficient points in general, i.e., without restricting the analysis to the points dominating the assessed DMU_0. Additionally, to do so, we will not resort to a big *M* and binary variables as Aparicio et al. [3] did, instead we will utilize a logical relationship that will be computationally implemented by means of a Special Ordered Set (SOS)[8] [11].

[8] SOS is a way to specify the number of nonzero solution values among a set of variables without the need of resorting to fixing a big *M*. The optimizers usually achieve it by using special branching strategies. Traditionally, SOS was used with discrete and integer variables, but modern optimizers, like for example CPLEX, use also SOS with continuous variables.

Theorem 1. *Let* $(x,y)\in R_+^{m+s}$. *Then,* $(x,y)\in\partial^s(T_{CRS})$ *if and only if* $\exists\lambda_j, d_j\ge 0,\quad j\in E,\quad with \nu_i\ge 1,\quad i=1,\dots,m and\quad \mu_r\ge 1,\quad r=1,\dots,s,$ *such that.*

$$\begin{aligned}
&x_i=\sum_{j\in E}\lambda_j x_{ij}, && i=1,\dots,m\\
&y_r=\sum_{j\in E}\lambda_j y_{rj}, && r=1,\dots,s\\
&\sum_{i=1}^{m}\nu_i x_{ij}-\sum_{r=1}^{s}\mu_r y_{rj}-d_j=0, && j\in E\\
&\text{In}(\lambda_j,d_j)\ \text{only one variable can be strictly positive,} && j\in E
\end{aligned}\tag{15}$$

where the set *E* contains the indexes of the Pareto-efficient DMUs.

Proof. See the proof of the Theorem on page 211 in Aparicio et al. [3] and note also that constraints (t.6) and (t.7) in Aparicio et al. [3] have as objective to avoid that the two variables in the pair (λ_j,d_j) are strictly positive at the same time. ∎

Once the strongly efficient frontier has been mathematically characterized, the introduction of a new graph measure based on the Principle of Least Action to be used for estimating productivity change is possible. To do that, we will use a new version of a weighted additive model. In particular, we will resort to the well-known Measure of Inefficiency Proportions (MIP) by Cooper et al. [28]. In this paper, however, we define the MIP under the application of the Principle of Least Action, allowing to calculate the least distance from the evaluated unit to the Pareto-efficient frontier of the reference technology. Next, we slightly change the used notation in order to accommodate the possibility of cross-evaluation. In this way, the 'compact' format of the new measure will be as follows:

$$WA_k^{PLA}\left(x_0^h,y_0^h\right)=Min\left\{\begin{array}{l}\sum_{i=1}^{m}\frac{\left|s_{i0}^-\right|}{x_{i0}^h}+\sum_{r=1}^{s}\frac{\left|s_{r0}^+\right|}{y_{r0}^h}:\\ (x_{10}^h-s_{10}^-,\dots,x_{m0}^h-s_{m0}^-,y_{10}^h+s_{10}^+,\dots,y_{s0}^h+s_{s0}^+)\in\partial^s\left(T_{CRS}^k\right)\end{array}\right\}.\tag{16}$$

In (16), we consider all the possibilities with respect to indexes *k* and *h*: ($k = t$ *and* $h = t$), ($k = t$ *and* $h = t+1$), ($k = t+1$ *and* $h = t$) and, finally, ($k = t+1$ *and* $h = t+1$). If we compare (16) versus the traditional MIP, some significant differences can be found. First, 'Max' has been substituted by 'Min'. This implies that (16) seeks slacks as close to zero as possible and, consequently by the constraint, (16) determines the closest Pareto-efficient targets to (x_0^h,y_0^h). Second, the slacks are free variables in (16), whereas the slacks were non-negative in the traditional MIP. Third, the objective function uses the absolute value function to aggregate slacks, something that did not happen with the original measure. These three differences are needed for dealing with the possibility of projecting the evaluated unit onto the Pareto-efficient set following a non-monotone procedure of adjusting inputs and outputs so as to reach the strongly efficient frontier.

Fundamentally, for evaluated units located outside the reference technology, this means that our methodology could generate targets that are not dominated by the assessed DMU_0. We illustrate this situation using again Fig. 1. In this graphical example units At+1=(2,3,1) and Bt+1=(3,2,1) determine the strongly efficient frontier of the production possibility set. If the distance from the unit At=(4,1,1), located outside the production possibility set, is evaluated with respect to the frontier of T_{CRS} in time t+1, we find that usual approaches, as those mentioned in Section 2, would project At following a monotone scheme, looking for a target point on the frontier with more or equal quantity of input and less or equal quantity of output. However, a movement of this nature avoids projecting unit At onto the Pareto-efficient subset of the frontier (consisting of the convex combinations of the points on

Please cite this article as: Aparicio J, et al. Graph productivity change measure using the least distance to the pareto-efficient frontier in data envelopment analysis. Omega (2016), http://dx.doi.org/10.1016/j.omega.2016.10.005

the ray that passes through At+1 and the ray that passes through Bt+1). In our model, the projection of DMU At onto unit Bt+1 is completely feasible, even meaning a decrease in the first input for DMU_0. The case of unit Bt=(3,3,1) is different. Bt is located inside the production possibility set in t+1. For this type of unit, it is not difficult to prove that there is always at least a Pareto-efficient point that dominates it (for example units At+1 or Bt+1, in the case of Bt). So, approaches like those proposed by Portela and Thanassoulis [50] or Tone [55] would generate a Pareto-efficient point as a target following a monotone scheme, decreasing the input and increasing the output. Our approach will also produce a Pareto-efficient target for unit Bt, but in our case the projection will follow a monotone scheme if and only if the strongly efficient target determined following that 'direction' (i.e. decreasing inputs and increasing outputs) is really the closest to the evaluated DMU. Therefore, we highlight that the projection generated by our model could follow a non-monotone scheme for units as Bt. Regarding this point, we are not the first to introduce a measure that yields this type of targets for interior DMUs. Ando et al. [2], Zofio et al. [58] and Fukuyama et al. [33] are recent examples of this kind of approach.

Model (16) is not a standard mathematical program, making its implementation difficult in practice. In this sense, the first step to write an equivalent standard 'linear' program consists of applying Theorem 1. In this way, (16) can be rewritten as program (17).

$$
\begin{array}{llll}
WA_k^{PLA}(x_0^h, y_0^h) = & Min & \left(\sum_{i=1}^{m} \frac{|s_{i0}^-|}{x_{i0}^h} + \sum_{r=1}^{s} \frac{|s_{r0}^+|}{y_{r0}^h} \right) & (17.1) \\
& s.t. & & \\
& & \sum_{j \in E^k} \lambda_{j0} x_{ij}^k = x_{i0}^h - s_{i0}^-, \quad i=1,\dots,m & (17.2) \\
& & \sum_{j \in E^k} \lambda_{j0} y_{rj}^k = y_{r0}^h + s_{r0}^+, \quad r=1,\dots,s & (17.3) \\
& & \sum_{i=1}^{m} \nu_{i0} x_{ij}^k - \sum_{r=1}^{s} \mu_{r0} y_{rj}^k - d_{j0} = 0, \quad j \in E^k & (17.4) \\
& & \nu_{i0} \ge 1, \quad i=1,\dots,m & (17.5) \\
& & \mu_{r0} \ge 1, \quad r=1,\dots,s & (17.6) \\
& & (\lambda_{j0}, d_{j0}) \quad SOS \quad j \in E^k & (17.7) \\
& & \lambda_{j0}, \nu_{i0}, \mu_{r0}, d_{j0} \ge 0, \quad \forall i,r,j & (17.8)
\end{array}
\tag{17}
$$

The measure derived from the optimal value of model (17) satisfies several interesting properties. Among them, $WA_k^{PLA}(x_0^h, y_0^h) = 0$ if and only if $(x_0^h, y_0^h) \in \partial^s\left(T_{CRS}^k\right)$ and (17) is always feasible since $\partial^s\left(T_{CRS}^k\right) \neq \varnothing$.

Program (17) determines the least distance between (x_0^h, y_0^h) and $\partial^s\left(T_{CRS}^k\right)$ and always generates Pareto-efficient targets. However, it cannot be considered a distance function, as for example the directional distance function, because the value of the model is always positive regardless of whether the assessed unit is inside or outside the reference technology. To define a suitable distance function from model (17), we first need to determine whether or not (x_0^h, y_0^h) belongs to T_{CRS}^k from any real distance function. To determine that, one can compute the directional distance function of Chambers et al. [24], model (5), resorting, for example, to the reference vector $g = (1_m, 1_s)$, where 1_p is a vector of p ones. In particular, $D^k(x_0^h, y_0^h; 1_m, 1_s) < 0$ if and only if $(x_0^h, y_0^h) \notin T_{CRS}^k$.[9] In this way, from the optimal value of (17) we can derive the following additive-type distance function:

$$
D^k\left(x_0^h, y_0^h\right) = \begin{cases} WA_k^{PLA}(x_0^h, y_0^h), & \text{if } (x_0^h, y_0^h) \in T_{CRS}^k \\ -WA_k^{PLA}(x_0^h, y_0^h), & \text{Otherwise} \end{cases} \tag{18}
$$

Another interesting property that merits attention is the possible existence of a relationship between the approach based on the determination of the least distance to the Pareto-efficient frontier and a function with economical meaning, like the profit, cost or revenue functions. In this respect, as far as we are aware, there is not much literature. Applying Duality Theory and Convex Analysis, Briec and Lesourd [16] and Briec and Leleu [17] showed that Hölder distance functions have a dual relationship with the profit function. However, these authors focused their analysis on the weakly efficient frontier. Although the study of this association in our context is outside the scope of this paper, we have highlighted this topic as an interesting avenue for additional research at the end of the Conclusions Section.

3.2. Productivity change and its decomposition using a Luenberger-type indicator

Now we are ready to measure productivity change by defining a Luenberger-type indicator and decompose it into its usual components. The productivity change for unit 0 is measured by means of:

$$
\begin{aligned}
TFPCH_0(t, t+1) = \frac{1}{2}\Big[& (D^t(x_0^t, y_0^t) - D^t(x_0^{t+1}, y_0^{t+1})) \\
& + \left(D^{t+1}(x_0^t, y_0^t) - D^{t+1}(x_0^{t+1}, y_0^{t+1}) \right) \Big].
\end{aligned} \tag{19}
$$

The new Luenberger indicator may then be decomposed into efficiency change - catch-up (EFFCH) and frontier shift (TECHCH):

$$
\begin{aligned}
& EFFCH_0(t, t+1) = D^t(x_0^t, y_0^t) - D^{t+1}(x_0^{t+1}, y_0^{t+1}), \\
& TECHCH_0(t, t+1) = \frac{1}{2}\Big[\left(D^{t+1}(x_0^{t+1}, y_0^{t+1}) - D^t(x_0^{t+1}, y_0^{t+1}) \right) \\
& \qquad + \left(D^{t+1}(x_0^t, y_0^t) - D^t(x_0^t, y_0^t) \right) \Big].
\end{aligned} \tag{20}
$$

3.3. Least distance and the property of monotonicity

Some authors had paid attention to the satisfaction of interesting properties of DEA measures based on the Principle of Least Action and the determination of the least distance. In particular, Pastor and Aparicio [47], Ando et al. [2], Aparicio and Pastor [4], Fukuyama et al. [33], Aparicio and Pastor [6] and Fukuyama et al. [35] focused their attention on the property of monotonicity of the values of the technical inefficiency measure. In that context, monotonicity relates technical inefficiency to Pareto dominance. Specifically, if unit A dominates unit B in the Pareto sense (i.e., A consumes less inputs and produces more outputs than B), then the measure of inefficiency associated to A should be lower than the measure of inefficiency of B. In fact, two ways to formulate this property exist: a strong and a weak version.

Definition 1. [strong monotonicity]. Let $I: R_+^m \times R_+^s \to R_+$ be an inefficiency index. I satisfies strong monotonicity if $I(x,y) < I(\tilde{x}, \tilde{y})$ for all feasible vectors (x,y) and $(\tilde{x}, \tilde{y})$ with $(x,y) \neq (\tilde{x}, \tilde{y})$ and $(x, -y) \le (\tilde{x}, -\tilde{y})$.

Definition 2. [weak monotonicity]. Let $I: R_+^m \times R_+^s \to R_+$ be an inefficiency index. I satisfies weak monotonicity if $I(x,y) \le I(\tilde{x}, \tilde{y})$ for all feasible vectors (x,y) and $(\tilde{x}, \tilde{y})$ with $(x,y) \neq (\tilde{x}, \tilde{y})$ and $(x, -y) \le (\tilde{x}, -\tilde{y})$.

It is clear that if I satisfies strong monotonicity, then it is also weakly monotonic, but the opposite is not true. These same

[9] In fact, from a computational burden point of view, it is sufficient to estimate mixed period directional distance functions, because only in that case can occur that (x_0^h, y_0^h) does not belong to T_{CRS}^k.

notions can be applied to distance functions through Definitions 3 and 4.

Definition 3. Let $D: R^m_+ \times R^s_+ \to R$ be a distance function. D satisfies strong monotonicity if $D(x,y) < D(\tilde{x},\tilde{y})$ for all vectors $(x,y) \in R^m_+ \times R^s_+$ and $(\tilde{x},\tilde{y}) \in R^m_+ \times R^s_+$ with $(x,y) \neq (\tilde{x},\tilde{y})$ and $(x,-y) \leq (\tilde{x},-\tilde{y})$.

Definition 4. Let $D: R^m_+ \times R^s_+ \to R$ be a distance function. D satisfies weak monotonicity if $D(x,y) \leq D(\tilde{x},\tilde{y})$ for all vectors (x,y) and $(\tilde{x},\tilde{y})$ with $(x,y) \neq (\tilde{x},\tilde{y})$ and $(x,-y) \leq (\tilde{x},-\tilde{y})$.

It is well-known that Shephard's distance functions and directional distance functions are weakly monotonic (see, for example, Lemma 2.2 (d) and (e) in [24]). However, least distance measures satisfy neither weak nor strong monotonicity when the reference set is the strongly efficient frontier [2].[10] What is less known in the literature is that this problem can affect the interpretation of productivity change indexes. Next, we show these implications in the case of the Luenberger indicator.

Let us suppose that for unit 0 we have that $(x^t_0, -y^t_0) \geq (x^{t+1}_0, -y^{t+1}_0)$ with $(x^t_0, y^t_0) \neq (x^{t+1}_0, y^{t+1}_0)$ and $(x^t_0, y^t_0), (x^{t+1}_0, y^{t+1}_0) \in T^t_{CRS}$. It follows from $(x^{t+1}_0, y^{t+1}_0) \in T^t_{CRS}$ that we can observe the changes of unit 0 from (x^t_0, y^t_0) to (x^{t+1}_0, y^{t+1}_0) on the production possibility set T^t_{CRS}. Furthermore, we may also observe it on the production possibility set T^{t+1}_{CRS} since $(x^{t+1}_0, y^{t+1}_0) \in T^{t+1}_{CRS}$ and $(x^t_0, -y^t_0) \geq (x^{t+1}_0, -y^{t+1}_0)$ imply that $(x^t_0, y^t_0) \in T^{t+1}_{CRS}$. Under these hypotheses unit 0 improves its performance from period t to t+1 under either T^t_{CRS} or T^{t+1}_{CRS}. In this way, we conclude that $TFPCH_0(t,t+1)$ should indicate productivity growth for unit 0 in this context.

Now, if the distance function $D^k(x^h_0, y^h_0)$ would satisfy weak monotonicity, then, in the considered context, $D^t(x^t_0, y^t_0) \geq D^t(x^{t+1}_0, y^{t+1}_0)$ and $D^{t+1}(x^t_0, y^t_0) \geq D^{t+1}(x^{t+1}_0, y^{t+1}_0)$. Therefore, $TFPCH_0(t,t+1) = \frac{1}{2}\left[\left(D^t(x^t_0,y^t_0) - D^t(x^{t+1}_0,y^{t+1}_0)\right) + \left(D^{t+1}(x^t_0,y^t_0) - D^{t+1}(x^{t+1}_0,y^{t+1}_0)\right)\right] \geq 0$, indicating no regress for unit 0. This scenario corresponds to the directional distance function for the Luenberger indicator and the Shephard distance functions for the Malmquist index. In this last case, the Malmquist index would take a value greater or equal to one. Additionally, if $D^k(x^h_0, y^h_0)$ would meet strong monotonicity, then $D^t(x^t_0, y^t_0) > D^t(x^{t+1}_0, y^{t+1}_0)$ and $D^{t+1}(x^t_0, y^t_0) > D^{t+1}(x^{t+1}_0, y^{t+1}_0)$ and so, consequently, $TFPCH_0(t,t+1) > 0$, signaling productivity growth, as expected.

Unfortunately, the inefficiency measure derived from model (17) satisfies neither weak nor strong monotonicity, as would be expected for measures that determine closest targets and least distance. This implies that $TFPCH_0(t,t+1)$ may be inconsistent in a setting like that described above.[11] In order to correct this inconsistency, we may apply some of the solutions proposed in the literature for least distance measures. Fundamentally, we can find two lines of research. On the one hand, Aparicio and Pastor [4] and Aparicio and Pastor [6] suggest modifying the shape of the production possibility set in DEA, extending the full dimensional efficient facets (FDEF), if they exist. On the other hand, Ando et al. [2], Fukuyama et al. [33] and Fukuyama et al. [35] propose to alter the original definition of the technical inefficiency measure in order to fulfill monotonicity. In particular, Ando et al. [2] focused their attention on the satisfaction of the weak monotonicity property, while the remaining authors focused on the strong version of the property. In this paper, with the aim of illustrating a possible solution for the detected problem associated with model (17), we will resort to the Ando et al. approach, since it can be easily implemented from a computational point of view. Nevertheless, we would like to highlight that any of the previously mentioned approaches could also be utilized in practice for correcting the measure and deriving a technical index thereby also satisfying strong monotonicity.

Ando et al. [2] prove that the next technical inefficiency measure is weakly monotonic.

$$\hat{f}^{wp}(x_0,y_0) = \min\{\|((\hat{x},\hat{y})-(x',y'))Z(x_0,y_0)\|_p : (x',y') \in \partial^s(T_{CRS}), (\hat{x},\hat{y}) \in Q(x_0,y_0)\}, \quad (21)$$

where $\|z\|_p = \left(\sum_{j=1}^{g} |z_j|^p\right)^{1/p}$ for $p \in [1,\infty)$, $\|z\|_\infty = \max_{j=1,\dots,g}\{|z_j|\}$, $Z(x_0,y_0)$ is the matrix $\begin{bmatrix} diag(x_0)^{-1} & 0 \\ 0 & diag(y_0)^{-1} \end{bmatrix}$ and $Q(x_0,y_0) = \{(x_0+d^x, y_0-d^y) : d^x \geq 0, d^y \geq 0\}$. Regarding the used notation, z denotes a vector of dimension g and, following the notation of Ando et al. [2], $diag(z)^{-1}$ is the diagonal matrix $\begin{bmatrix} z_1^{-1} & 0 & \cdots & 0 \\ 0 & z_2^{-1} & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & z_g^{-1} \end{bmatrix}$.

Note that, in our context, the objective function of (17) coincides with $\sum_{i=1}^{m} \frac{|s^-_{i0}|}{x^h_{i0}} + \sum_{r=1}^{s} \frac{|s^+_{r0}|}{y^h_{r0}} = \|((x^h_0, y^h_o)-(x',y'))Z(x^h_0,y^h_o)\|_1$, which is in the line of the objective function of (21). Therefore, we need to modify (17) as follows.

If $(x^h_0, y^h_0) \in T^k_{CRS}$, then (22) should be computed.

$$\begin{aligned}
\hat{f}^{wp+}(x^h_0,y^h_0) = \ & Min \ \left(\sum_{i=1}^{m}\frac{|\hat{x}_i - x'_i|}{x^h_{i0}} + \sum_{r=1}^{s}\frac{|\hat{y}_r - y'_r|}{y^h_{r0}}\right) && (22.1)\\
& s.t. \\
& \sum_{j \in E^k} \lambda_{j0} x^k_{ij} = x'_i, \quad i=1,\dots,m && (22.2)\\
& \sum_{j \in E^k} \lambda_{j0} y^k_{rj} = y'_r, \quad r=1,\dots,s && (22.3)\\
& \hat{x}_i \geq x^h_{i0}, \quad i=1,\dots,m && (22.4)\\
& \hat{y}_r \leq y^h_{r0}, \quad r=1,\dots,s && (22.5)\\
& \sum_{i=1}^{m}\nu_{i0}x^k_{ij} - \sum_{r=1}^{s}\mu_{r0}y^k_{rj} - d_{j0} = 0, \quad j \in E^k && (22.6)\\
& \nu_{i0} \geq 1, \quad i=1,\dots,m && (22.7)\\
& \mu_{r0} \geq 1, \quad r=1,\dots,s && (22.8)\\
& (\lambda_{j0}, d_{j0}) \ \ SOS \quad j \in E^k && (22.9)\\
& x'_i, y'_r, \hat{x}_i, \hat{y}_r, \lambda_{j0}, \nu_{i0}, \mu_{r0}, d_{j0} \geq 0, \quad \forall i,r,j && (22.10)
\end{aligned} \quad (22)$$

By analogy, if $(x^h_0, y^h_0) \notin T^k_{CRS}$, then (23) should be solved, where (22.4) and (22.5) have been substituted by (23.4) and (23.5),

[10] In particular, Ando et al. [2] studied Hölder distance functions.

[11] The comments of one of the reviewers motivated Subsection 3.3 in this paper. Specifically, this reviewer pointed out that it is possible to add a new unit to the database utilized in the empirical section (Section 4) and show a situation like that described in these paragraphs. In particular, the 'virtual' unit to be added could be $(x^{2010-2011}, y^{2010-2011}) = (5,850,\ 570;\ 8,432,\ 55,000)$ and $(x^{2011-2012}, y^{2011-2012}) = (5,850,\ 570;\ 8,432,\ 55,236)$. So, this new unit satisfies that $(x^t_0, -y^t_0) \geq (x^{t+1}_0, -y^{t+1}_0)$ with $(x^t_0, y^t_0) \neq (x^{t+1}_0, y^{t+1}_0)$ and $(x^t_0, y^t_0), (x^{t+1}_0, y^{t+1}_0) \in T^t_{CRS}$. Consequently, $TFPCH_0(t,t+1)$ should be greater than zero. However, if expressions (17)–(19) are used for computing the Luenberger indicator for this new unit, we get that $TFPCH_0(t,t+1) = -0.0017 < 0$.

respectively.

$$\hat{f}^{wp-}(x_0^h, y_0^h) = \quad Min \quad \left(\sum_{i=1}^{m} \frac{|\hat{x}_i - x'_i|}{x_{i0}^h} + \sum_{r=1}^{s} \frac{|\hat{y}_r - y'_r|}{y_{r0}^h} \right) \quad (23.1)$$

$$\begin{array}{lll} s.t. & & \\ \sum_{j \in E^k} \lambda_{j0} x_{ij}^k = x'_i, & i = 1, \dots, m & (23.2) \\ \sum_{j \in E^k} \lambda_{j0} y_{rj}^k = y'_r, & r = 1, \dots, s & (23.3) \\ \hat{x}_i \le x_{i0}^h, & i = 1, \dots, m & (23.4) \\ \hat{y}_r \ge y_{r0}^h, & r = 1, \dots, s & (23.5) \\ \sum_{i=1}^{m} \nu_{i0} x_{ij}^k - \sum_{r=1}^{s} \mu_{r0} y_{rj}^k - d_{j0} = 0, & j \in E^k & (23.6) \\ \nu_{i0} \ge 1, & i = 1, \dots, m & (23.7) \\ \mu_{r0} \ge 1, & r = 1, \dots, s & (23.8) \\ (\lambda_{j0}, d_{j0}) \quad SOS & j \in E^k & (23.9) \\ x'_i, y'_r, \hat{x}_i, \hat{y}_r, \lambda_{j0}, \nu_{i0}, \mu_{r0}, d_{j0} \ge 0, & \forall i, r, j & (23.10) \end{array} \quad (23)$$

Now, we need to derive an additive-type distance function from the optimal values of (22) and (23), as we did with respect to (17):

$$\hat{D}^k\left(x_0^h, y_0^h\right) = \begin{cases} \hat{f}^{wp+}(x_0^h, y_0^h), & \text{if } (x_0^h, y_0^h) \in T_{CRS}^k \\ -\hat{f}^{wp-}(x_0^h, y_0^h), & \text{Otherwise} \end{cases} \quad (24)$$

Finally, a Luenberger-type indicator may be computed using expressions (19) and (20) by substituting $D^k(x_0^h, y_0^h)$ by $\hat{D}^k(x_0^h, y_0^h)$. This indicator does not suffer from the problem of inconsistency described above, at least in a 'weak' sense. In this respect, it is worth mentioning that the traditional Luenberger indicator based on the directional distance function satisfies exactly the same property, since this distance only satisfies weak monotonicity.

We illustrate the new methodology by an empirical example in the next section.

4. An empirical illustration

4.1. Data

The data on the Spanish Designation of Origin (DO) of wines used in the empirical application was collected from the Spanish Ministry of Agriculture, Food and the Environment (*Ministerio de Agricultura, Alimentación y Medio Ambiente*). In total, there are 69 DOs in Spain, which produce a variety of wines including white, red, rosé and sparkling wines. To make our sample homogenous, we delimit our analysis to DOs which engage in the production of red wine as the main activity. Moreover, the sample is homogenized in the way that we include DOs of similar size.[12] The final dataset used in the analysis consists of 21 DOs for three recent seasons of wine production: 2010–2011, 2011–2012 and 2012–2013 (balanced panel). This time span allows us to estimate productivity change for two periods (between the 2010–2011 and 2011–2012 seasons, and the 2011–2012 and 2012–2013 seasons).

Two outputs and two inputs are distinguished in the estimation of the new Luenberger indicator and its components. The two outputs include sales volume in domestic markets (in hectoliters) and the sales volume in foreign markets (in hectoliters). The two inputs consist in the surface area (in hectares) and the number of winegrowers. Table 1 shows the descriptive statistics for the input-output variables for the sample for the entire period analyzed.

[12] In particular, we look at the ratio of the value of total sales (domestic and international) over growing surface (in hectares). We removed these DOs for which this ratio is smaller than 10 (to remove very small DOs) and larger than 50 (to remove very large DOs). We also removed DOs where the number of winegrowers was equal to 1 as they were considerably deviating from the rest of the sample.

Table 1
Descriptive statistics of Spanish DO data, seasons 2010–2011, 2011–2012 and 2012–2013.

Statistics	Inputs		Outputs	
	Surface volume (ha)	Winegrowers	Sales in domestic market (hl)	Sales in international market (hl)
Mean	9533.714	2800.810	176810.651	117091.873
Standard deviation	13689.638	4135.773	382178.426	216919.202
Minimum	220	228	2273	12
Maximum	63330	17258	1763315	1021206

As the reported statistics show, despite focusing the sample on DOs of similar size, the dataset still entails considerable variability. In particular, the standard deviations relative to their respective means, are relatively high, pointing to the presence of some variability in the sample.

4.2. Results

This section focuses on reporting the results of the new Luenberger indicator developed in this paper based upon a least distance measure satisfying the property of weak monotonicity. To determine a reference in this application and compare the magnitude of the estimations with respect to the new Luenberger-type approach, we also calculate the traditional Luenberger indicator of Chambers et al. [22] and Chambers and Pope [23] (with $g^I = x_0$ and $g^O = y_0$),[13] as well as computing the indicator derived from Tone´s slacks-based measure.[14] It is worth noticing that the slacks-based measure is traditionally used in the context of Malmquist-type indexes, for example by Tone [55]. Because the comparison of magnitudes between the Malmquist index and the Luenberger indicator is not straightforward (see for example, the discussion by Boussemart et al. [12]), we transform the slacks-based measure into an inefficiency measure (that is we apply the formula: inefficiency = 1-efficiency) and then using such inefficiency measure we derive the corresponding Luenberger-type index.

Table 2 summarizes the results of distance functions that form the basis of the three indicators described above, that is single period and mixed period distance functions for the seasons 2010–2011 and 2011–2012 for each individual DO as well as the average for the whole sample. Table 3 records the results of productivity change (TFPCH) and its decomposition into efficiency change (EFFCH) and technical change (TECHCH) with regard to the three approaches described above between the 2010–2011 and 2011–2012 seasons for each individual DO as well as the average for the whole sample. These two tables also show the results of the Simar and Zelenyuk [53] test (denoted further as S-Z test) that allows the assessment of the statistical significance of the differences between distance functions as well as productivity change measures and their decomposition indicators.[15] In addition, Table 4 reports correlations between these three Luenberger-based indicators.

[13] As shown by Briec and Kerstens [18] the advantage of the choice of actual input and output values as directional vectors is that infeasibilities are avoided.

[14] The slacks-based measure is based on further targets but it is comparable with the new approach in the sense that both models take into account input and output slacks. The Luenberger indicator based on the directional distance function neglects slacks but it is the traditional reference when productivity change is estimated in Data Envelopment Analysis in an additive way.

[15] This test adapts the nonparametric test of the equality of two densities developed by Li [41] and is based on the computation and bootstrapping the Li statistic.

Please cite this article as: Aparicio J, et al. Graph productivity change measure using the least distance to the pareto-efficient frontier in data envelopment analysis. Omega (2016), http://dx.doi.org/10.1016/j.omega.2016.10.005

Table 2
Distance functions for seasons 2010–2011 and 2011–2012.

DO	Traditional Luenberger indicator				Indicator derived from the slacks-based measure				New indicator			
	$D^t(x^t,y^t)$	$D^t(x^{t+1},y^{t+1})$	$D^{t+1}(x^{t+1},y^{t+1})$	$D^{t+1}(x^t,y^t)$	$D^t(x^t,y^t)$	$D^t(x^{t+1},y^{t+1})$	$D^{t+1}(x^{t+1},y^{t+1})$	$D^{t+1}(x^t,y^t)$	$D^t(x^t,y^t)$	$D^t(x^{t+1},y^{t+1})$	$D^{t+1}(x^{t+1},y^{t+1})$	$D^{t+1}(x^t,y^t)$
ALICANTE	0.244	0.313	0.290	0.224	0.665	0.724	0.784	0.734	0.972	1.095	0.895	0.699
BIERZO	0.137	0.376	0.402	0.225	0.804	0.752	0.800	0.823	1.450	1.227	1.251	1.097
BULLAS	0.867	0.811	0.813	0.869	0.957	0.911	0.927	0.965	1.854	1.737	1.721	1.861
CALATAYUD	0.372	0.206	0.265	0.425	0.673	0.735	0.740	0.695	0.742	0.633	0.660	0.688
CAMPO DE BORJA	0.144	0.148	0.238	0.235	0.458	0.457	0.521	0.522	0.394	0.379	0.447	0.460
CARIÑENA	0.000	−0.112	0.000	0.075	0.000	−0.184	0.000	0.163	0.000	−0.311	0.000	0.190
COSTERS DEL SEGRE	0.465	0.330	0.273	0.445	0.726	0.693	0.750	0.776	1.192	0.993	0.803	1.135
EMPORDÀ	0.244	0.256	0.227	0.211	0.713	0.756	0.812	0.777	1.130	1.273	0.718	0.679
MONTSANT	0.213	0.268	0.301	0.255	0.603	0.615	0.622	0.626	0.483	0.653	0.596	0.461
NAVARRA	0.016	0.019	0.023	0.012	0.064	0.069	0.152	0.138	0.044	0.053	0.054	0.083
PRIORAT	0.533	0.377	0.403	0.550	0.735	0.619	0.677	0.782	1.339	0.991	0.981	1.340
RIBEIRA SACRA	0.360	0.315	0.410	0.452	0.993	0.990	0.989	0.993	1.974	1.961	1.893	1.927
RIBERA DEL DUERO	0.048	−0.109	0.000	0.143	0.712	−0.106	0.000	0.736	1.365	−0.196	0.000	0.364
RIOJA	0.000	−0.017	0.000	0.002	0.000	−0.016	0.000	0.028	0.000	−0.048	0.000	0.010
SOMONTANO	0.000	0.048	0.000	−0.083	0.000	0.185	0.000	−0.083	0.000	0.137	0.000	−0.152
TORO	0.403	0.466	0.459	0.432	0.626	0.775	0.817	0.693	1.013	1.264	1.274	1.012
UCLÉS	0.463	0.560	0.575	0.413	0.827	0.787	0.801	0.859	1.418	1.394	1.380	1.149
VALDEPEÑAS	0.186	0.128	0.138	0.168	0.438	0.292	0.430	0.547	0.513	0.367	0.375	0.475
VALENCIA	0.000	0.013	0.000	−0.033	0.000	0.078	0.000	−0.031	0.000	0.071	0.000	−0.065
YCODEN-DAUTE-ISORA	0.486	0.486	0.565	0.565	0.988	0.998	0.998	0.987	1.951	1.991	1.977	1.865
YECLA	0.276	0.230	0.330	0.373	0.832	0.892	0.889	0.823	0.878	1.277	1.318	0.911
Average	0.260	0.243	0.272	0.284	0.563	0.525	0.558	0.598	0.891	0.807	0.778	0.771
S-Z test	a, b, c	a, b, c	a, b, c	a, b, c								

a denotes significant differences between distance functions for the traditional Luenberger indicator and for the slacks-based measure at the critical 5% level (results for $D^t(x^t,y^t)$ are reported in the first column, for $D^t(x^{t+1},y^{t+1})$ in the second column, for $D^{t+1}(x^{t+1},y^{t+1})$ in the third column, and for $D^{t+1}(x^t,y^t)$ in the fourth column).
b denotes significant differences between distance functions for the traditional Luenberger indicator and for the new indicator at the critical 5% level.
c denotes significant differences between distance functions for the new indicator and for the slacks-based measure at the critical 5% level.

Table 3
Productivity change and its decomposition between seasons 2010–2011 and 2011–2012.

DO	Traditional Luenberger indicator			Indicator derived from the slacks-based measure			New indicator		
	TFPCH	EFFCH	TECHCH	TFPCH	EFFCH	TECHCH	TFPCH	EFFCH	TECHCH
ALICANTE	−0.067	−0.046	−0.021	−0.054	−0.119	0.065	−0.160	0.077	−0.237
BIERZO	−0.208	−0.265	0.057	0.038	0.004	0.034	0.034	0.199	−0.165
BULLAS	0.056	0.054	0.002	0.042	0.030	0.012	0.128	0.133	−0.005
CALATAYUD	0.163	0.107	0.056	−0.054	−0.067	0.013	0.069	0.082	−0.013
CAMPO DE BORJA	−0.004	−0.095	0.091	0.000	−0.064	0.064	0.015	−0.052	0.067
CARIÑENA	0.094	0.000	0.094	0.174	0.000	0.174	0.250	0.000	0.250
COSTERS DEL SEGRE	0.153	0.192	−0.039	0.029	−0.024	0.053	0.265	0.388	−0.123
EMPORDÀ	−0.014	0.017	−0.031	−0.039	−0.099	0.060	−0.091	0.412	−0.503
MONTSANT	−0.051	−0.088	0.037	−0.004	−0.019	0.015	−0.153	−0.113	−0.040
NAVARRA	−0.007	−0.007	0.000	−0.010	−0.089	0.079	0.010	−0.010	0.020
PRIORAT	0.151	0.130	0.021	0.110	0.058	0.052	0.354	0.359	−0.005
RIBEIRA SACRA	0.044	−0.049	0.093	0.004	0.004	0.000	0.023	0.080	−0.057
RIBERA DEL DUERO	0.150	0.048	0.102	0.777	0.712	0.065	0.963	1.365	−0.402
RIOJA	0.010	0.000	0.010	0.022	0.000	0.022	0.029	0.000	0.029
SOMONTANO	−0.066	0.000	−0.066	−0.134	0.000	−0.134	−0.145	0.000	−0.145
TORO	−0.045	−0.056	0.011	−0.136	−0.191	0.055	−0.256	−0.261	0.005
UCLÉS	−0.130	−0.113	−0.017	0.049	0.026	0.023	−0.103	0.038	−0.141
VALDEPEÑAS	0.044	0.048	−0.004	0.131	0.008	0.123	0.123	0.138	−0.015
VALENCIA	−0.023	0.000	−0.023	−0.054	0.000	−0.054	−0.068	0.000	−0.068
YCODEN-DAUTE-ISORA	0.000	−0.079	0.079	−0.010	−0.010	0.000	−0.076	−0.026	−0.050
YECLA	0.044	−0.054	0.098	−0.064	−0.058	−0.006	−0.403	−0.440	0.037
Average	0.014	−0.012	0.026	0.039	0.005	0.034	0.038	0.113	−0.074
S-Z test	-, b, c	-, -, c	-, b, c						

a denotes significant differences between component for the traditional Luenberger indicator and for the slacks-based measure at the critical 10% level (results for TFPCH are reported in the first column, for EFFCH in the second column, and for TECHCH in the third column).
b denotes significant differences between component for the traditional Luenberger indicator and for the new indicator at the critical 10% level.
c denotes significant differences between component for the new indicator and for the slacks-based measure at the critical 10% level.

Please cite this article as: Aparicio J, et al. Graph productivity change measure using the least distance to the pareto-efficient frontier in data envelopment analysis. Omega (2016), http://dx.doi.org/10.1016/j.omega.2016.10.005

Table 4
Correlations between indicators, data between seasons 2010–2011 and 2011–2012.

	Traditional Luenberger indicator	Indicator derived from the slacks-based measure	New indicator
TFPCH			
Traditional Luenberger indicator	1.000	0.410	0.580
Indicator derived from slacks-based measure	0.410	1.000	0.897
New indicator	0.580	0.897	1.000
EFFCH			
Traditional Luenberger indicator	1.000	0.167	0.351
Indicator derived from slacks-based measure	0.167	1.000	0.840
New indicator	0.351	0.840	1.000
TECHCH			
Traditional Luenberger indicator	1.000	0.289	0.306
Indicator derived from slacks-based measure	0.289	1.000	0.182
New indicator	0.306	0.182	1.000

Table 5
Distance functions for seasons 2011–2012 and 2012–2013.

DO	Traditional Luenberger indicator				Indicator derived from the slacks-based measure				New indicator			
	$D^t(x^t,y^t)$	$D^t(x^{t+1},y^{t+1})$	$D^{t+1}(x^{t+1},y^{t+1})$	$D^{t+1}(x^t,y^t)$	$D^t(x^t,y^t)$	$D^t(x^{t+1},y^{t+1})$	$D^{t+1}(x^{t+1},y^{t+1})$	$D^{t+1}(x^t,y^t)$	$D^t(x^t,y^t)$	$D^t(x^{t+1},y^{t+1})$	$D^{t+1}(x^{t+1},y^{t+1})$	$D^{t+1}(x^t,y^t)$
ALICANTE	0.290	0.362	0.362	0.290	0.784	0.183	0.820	0.772	0.895	1.085	1.267	1.137
BIERZO	0.402	0.363	0.324	0.372	0.800	0.212	0.786	0.797	1.251	1.240	1.147	1.159
BULLAS	0.813	0.663	0.662	0.809	0.927	0.148	0.847	0.926	1.721	1.589	1.590	1.706
CALATAYUD	0.265	0.353	0.366	0.278	0.740	0.161	0.841	0.743	0.660	1.206	1.176	0.636
CAMPO DE BORJA	0.238	0.247	0.244	0.249	0.521	0.462	0.520	0.485	0.447	0.429	0.416	0.416
CARIÑENA	0.000	−0.035	0.000	−0.059	0.000	1.035	0.000	−0.097	0.000	−0.067	0.000	−0.176
COSTERS DEL SEGRE	0.273	0.433	0.422	0.272	0.750	0.731	0.728	0.754	0.803	0.882	0.832	0.965
EMPORDÀ	0.227	0.092	0.092	0.226	0.812	0.719	0.713	0.799	0.718	0.452	1.156	1.307
MONTSANT	0.301	0.393	0.393	0.294	0.622	0.676	0.664	0.630	0.596	1.030	1.011	0.548
NAVARRA	0.023	0.057	0.058	0.025	0.152	0.233	0.233	0.146	0.054	0.141	0.152	0.070
PRIORAT	0.403	0.532	0.537	0.406	0.677	0.769	0.763	0.666	0.981	1.288	1.261	0.923
RIBEIRA SACRA	0.410	0.490	0.400	0.311	0.989	0.993	0.992	0.989	1.893	1.922	1.972	1.962
RIBERA DEL DUERO	0.000	0.176	0.087	−0.113	0.000	0.725	0.734	−0.110	0.000	0.478	1.290	−0.203
RIOJA	0.000	−0.018	0.000	−0.002	0.000	−0.024	0.000	0.006	0.000	−0.040	0.000	−0.004
SOMONTANO	0.000	−0.003	0.000	−0.007	0.000	−0.003	0.000	−0.009	0.000	−0.006	0.000	−0.017
TORO	0.459	0.480	0.480	0.459	0.817	0.801	0.804	0.821	1.274	1.244	1.179	1.243
UCLÉS	0.575	0.561	0.569	0.573	0.801	0.810	0.810	0.796	1.380	1.387	1.366	1.364
VALDEPEÑAS	0.138	0.096	0.074	0.117	0.430	0.359	0.262	0.377	0.375	0.251	0.205	0.339
VALENCIA	0.000	0.030	0.000	−0.034	0.000	0.130	0.000	−0.035	0.000	0.100	0.000	−0.075
YCODEN-DAUTE-ISORA	0.565	0.426	0.330	0.483	0.998	0.987	0.986	0.998	1.977	1.865	1.952	1.992
YECLA	0.330	0.255	0.286	0.360	0.889	0.921	0.915	0.884	1.318	1.537	1.520	1.294
Average	0.272	0.283	0.271	0.253	0.558	0.525	0.591	0.540	0.778	0.858	0.928	0.790
S-Z test	a, b, c	a, b, c	a, b, c	a, b, c								

a denotes significant differences between distance functions for the traditional Luenberger indicator and for the slacks-based measure at the critical 5% level (results for $D^t(x^t,y^t)$ are reported in the first column, for $D^t(x^{t+1},y^{t+1})$ in the second column, for $D^{t+1}(x^{t+1},y^{t+1})$ in the third column, and for $D^{t+1}(x^t,y^t)$ in the fourth column).
b denotes significant differences between distance functions for the traditional Luenberger indicator and for the new indicator at the critical 5% level.
c denotes significant differences between distance functions for the new indicator and for the slacks-based measure at the critical 5% level.

The results of distance functions in Table 2 show that all measures are significantly different between them as confirmed by the results of S-Z test. Turning to the results of productivity change and its components in Table 3, we can observe that in all three approaches there are a similar number of DOs experiencing productivity decrease and productivity growth between the 2010–2011 and 2011–2012 seasons. This translates into the average values for the whole sample which indicate productivity increases in the analyzed period for all analyzed approaches. The analysis of the components of productivity change finds that, on average, for the traditional Luenberger approach, efficiency change has a negative contribution to productivity growth, whereas technical change offers a positive contribution. The opposite pattern is observed for the Luenberger indicator based on the least distance to the Pareto-efficient frontier, that on average shows how efficiency change increases and technical change decreases in the analyzed period. The indicators derived from the slacks-based measure show, on average, the positive contributions of efficiency change and technical change to productivity growth. The results in Table 3 further indicate that for many individual DOs the results of efficiency and technical changes are opposite between the three approaches. For example, the DO "Bierzo" experiences efficiency decline and technical progress over the analyzed period according to the traditional Luenberger indicator, progress in efficiency and technology for the indicator derived from the slacks-based measure, while the new indicator signals the efficiency increase and technical regress for this DO. Worthy of note, however, is that as shown by S-Z test results, while we can confirm that the differences in productivity change and technical change between the traditional Luenberger indicator and the new approach, and the

Table 6
Productivity change and its decomposition between seasons 2011–2012 and 2012–2013.

DO	Traditional Luenberger indicator			Indicator derived from the slacks-based measure			New indicator		
	TFPCH	EFFCH	TECHCH	TFPCH	EFFCH	TECHCH	TFPCH	EFFCH	TECHCH
ALICANTE	−0.072	−0.072	0.000	0.276	−0.036	0.312	−0.160	−0.372	0.212
BIERZO	0.044	0.078	−0.034	0.299	0.013	0.286	0.013	0.105	−0.092
BULLAS	0.149	0.151	−0.002	0.429	0.080	0.349	0.124	0.131	−0.007
CALATAYUD	−0.088	−0.101	0.013	0.240	−0.101	0.341	−0.544	−0.517	−0.027
CAMPO DE BORJA	−0.002	−0.006	0.004	0.012	0.001	0.011	0.008	0.030	−0.022
CARIÑENA	−0.012	0.000	−0.012	−0.566	0.000	−0.566	−0.055	0.000	−0.055
COSTERS DEL SEGRE	−0.156	−0.150	−0.006	0.022	0.022	0.000	0.028	−0.028	0.056
EMPORDÀ	0.135	0.135	0.000	0.090	0.099	−0.009	0.209	−0.438	0.647
MONTSANT	−0.095	−0.092	−0.003	−0.044	−0.042	−0.002	−0.449	−0.415	−0.034
NAVARRA	−0.033	−0.035	0.002	−0.084	−0.081	−0.003	−0.085	−0.099	0.014
PRIORAT	−0.130	−0.134	0.004	−0.095	−0.086	−0.009	−0.322	−0.280	−0.042
RIBEIRA SACRA	−0.084	0.010	−0.094	−0.003	−0.003	0.000	−0.020	−0.079	0.059
RIBERA DEL DUERO	−0.188	−0.087	−0.101	−0.785	−0.734	−0.051	−0.985	−1.290	0.305
RIOJA	0.008	0.000	0.008	0.015	0.000	0.015	0.018	0.000	0.018
SOMONTANO	−0.002	0.000	−0.002	−0.003	0.000	−0.003	−0.005	0.000	−0.005
TORO	−0.021	−0.021	0.000	0.016	0.013	0.003	0.048	0.095	−0.047
UCLÉS	0.009	0.006	0.003	−0.011	−0.009	−0.002	−0.005	0.014	−0.019
VALDEPEÑAS	0.042	0.064	−0.022	0.093	0.168	−0.075	0.129	0.170	−0.041
VALENCIA	−0.032	0.000	−0.032	−0.083	0.000	−0.083	−0.087	0.000	−0.087
YCODEN-DAUTE-ISORA	0.146	0.235	−0.089	0.012	0.012	0.000	0.076	0.025	0.051
YECLA	0.074	0.043	0.031	−0.032	−0.026	−0.006	−0.222	−0.202	−0.020
Average	−0.015	0.001	−0.016	−0.010	−0.034	0.024	−0.109	−0.150	0.041
S-Z test	-, -, -	-, -, c	-, b, c						

a denotes significant differences between component for the traditional Luenberger indicator and for the slacks-based measure at the critical 10% level (results for TFPCH are reported in the first column, for EFFCH in the second column, and for TECHCH in the third column).
b denotes significant differences between component for the traditional Luenberger indicator and for the new indicator at the critical 10% level.
c denotes significant differences between component for the new indicator and for the slacks-based measure at the critical 10% level.

Table 7
Correlations between indicators, data between seasons 2011–2012 and 2012–2013.

	Traditional Luenberger indicator	Indicator derived from the slacks-based measure	New indicator
TFPCH			
Traditional Luenberger indicator	1.000	0.449	0.674
Indicator derived from slacks-based measure	0.449	1.000	0.533
New indicator	0.674	0.533	1.000
EFFCH			
Traditional Luenberger indicator	1.000	0.400	0.401
Indicator derived from slacks-based measure	0.400	1.000	0.850
New indicator	0.401	0.850	1.000
TECHCH			
Traditional Luenberger indicator	1.000	0.130	-0.178
Indicator derived from slacks-based measure	0.130	1.000	0.042
New indicator	−0.178	0.042	1.000

differences in productivity change, efficiency change and technical change between the new approach and the indicator derived from the slacks-based measure are statistically significant, however, the same cannot be said for the differences in productivity change, efficiency change and technical change between traditional Luenberger indicator and slacks-based measure, and for efficiency change between the traditional Luenberger indicator and the new indicator. The correlations contained in Table 4 provide additional insights into the differences and similarities between indicators derived from the three approaches considered. According to these results, the new approach derives the measures that are mostly similar to the slacks-based approach, at least with regard to efficiency change and productivity change since correlations for technical change component are relatively low.

Tables 5 and 6 show the results for distance functions and indicators, respectively, derived using the three approaches for the 2011–2012 and 2012–2013 seasons for each individual DO as well as the average for the whole sample together with the S-Z test results for the differences between all measures. Table 7 pictures the correlations between the three productivity measures considered and their decomposition indicators.

The results in Table 5 indicate, similarly to those of the previous period, that all differences between distance functions are statistically significant. Table 6 reveals, on average, the negative productivity change experienced by DOs in the sample between the 2011–2012 and 2012–2013 seasons in all three approaches. Although the new approach estimates the largest values of productivity decline, the results of S-Z test show that these differences between all three indicators are not statistically significant. From the results, we also observe that the average indicators of efficiency change and technical change for the new indicator and the indicator derived from the slacks-based measure present similar patterns, suggesting negative efficiency change and technical progress, while the traditional Luenberger indicator reports the opposite result, that is efficiency increase and technical regress. The results of correlations contained in Table 7 suggest that the only relatively high correlations observed are those between the new approach and the indicator derived from the slacks-based measure and only with regard to efficiency change.

Overall, these are interesting findings since we are able to demonstrate the differences between the values of productivity indicators obtained using three different approaches. Hence, depending on the method the practitioner chooses, fulfilling some desired characteristics and properties, the results obtained will tell a different story. This heterogeneity in findings arise from, among others: 1) the differences in the objective functions between approaches, 2) each approach uses a different set as reference targets (weakly versus strongly efficient frontier), 3) the new approach does not follow the monotone procedures of projection as the indicators derived from the directional distance function and the slacks-based measure do and 4) the differences in the philosophy followed to project units onto the frontier (closest versus furthest targets).

5. Conclusions

This paper contributed to the literature by developing a new approach to the productivity change assessment over time in the full input-output space. This new approach is based on the idea of the measurement of the least distance to the Pareto-efficient frontier and exploits the Luenberger-type productivity change measure. The new approach was empirically illustrated with a recent dataset on the Spanish Designation of Origin wines sector.

The new approach to the productivity change measurement proposed in this paper can be useful from the point of view of practice, for example to managers in their decision making. It is especially important for firms which seek to achieve superior performance results as soon as possible. Indeed, the efficiency measure which forms the basis of the productivity change estimator developed in this paper generates the targets that are easily achievable by firms. Moreover, the benchmarking exercise of this approach is based on the comparisons with the best firms, which is guaranteed by the projection onto the Pareto-efficient part of the frontier of best practices. Overall, these characteristics can have important implications for firms to attain their competitive advantage. The new approach is also of practical use to regulators. Because it does not suffer from the infeasibility problem, the productivity change is estimated for all firms, which makes it a suitable approach, for example, in the regulatory setting.

The results of the empirical application of this paper show the large differences between the obtained values of productivity, efficiency and technical changes using the traditional Luenberger indicator, indicator derived from the slacks-based measure and the Luenberger indicator proposed in this paper that seeks the least distance to the Pareto-efficient frontier. In many cases the results found were opposite between approaches, both in average terms for the whole sample and when looking at the individual DOs. Nevertheless, the largest correlations were found between the new approach and the indicator derived from the slacks-measure and mainly with regard to efficiency change. Yet to have definite conclusions regarding similarities between approaches, one would need to undertake additional calculations using a sufficiently large battery of different datasets.

Despite the findings of this paper, due to the strength of its conclusions, it would be crucial for future research to apply the new productivity change measure developed in this paper to other real-life datasets. We also believe that an interesting extension of this study would be to develop a more extended decomposition of a new productivity change measure to account for pure efficiency change, scale efficiency change, pure technical change and scale change of technology, similar to the one proposed in Simar and Wilson [52] and Zofio [57] in the context of the Malmquist index. Yet another topic that clearly deserves further research is the application of the new method to the production framework that accounts for undesirable outputs. Another good avenue for additional research is the comparison of the performance of all the existing approaches for estimating productivity change over time, in the full input-output space, utilizing a battery of simulated datasets, taking into account the different nature of each methodology (furthest vs closest targets, Variable Returns to Scale vs Constant Returns to Scale, slacks-based measures vs non slacks-based measures, …). Finally, it seems also interesting to study the relationship between the least distance to the Pareto-efficient frontier and some support function (profit, revenue and cost functions) in the context of the measurement of productivity change over time.

Acknowledgements

We thank three anonymous referees for providing constructive comments and help. The calculations of adapted Li test were made at the Wroclaw Centre for Networking and Supercomputing (www.wcss.wroc.pl), grant no. 286. Additionally, J. Aparicio and Jesus T. Pastor would like to express their gratitude to the Spanish Ministry for Economy and Competitiveness for supporting this research through grant MTM2013-43903-P.

Appendix A. Supplementary material

Supplementary data associated with this article can be found in the online version at http://dx.doi.org/10.1016/j.omega.2016.10.005.

References

[1] Amirteimoori A, Kordrostami S. A Euclidean distance-based measure of efficiency in data envelopment analysis. Optimization 2010;59:985–96.
[2] Ando K, Kai A, Maeda Y, Sekitani K. Least distance based inefficiency measures on the Pareto-efficient frontier in DEA. J Oper Res Soc Jpn 2012;55:73–91.
[3] Aparicio J, Ruiz JL, Sirvent I. Closest targets and minimum distance to the Pareto-efficient frontier in DEA. J Prod Anal 2007;28:209–18.
[4] Aparicio J, Pastor JT. A well-defined efficiency measure for dealing with closest targets in DEA. Appl Math Comput 2013;219:9142–54.
[5] Aparicio J, Pastor JT. On how to properly calculate the Euclidean distance-based measure in DEA. Optimization 2014;63(3):421–32.
[6] Aparicio J, Pastor JT. Closest targets and strong monotonicity on the strongly efficient frontier in DEA. Omega 2014;44:51–7.
[7] Aparicio J, Mahlberg B, Pastor JT, Sahoo BK. Decomposing technical inefficiency using the principle of least action. Eur J Oper Res 2014;239:776–85.
[8] Aparicio J, Borras F, Ortiz L, Pastor JT. Benchmarking in healthcare: an approach based on closest targets. In: Emrouznejad Ali, Cabanda Emilyn, editors. Managing service, 215. Springer; 2014. p. 67–91 [International Series in Operations Research &[amp; Management Science].
[9] Aparicio J, Kapelko M, Mahlberg B and Sainz-Pardo JL. Measuring input-specific productivity change through a Luenberger indicator based on the principle of least action. Paper presented at 14th European Workshop on Efficiency and Productivity Analysis, Helsinki, Finland; 2015.
[10] Baek C, Lee J. The relevance of DEA benchmarking information and the least-distance measure. Math Comput Model 2009;49:265–75.
[11] Beale EML, Tomlin JA. Special facilities in a general mathematical programming system for non-convex problems using ordered sets of variables. In: Lawrence J, editor. Proceedings of the fifth international conference on operational research. London: Tavistock Publications; 1970. p. 447–54.
[12] Boussemart J-P, Briec W, Kerstens K, Poutineau J-C. Luenberger and Malmquist productivity indices: theoretical comparisons and empirical illustration. Bull Econ Res 2003;55:391–405.
[13] Briec W. Minimum distance to the complement of a convex set: duality result. J Optim Theory Appl 1997;93(2):301–19.
[14] Briec W. Hölder distance function and measurement of technical efficiency. J Prod Anal 1998;11:111–31.
[15] Briec W, Lemaire B. Technical efficiency and distance to a reverse convex set. Eur J Oper Res 1999;114:178–87.

[16] Briec W, Lesourd JB. Metric distance function and profit: some duality results. J Optim Theory Appl 1999;101(1):15–33.
[17] Briec W, Leleu H. Dual representations of non-parametric technologies and measurement of technical efficiency. J Prod Anal 2003;20:71–96.
[18] Briec W, Kerstens K. Infeasibility and directional distance functions with application of determinateness of the Luenberger productivity indicator. J Optim Theory Appl 2009;141:55–73.
[19] Briec W, Kerstens K. The Luenberger productivity indicator: an economic specification leading to infeasibilities. Econ Model 2009;26:597–600.
[20] Caves D, Christensen L, Diewert W. The economic theory of index numbers and the measurement of input, output and productivity. Econometrica 1982;50(6):1393–414.
[21] Cherchye L, Van Puyenbroeck T. A comment on multi-stage DEA methodology. Oper Res Lett 2001;28:93–8.
[22] Chambers RG, Färe R, Grosskopf S. Productivity growth in APEC countries. Pac Econ Rev 1996;1:181–90.
[23] Chambers RG, Pope RD. Aggregate productivity measures. Am J Agric Econ 1996;78(5):1360–5.
[24] Chambers RG, Chung Y, Färe R. Profit, directional distance functions, and Nerlovian efficiency. J Optim Theory Appl 1998;98:351–64.
[25] Chambers R.G. Exact nonradial input, output, and productivity measurement, Econ Theory 20, 2002, 751-765.
[26] Charnes A. Cooper W.W. Golany B. Seiford L. and Stutz J. Foundations of data envelopment analysis for Pareto-Koopmans efficient empirical productions functions J Econ 30, 1985, 91-107.
[27] Coelli T. A multi-stage methodology for the solution of orientated DEA models. Oper Res Lett 1998;23:143–9.
[28] Cooper WW, Park KS, Pastor JT. RAM: a range adjusted measure of inefficiency for use with additive models, and relations to others models and measures in DEA. J Prod Anal 1999;11:5–42.
[29] Cuesta R, Zofío JL. Hyperbolic efficiency and parametric distance functions: with application to Spanish savings banks. J Prod Anal 2005;24(1):31–48.
[30] Färe R, Grosskopf S, Lovell CAK. The measurement of efficiency of production. Boston: Kluwer Nijhof Publishing; 1985.
[31] Färe R, Grosskopf S, Lindgren B, Roos P. Productivity changes in Swedish pharmacies 1980–1989: a non-parametric Malmquist approach. J Prod Anal 1992;3(1–2):85–101.
[32] Frei FX, Harker PT. Projections onto efficient frontiers: theoretical and computational extensions to DEA. J Prod Anal 1999;11:275–300.
[33] Fukuyama H, Maeda Y, Sekitani K, Shi J. Input-output substitutability and strongly monotonic p-norm least-distance DEA measures. Eur J Oper Res 2014;237:997–1007.
[34] Fukuyama H, Masaki H, Sekitani K, Shi J. Distance optimization approach to ratio-form efficiency measures in data envelopment analysis. J Prod Anal 2014;42:175–86.
[35] Fukuyama H, Hougaard JL, Sekitani K, Shi J. Efficiency measurement with a nonconvex free disposal hull technology. J Oper Res Soc 2016;67(1):9–19.
[36] Gonzalez E, Alvarez A. From efficiency measurement to efficiency improvement: the choice of a relevant benchmark. Eur J Oper Res 2001;133:512–20.
[37] Jahanshahloo GR, Mehdiloozad M, Roshdi I. Directional closest-target based measures of efficiency: hölder norms approach. Int J Ind Math 2013;5:31–9.
[38] Joro T, Korhonen P, Walleniuss J. Structural comparison of data envelopment analysis and multiple objective linear programming. Manag Sci 1998;44 (7):962–70.
[39] Kerstens K, Hachem BAM and Van de Woestyne I. Malmquist and Hicks-Moorsteen productivity indices: An empirical comparison focusing on infeasibilities. Working Paper Series of Hogeschool-Universiteit Brussel, Faculteit Economie en Management, number 2010/31. 〈https://lirias.kuleuven.be/bitstream/123456789/409560/1/10HRP31.pdf〉; 2010.
[40] Koopmans TC. An analysis of production as an efficient combination of activities. In: Koopmans TC, editor. Activity analysis of production and allocation. New York: Wiley; 1951.
[41] Li Q. Nonparametric testing of closeness between two unknown distribution functions. Econ Rev 1996;15:261–74.
[42] Lovell CAK. The decomposition of Malmquist productivity indexes. J Prod Anal 2003;20:437–58.
[43] Lozano S, Villa G. Determining a sequence of targets in DEA. J Oper Res Soc 2005;56:1439–47.
[44] Luenberger DG. New optimality principles for economic efficiency and equilibrium. J Optim Theory Appl 1992;75(2):221–64.
[45] Luenberger DG. Microeconomic theory. New York: McGraw-Hill; 1995.
[46] Pastor JT, Ruiz JL, Sirvent I. An enhanced DEA Russell graph efficiency measure. Eur J Oper Res 1999;115(3):596–607.
[47] Pastor JT, Aparicio J. The relevance of DEA benchmarking information and the Least-Distance Measure: comment. Math Comput Model 2010;52:397–9.
[48] Pastor JT, Lovell CAK, Aparicio J. Families of linear efficiency programs based on Debreu's loss function. J Prod Anal 2012;38:109–20.
[49] Portela MCAS, Castro P, Thanassoulis E. Finding closest targets in non-oriented DEA models: the case of convex and non-convex technologies. J Prod Anal 2003;19:251–69.
[50] Portela MCAS, Thanassoulis E. Malmquist indexes using a geometric distance function (GDF). Application to a sample of Portuguese bank branches. J Prod Anal 2006;25:25–41.
[51] Shephard RW. Cost and production functions. Princeton: Princeton University Press; 1953.
[52] Simar L, Wilson PW. Productivity growth in industrialized countries. Belgium: Universite Catholique de Louvain; 1998 [Discussion Paper No. 9810].
[53] Simar L, Zelenyuk V. On testing equality of distributions of technical efficiency scores. Econ Rev 2006;25(4):497–522.
[54] Tone K. A Slacks-based measure of efficiency in data envelopment analysis. Eur J Oper Res 2001;130(3):498–509.
[55] Tone K. Malmquist productivity index. Efficiency change over time. In: Cooper WW, Seiford LM, Zhu J, editors. Handbook on data envelopment analysis. Boston: Kluwer Academic Publishers; 2004.
[56] Zofio JL, Lovell CAK. Graph efficiency and productivity measures: an application to US agriculture. Appl Econ 2001;33:1433–42.
[57] Zofio JL. Malmquist productivity index decompositions: a unifying framework. Appl Econ 2007;39:2371–87.
[58] Zofio JL, Pastor JT, Aparicio J. The directional profit efficiency measure: on why profit inefficiency is either technical or allocative. J Prod Anal 2013;40:257–66.

Please cite this article as: Aparicio J, et al. Graph productivity change measure using the least distance to the pareto-efficient frontier in data envelopment analysis. Omega (2016), http://dx.doi.org/10.1016/j.omega.2016.10.005