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On competition and welfare enhancing policies in a mixed oligopoly

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Abstract In a mixed quantity-setting oligopoly with an inefficient public firm, we investigate the optimal government intervention contrasting two different regulatory measures; (possibly partial) privatization and an output subsidy. We find that the effects of the policy implemented crucially depend on the decision timing. Using an interdependent payoff structure in the fashion of a delegation contract to model imperfect competition, we show that privatization incentives are generally larger if it takes place before private firms determine the degree of competition since, in this case, the private firms' output is higher. On the contrary, if the regulator incorporates a production subsidy after the degree of competition is set, the private sector benefits from a high subsidy and achieves perfect collusion.

Keywords Imperfect competition · Mixed oligopoly · Partial privatization · Subsidies

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1 Introduction

Public or at least to some extent state-owned firms are very common in many markets such as airlines, telecommunications, railways, electricity, banking, broadcasting, and education. Consequently, the literature on mixed oligopolies where a public or semi-public firm competes against one or more private profit-maximizing firms is also extensive. Many papers have also examined mixed oligopolies considering whether privatization or subsidization is optimal. Classical examples include De Fraja and Delbono (1989, 1990) and ensuing literature showing that, with symmetric convex cost functions, in some cases, a public firm should be privatized and should maximize profits rather than welfare. The intuition is that privatization might increase welfare since with a large number of private firms the public firm must produce a very high level of output, driving private profits to a very low level. More recently, Matsumura and Kanda (2005) have shown that while partial privatization might be the optimal policy in the short-run, full nationalization becomes optimal in the long-run only with free entry among private firms. With respect to the effects of subsidization, one line of research has shown that there are no consequences from privatization of a public firm whenever a subsidy ensures the first-best allocation. For instance, White (1996) shows that in a Cournot setting both in the private and in the mixed oligopoly, the optimal subsidy is identical. This has often been referred to as the privatization neutrality theorem. Another line of research, however, has derived a non-neutral result of privatization. For instance, Matsumura and Tomaru (2012, 2013) have respectively shown that privatization neutrality does not hold if there are foreign competitors or when an excess burden of taxation is introduced.

However, previous studies on policy issues in mixed oligopolies have ignored the effects of the capacity of the regulator to assess the competitive (or collusive) reaction of private firms to the policy measures.[1] More precisely, little attention has been paid to investigate the question of how the timing of policy measures affects the degree of competition, the policy outcomes as well as welfare in a mixed oligopoly. Among the few exceptions are the recent papers by Matsumura and Okamura (2015) and Lee et al. (2018) with which the present work is most closely related. In the first work, the authors introduce an interdependent payoff structure into a mixed oligopoly assuming that firms consider their own and other firms' profits. It is basically obtained that the optimal degree of privatization is higher when there is less market competition. On the other hand, Lee et al. (2018) uses a model of mixed oligopolies with free-entry where private firms enter the market, and then the government chooses the degree of privatization of the public firm to show that the timing of privatization affects the equilibrium degree of privatization according to whether private firms are domestic or foreign. This assumption differs from that employed in the papers cited above where privatization policies are set before the entries of private firms. As argued by Lee and coauthors, this consideration is important because even if the government partially

[1] Recently, some papers though have considered the possibility that private firms achieve a collusive agreement in a mixed oligopoly. Among the examples are the papers by Correia-da-Silva and Pinho (2018), Delbono and Lambertini (2016), and Colombo (2016) where it is generally found that the presence of a public firm makes collusion among private firms harder to sustain.

privatizes a public firm before opening up the market, the government may be unable to commit to a privatization policy before private firms enter the market or determine their outputs.[2] There are, however, some important differences between the present paper and above-mentioned papers. To begin with, Matsumura and Okamura consider that privatization always takes place after the competitiveness of the market is determined. Consequently, they do not analyze which is the effect of the timing of decisions. On the other hand, Lee and coauthors restrict their attention to Cournot competition ignoring, therefore, the effects of (or on) the intensity of market competition. Moreover, these papers do not consider either the output subsidization.

Summarizing, the main goal of the present article is precisely to study the optimal government intervention contrasting two different regulatory timings; a Social Planner (hereafter, SP) that does not consider that a policy measure applied might affect market competition (namely, decides after the degree of market competition is set), and a SP that is able to anticipate that the degree of market competition depends on the policy measure implemented and, therefore, the policy measures are implemented before competition among private firms takes place. To that extent, we firstly develop as a benchmark model a mixed oligopoly where private firms simultaneously compete in quantities with a relatively more inefficient and partially welfare-maximizing public firm. Additionally, and following (among many others) Matsumura and Okamura (2015), we introduce an interdependent payoff structure assuming that private firms maximize the sum of their own profits and a fraction of the other private firms' profits.[3] As a consequence, this fraction may be considered as the degree of competition which implies that firms can agree on a distribution of the output quotas different to that arising from a perfect joint profit maximization agreement. The present model is mathematically equivalent to a model of standard strategic delegation in which in the first stage, profit-maximizing firms make a commitment not to maximize their own profits while in the second stage, these firms face Cournot competition. The literature on the delegation game is quite extensive (see, for instance, Fershtman 1985, or Vickers 1985 and subsequent contributions) and has reached consensus on the fact that players who care not only about their own payoffs but also about their payoffs relative to others earn strictly higher payoffs than do the standard payoff maximizers.[4]

Accordingly, we obtain that private firms always prefer a larger output than the joint profit-maximizing level which can also be interpreted as collusion profitability being limited by the output expansion of the public firm intended to maximize welfare. Secondly, we also analyze whether the welfare may be enhanced by using an optimal privatization scheme or by subsidizing production contingent on the cost inefficiency of the public firm. Our most important result is that the success of the policy implemented

[2] In their paper, some examples are provided of governments changing (both increasing or decreasing) its ownership of partially privatized firms after observing the behavior of private firms. This was the case of the Japanese government plans to sell its share in Japan Post or the when the French government increased its ownership of Renault.

[3] A general discussion of this approach can be found in Matsumura et al. (2013) and Nakamura (2015).

[4] In this line, Koçkesen et al. (2000) identify sufficient conditions under which players prefer to be relative payoff maximizers. They also discuss the implications for the evolutionary theory of preference formation and the theory of strategic delegation.

and consequently also the level of welfare depends on the timing of the policy. We show that privatization incentives are generally larger if a SP privatizes before the private firms determine the degree of competition since, in this case, the SP is aware of the fact that by keeping a larger amount of shares in a partially privatized firm, private firms voluntarily reduce the degree of imperfect collusion.[5]

On the other hand, if the regulator incorporates a production subsidy after the degree of competition is set, the private sector benefits from a high subsidy and achieves perfect collusion. In this case, the government incurs a higher expenditure. On the contrary, a SP subsidizing before market competition offers a lower subsidy reducing, therefore, firms' incentives to reduce competition and yielding higher welfare. The results of the present paper thus prove useful to highlight that in a plausible scenario of imperfect competition among private firms, the welfare effects of a policy measure depend on the capacity of the regulator to assess not only the current but also the subsequent level of market competition.[6]

Although we focus on a mixed oligopoly from a theoretical viewpoint, our paper is also motivated by some real cases. Mixed public–private firms are increasingly used in several European countries and regulators make use of mixed firms following cost considerations or financial constraints. Some empirical studies have also analyzed how weak competition makes it difficult for local governments to obtain benefits from contracting out. For instance, in the Netherlands, Dijkgraaf and Gradus (2007) investigate whether collusion exists and what the impact is on tariffs for waste collection. Their results indicate the existence of collusion between private firms and that the presence of competing public firms might be essential to ensure more and fair competition. The electricity market also provides several interesting examples; in the 90's United States, Chile, and other countries within EU (Great Britain, Spain, Germany, among others) begin to implement reforms aimed to privatize and restructure the electric power industry. Several authors have found that wholesale prices increased well above marginal cost and that some degree of collusion could be observed in the market. For instance, Wolfram (1999) presents an empirical study of market power in the British electricity industry or Borenstein and Bushnell (1999) analyze how the wave of liberalization and restructuring in the US electricity market has modified the incentives to exert horizontal market power. The Spanish power electricity market is analyzed by Fabra and Toro (2005) where it is found that the post-privatization performance of this market is not consistent with the predictions of models of individual profit-maximizing behavior.

[5] Imperfect collusion is often used to describe situations where firms collude without reaching the Pareto frontier of profits and it is different to partial collusion which refers to a market where only a subset of firms colludes. Levenstein and Suslow (2006) report several case studies of cartels in industries of Europe and the USA, like Bromine, Cement, Diamonds, Oil, Steel, and Sugar that were not always able to raise the price substantially. In addition, in a recent paper, Okullo and Reynès (2016) show that OPEC behavior is consistent with imperfect collusion.

[6] Recently, Lin and Matsumura (2018) have shown that the neutrality result does not hold unless public and private firms have the same cost function. In this respect, our results also confirm that when there is a cost difference between public and private firms, the privatization neutrality does not hold either when private firms do not necessarily compete in a Cournot fashion.

Despite the trend toward economic liberalization and privatization of public firms, we can observe that some governments still hold a large share in public firms and that privatization has often occurred gradually over recent decades (see, for instance, Lee 2006). Consequently, studying how the timing of the policies affects the market outcomes may provide new insight into the optimality of privatization and output subsidization. For instance, an interesting World Bank policy research report (see World Bank 1995) argued that the historical evidence on the timing of reforms shows that the degree of political intervention depends on the opportunity cost of the public firms' inefficiency which, in turn, also depends on the actual and potential market competition.

The rest of the paper is structured as follows. Section 2 describes the imperfectly collusive market in the presence of a public firm. Section 3 presents two different policy measures in order to enhance the welfare and compares two different timings. Section 4 presents an extension of the model showing that, if we allow for multiple public firms, a sufficiently high number of public firms makes collusion among private firms unprofitable. Section 5 concludes. All proofs are grouped together in the "Appendix".

2 The benchmark model: imperfect competition in a mixed oligopoly

We consider an industry with $N+1$ firms simultaneously producing a homogeneous product. N firms ($N \geq 2$) indexed by $i = 2, 3, \ldots, N+1$, are profit-maximizing private firms that produce a quantity q_i with a quadratic cost function given by $c_i(q_i) = \frac{1}{2}q_i^2$. A welfare-maximizing public firm indexed by 1 produces a quantity q_1 with a quadratic cost function given by $c_1(q_1) = \frac{c}{2}q_1^2$, with $c \geq 1$.[7] Therefore, c accounts for the cost asymmetry between public and private firms.[8] Welfare (W) accounts for cumulative firm's profits $\sum_{i=1}^{N+1} \Pi_i$ plus consumer surplus CS, where Π_i denotes profit of firm i. Industry inverse demand is piecewise linear $p(Q) = \max(0, a - Q)$, where $Q = \sum_{i=1}^{N+1} q_i$ is the industry output, p is the output price, and $a > 0$. Throughout the paper, we focus on the short-run equilibrium in which entry and exit in the market are not possible.

[7] As stated in De Fraja and Delbono (1990), if each firm's marginal cost is constant the public firm will impose the rule of pricing at marginal cost. This is true independently of the relative efficiency of private and public firms. Then, if there were any fixed costs, the public firm would be unable to cover the losses which would then need to be funded by the taxpayer. We abstract from this issue by considering increasing marginal costs.

[8] Several papers address the question of whether a public or a private firm produces more efficiently. Theoretical works reveal mixed results. On the one hand, some works conducted in agency settings show that productivity need not be higher in a private firm and that might depend, for example, on properties of the production function (see, for instance, Corneo and Rob 2003). On the other hand, and following the property-rights argument of Alchian and Demsetz (1972), a public ownership might be less efficient than private ownership because, for instance, the public firm is more lax with its workers, which results in lower productivity. In addition, if cost-reducing activities are considered, the private firms' costs may also become lower than the public firms' since private firms engage in excessive strategic cost-reducing activities (see Matsumura and Matsushima 2004). Empirical studies though have often shown that private firms produce at lower costs (see, for instance, Bös 1991 or Megginson and Netter 2001) because the extent to which pay is linked to measured performance is stronger in private firms. We believe thus that the assumption of a more inefficient public firm is both realistic and reasonable.

In many cases, though, a mixture of private and public ownership where the government still holds a non-negligible proportion of shares in privatized firms can be observed. Consequently, these semi-privatized firms must respect the interests of private shareholders and they cannot be pure welfare maximizers. Under this partial privatization structure, the unique semi-public firm maximizes the weighted sum of own profit and welfare: $\beta(\sum_{i=1}^{N+1}\Pi_i + CS) + (1-\beta)\Pi_1$ where $\beta \in [0, 1]$. Therefore, the present formulation encompasses the model with one pure public firm and N private firms if $\beta = 1$ and the quantity competition among $N + 1$ private firms if $\beta = 0$. To put it differently, if the shares owned by the government increase then also β increases.

As mentioned in the introduction, we characterize the degree of competition among private firms considering a particular model where the output is determined in such a way that the symmetric N private firms maximize the sum of their own profits and a fraction of the other private firms' profits, $\Pi_i + \alpha(\sum_{j\neq i}^{N+1}\Pi_j)$ where $\alpha \in [-\frac{1}{N-1}, 1]$ is assumed to be symmetric and constant. This parameter can be interpreted as representing the competitiveness of the market, with a smaller α indicating a more competitive market.[9] This interdependent payoff approach enables us to treat the degree of market competition as a continuous variable. In this model, the equilibrium outcome converges to a competitive outcome (Walrasian) in which all private firms are price takers when α approaches $-\frac{1}{N-1}$, becomes the Cournot equilibrium when $\alpha = 0$, and the private firms' joint profit-maximizing allocation when $\alpha = 1$. Therefore, a direct link between a positive α and the degree of collusion can be established.[10]

Throughout the paper we assume exogenously given values of c and N. It is a straightforward exercise to obtain the equilibrium,

$$q_1(\alpha,\beta) = \frac{a(2+(N-1))}{(1+\Delta)(2+(N-1)\alpha)+N\Delta}, \quad q_i(\alpha,\beta) = \frac{a\cdot\Delta}{(1+\Delta)(2+(N-1)\alpha)+N\Delta}, \tag{1}$$

where $\Delta \equiv 1 + c - \beta$. The associated level of profits for the (semi) public and private firms are $\Pi_1(\alpha,\beta) = [(1+\Delta)/2]\cdot q_1^2(\alpha,\beta)$ and $\Pi_i(\alpha,\beta) = [(3+2(N-1)\alpha)/2]\cdot q_i^2(\alpha,\beta)$, respectively. It can be easily checked that public firm's output decreases with c and increases with α and β while the reverse is true for private firms.

As explained in the introduction, the present model is mathematically equivalent to a model of standard strategic delegation. In this case, and as long as firms produce substitute goods, it is well-known that a firm has a strategic incentive for committing to a larger output than the profit-maximizing level in order to reduce the rivals' outputs.[11] Additionally, the output expansion by the public firm also constrains the incentives of

[9] Escrihuela-Villar (2015) shows that the present formulation and the conjectural variations approach lead to equivalent closed-form solutions.

[10] Admittedly, we assume imperfect competition only among private firms whereas the (semi) public firm is concerned with social welfare and individual profits. Arguably, welfare-improving collusion between public and private firms might also be formed assuming, for instance, that firms are also concerned with corporate social responsibility (see, for instance, Haraguchi and Matsumura 2017).

[11] An example can be found in Sklivas (1987) where, in a Cournot model where owners delegate decisions to managers, firms behave more aggressively than profit maximizers.

private firms to reduce competition in our model and, therefore, the Pareto frontier of profits for private firms is not reached when they maximize joint profits.

Proposition 1 *The symmetric profit-maximizing degree of imperfect competition by private firms is* $\widehat{\alpha} = \frac{N\Delta-(1+\Delta)}{(N-1)(1+\Delta)} < 1$ *where* $\widehat{\alpha}$ *increases with* c *and* N *and decreases with* β*. In addition,* $\lim_{c\to\infty} \widehat{\alpha} = 1$.

By replacing $\widehat{\alpha}$ in (1) we can obtain the following equilibrium quantities and profits:

$$q_1(\beta) = \frac{a((1+\Delta)+N\Delta)}{(1+\Delta)((1+\Delta)+2N\Delta)}, \quad q_i(\beta) = \frac{a\Delta}{(1+\Delta)+2N\Delta},$$
$$\Pi_1(\beta) = \left[\frac{(1+\Delta-\beta)}{2}\right] \cdot q_1^2(\beta), \quad \Pi_i(\beta) = \left[\frac{a\Delta}{2(1+\Delta)}\right] \cdot q_i(\beta) \tag{2}$$

Imperfect competition among private firms' consists of a reduction in their output in order to increase the market price. However, the output expansion of the public firm in response to the collusive behavior of private firms limits the scope of cooperation and thus, only (some) imperfect collusion is profitable. If the public firm becomes more inefficient, the aforementioned output expansion is also reduced since $\partial q_1(\widehat{\alpha}, \beta)/\partial c < 0$. As a consequence, the scope for the cooperation of private firms is enhanced by public firm's cost inefficiency. Besides, an increase in the number of private firms also reduces competition among private firms since an increase in the number of private firms mitigates the output expansion of the public firm in response to private firms' cooperation. Finally, with a smaller β where the public firm cares less about welfare but more about its individual profits, the output expansion effect of the public firm is also mitigated.

3 Enhancing welfare

In what follows we investigate the extent to which a welfare maximizing SP can mitigate the negative effect that the degree of imperfect competition among private firms has in welfare. Since the effect of the number of private firms on the optimal privatization has already been stated among others by De Fraja and Delbono (1989), we assume hereafter to simplify that $N = 2$. We discuss two possible different policy measures to enhance welfare in our setting: (i) (possibly partial) privatization of the public firm that consists of choosing the value of β that maximizes welfare and (ii) the implementation of an optimal production subsidy (that we denote by s) for all firms where, in this case, welfare must also incorporate the cost of the subsidy: $W = \sum_{i=1}^{3} \Pi_i + CS - s(\sum_{i=1}^{3} q_i)$. In both cases, we consider two different scenarios. First, welfare is maximized by a SP that treats α as an exogenous parameter; that is, the regulator does not consider that a change in the policy measure applied might affect the degree of competition. Second, the SP is able to correctly anticipate that the degree of competition among private firms depends on the policy measure implemented. In other words, in the latter scenario SP knows that the policy measure will be followed by private firms choosing a profit-maximizing degree of competition contingent on the policy measure implemented. We will refer hereafter to these scenarios as Model 1 and Model 2 respectively.

3.1 Optimal privatization

Regarding the optimal degree of privatization, Model 1 can be formulated as the following three-stage game: at the first stage, private firms simultaneously make the commitment to a particular (common and symmetric) value of α that maximizes their individual profits. In a second stage, the SP decides how many shares of the public firm must be sold for the value of α previously decided without taking into account a possible later reaction of private firms in terms of α, once β has been determined. At the third stage, assuming that α and β are known to all interested parties, all (public and private) firms simultaneously produce. We also note that at the first stage, private firms choose α considering its effect on the value of β that will be determined at the second stage. Consequently, at the third stage firms will produce the outputs described in (1) for the α and β previously determined.

Equivalently, the Model 2 can also be formulated as a three-stage game. At the first stage, the SP decides how many shares of the public firm must be sold (namely, the value of β). In a second stage, private firms observe β and make the commitment to the value of α that maximizes individual profits. The point here, and unlike the case of Model 1, is that regardless of the level of market competition that could be previously observed by the SP before setting β, the regulator anticipates that a different α might follow the level of β determined. In other words, the regulator now correctly predicts that the private firms' commitment on α depends on β. From Proposition 1 we know that private firms' profits are maximized at $\widehat{\alpha}$ which is, therefore, the Nash equilibrium of the second stage of the game in the Model 2. At the third stage, quantity competition among public and private firms takes place inducing firms to produce the outputs given in (2) for the value of β previously determined.[12]

Regarding the Model 1, the following lemma describes the equilibrium choice of β in the second stage of the game.

Lemma 1 *In the Model 1, the optimal privatization policy of is* $\beta(\alpha) = 1 - \frac{2c(1+\alpha)}{6+\alpha(4+\alpha)}$ *if* $1 \leq c < 3$ *whereas if* $c \geq 3$, $\beta(\alpha) = 0$.

The intuition behind Lemma 1 is important to understand the solution of the three-stage game proposed in the Model 1. When $1 \leq c < 3$, the value of $\beta(\alpha)$ decreases with α; i.e. the optimal privatization policy always decreases if cooperation between private firms is larger. The intuition is that private firms' profits, as a proportion of total welfare, are larger if α increases and thus, a lower β is called for in order to maximize welfare.

Using backward induction, we are now ready to present the solution of both models. We denote by $\beta_1^*, \beta_2^*, \alpha_1^*$, and α_2^* the optimal privatization policies and the profit-maximizing degrees of imperfect competition by private firms in the Models 1 and 2 respectively.

[12] Conceivably, one could think that any commitment about α is not necessarily credible, and possible subsequent deviations should also be considered markedly extending the set of strategies available to firms. However, we interpret here that α merely parameterizes the degree of imperfect competition and its choice cannot be dissociated from firms' production. This extension has been left for future research.

Proposition 2 *In our mixed oligopoly with imperfect competition, (a) If* $1 \leq c < 3$*, then* $0 < \beta_1^* < \beta_2^* < 1$*, and* $\alpha_2^* < \alpha_1^* < 1$*; (b) if* $3 \leq c < 4 + 2\sqrt{7}$*, then* $1 > \beta_2^* > \beta_1^* = 0$*, and* $\alpha_1^* = \frac{c}{2+c} > \alpha_2^*$*; and (c) when* $c \geq 4 + 2\sqrt{7}$*,* $\beta_2^* = \beta_1^* = 0$ *and* $\alpha_1^* = \alpha_2^* = \frac{c}{2+c}$*. Furthermore,* α_1^* *and* α_2^* *increase with c whereas* β_2^* *and* β_1^* *decrease with c whenever they are positive.*

Proposition 2 establishes that full privatization is advisable only if the public firm is inefficient enough compared to private firms. The intuition is fairly simple. Profits of both private and public firm are maximized when $\beta = 0$. However, CS is maximized when $\beta = 1$. Then, if c increases beyond a certain level, the effect of β on CS is too small to boost total output as long as the inefficient public firm's market share becomes also very small. In this case, a positive β barely increases CS but decreases private and public firm's profits compared to $\beta = 0$. In general, an inverted-U relationship between welfare and β is obtained for a given α; hence, partial privatization seems to be the best policy unless the public firm is markedly inefficient. If β exceeds the critical positive value (β_1^* or β_2^*, respectively) a further nationalization would imply that the decline in private and public firms' profits would offset the increase in CS. Moreover, β_1^* is lower than β_2^* because from Lemma 1 we know that the former privatization policy always decreases with α. Then, in a SPNE, private firms are aware of this effect which allows them to further expand collusion.

Regarding the Model 2, privatization is usually less desirable than in the Model 1. Intuitively, we can distinguish two different effects. On the one hand, if c is not too large, a (to some extent) larger β might increase welfare due to the output expansion of a not markedly inefficient public firm. On the other hand, there is an additional effect that the SP in the Model 2 also takes into account: as Proposition 1 states, a larger β also implies that the scope for the cooperation of private firms is reduced. Proposition 2 shows that by incorporating the second effect, privatization incentives of the SP in the Model 2 are reduced compared to the ones of the SP in the Model 1. The comparison between both optimal privatization policies reveals that, in some cases, in the Model 2 only a partial privatization scheme is advisable whereas the SP of the Model 1 (that ignores the effects of privatization on the degree of imperfect competition) would fully privatize. In other words, Proposition 2 states that privatization is generally less desirable in the Model 2 than in the Model 1 since, in the former case, a further output expansion from the public firm (namely a larger β) successfully curtails the incentives from private firms to reduce competition. We can thus expect less privatization in a more competitive market (Model 2) compared to a larger degree of privatization in a less competitive market (Model 1). As explained in the introduction, Matsumura and Okamura (2015) obtained that the optimal degree of privatization is higher when there is less market competition which coincides with the result stated in our Lemma 1. In fact, in their paper, the timing assumed corresponds with the one in our Model 1 because the degree of competition is assumed to be given without firms being able to adapt the intensity of the market competition to the nature of the privatization policy implemented. Our analysis thus proves useful to identify that the main result in Matsumura and Okamura crucially relies on the timing assumed.[13] We also obtain though that, in both models, a more inefficient public firm should be further privatized. Interestingly enough, it can also be easily checked that welfare is always larger in the Model 2 than in the Model 1. This is true since, in the former case, a lower privatization level (and a subsequent lower degree of collusion) increases CS and compensates the reduction in private and public firms' profits. This highlights the importance for the success of regulation that a SP also takes into account the market reaction to the regulation. Finally, it is also worth noting that a pure public firm is never optimal (namely, β_2^* and β_1^* are always smaller than 1) since the increase in the degree of competition that a "too large" β might represent is more than offset by the excessive output expansion of an inefficient public firm.

3.2 Optimal subsidization

In this subsection, we study the effect on welfare of an optimal subsidy assuming that the degree of privatization β is exogenously given. The subsidy is included in the total welfare as part of the public and private firms' profits but also as an equivalent expenditure. As in the previous subsection, we also consider the scenarios referred to as Model 1 and Model 2. Therefore, the description of the game presented above carries over to the present case with a SP deciding on s instead of β. More precisely, regarding the Model 1, the game at hand can be modeled as follows: at the first stage, the private firms simultaneously decide the value of α that maximizes their individual profits and later, at the second stage, the SP observes the value of α decided by private firms and decides the level of the subsidy s. Finally, at the third stage, simultaneous quantity competition among all firms takes place for the values of s and α previously determined. Conversely, in the Model 2, the first and second stages are reversed in such a way that firstly the SP decides the level of the subsidy and afterward, private firms decide on α.

By backward induction, we can obtain the imperfectly competitive equilibrium at the third stage for given values of α, β, and s, which is the same irrespective of the model considered:

$$q_1(\alpha, \beta, s) = \frac{(a+s)(2+\alpha)-\beta s(4+\alpha)}{(2+\alpha)+\Delta}, \quad q_i(\alpha, \beta, s) = \frac{a\Delta+s(1+c)}{(2+\alpha)+\Delta}. \tag{3}$$

In (3) we can easily check that the subsidy positively affects private firms' output whereas it has a negative effect on the output of the public firm. Lemma 2 presents the solution of the second stage of the game for both models.

Lemma 2 *In the Model 2, private firms choose* $\alpha = (c - \beta)/(1 + \Delta)$ *which does not depend on the subsidy s, whereas in the Model 1, the optimal subsidy is given by* $s(\alpha) = \frac{a[(c(2+\alpha)+\Delta)(\beta(4+\alpha)-(2+\alpha))-4\Delta(1+c)]}{(1+c)[18+6c-\beta(2+\beta)(4+\alpha)^2+\alpha(8+\alpha)]}$, *where* $s(\alpha)$ *increases with* α.

[13] Actually, our results are in line with the results of De Fraja and Delbono (1989) where the optimal degree of privatization is higher in more competitive markets (namely, in a market with a larger number of private firms). Conversely, in the present work, the degree of competition in a mixed oligopoly is captured by α and not by the number of private firms.

The first part of this lemma points out that the degree of competition chosen by private firms does not depend on the subsidy in the Model 2. The intuition behind is that private firms are symmetrically affected by the subsidy in such a way that the output reduction of a firm due to an increase in α only depends on the relative inefficiency of the public firm for a given value of s and, consequently, the subsidy does not affect the profit-maximizing degree of imperfect competition. The second part states that, in the Model 1, the level of the subsidy increases if α increases as long as the subsidy is intended to mitigate the negative effect that low competition has in total output (which decreases CS and, consequently, welfare).

Now we are ready to present the main result of this subsection. We denote by s_1^*, s_2^*, α_1^* and α_2^* the optimal privatization policies and the profit-maximizing degrees of imperfect competition in the Model 1 and 2 respectively.

Proposition 3 *For given values of c and β, $s_1^* > s_2^* > 0$. Furthermore, private firms perfectly collude in the Model 1 (namely, $\alpha_1^* = 1$) whereas in the Model 2, $\alpha_2^* = (c - \beta)/(1 + \Delta) < 1$ where α_2^* increases with c.*

Proposition 3 states that in the Model 1, private firms produce the output at which joint profits are maximized because, for all $\alpha < 1$, an additional reduction in the degree of competition is always rewarded with a larger subsidy intended to encourage production. It can be also checked that, in both models, the subsidy generally increases if the public firm is more inefficient. The intuition is that even though subsidization is symmetric (s is the same for public and private firms)[14] if the public firm becomes more inefficient, a higher subsidy is intended to offset the negative effect of the associated decrease in the degree of competition. Consequently, in the Model 2, a more inefficient public firm induces a higher subsidy which also increases private firms' incentives to reduce competition.

Analogously to the previous subsection, it can also be easily checked that welfare is larger in the Model 2 than in the Model 1. Intuitively, when comparing both models in welfare terms we can observe that there is a trade-off between the level of the subsidy (and hence, competition) and the level of firms' profits: the larger the subsidy (which is an expenditure for the government) the higher the level of firms' profits, including the public firm. In other words, compared to the SP in the Model 2, the SP in the Model 1 basically subsidizes private firms' cooperation at the cost of a higher expenditure and a lower *CS*.

4 Extension

Some previous studies, such as Bose and Gupta (2013) or Matsumura and Shimizu (2010), have allowed for multiple public firms. We check here the effect of an increase in the number of public firms on the profit-maximizing degree of imperfect competition

[14] We have also checked the asymmetric case where the SP only subsidizes private firms. In this case, the results do not significantly change. The main difference is that profits for the public firm are lower than in the symmetric case and, consequently, private firms' profits increase.

by assuming that, in the basic model presented in Sect. 2, $K > 1$ symmetric welfare-maximizing public firms simultaneously set outputs competing with N private firms. To simplify, we also assume that $\beta = 1$.

Proposition 4 *The symmetric profit-maximizing degree of imperfect competition by private firms is* $\hat{\alpha} \equiv \frac{c(N-1)-K}{(K+c)(N-1)}$ *where* $\hat{\alpha}$ *decreases with* K*. Therefore, when* $N = 2$*,* $\hat{\alpha} \leq 0$ *if* $c < 2$ *and when* $N > 2$*,* $\hat{\alpha} \leq 0$ *if* $K \geq c(N-1)$.

Proposition 4 shows that the output expansion of public firms increases with K. Consequently, if the number of public firms is large enough, private firms voluntarily produce above the Cournot level since an output contraction of private firms would be followed by an output expansion of public firms turning private firms' profits below the ones obtained in the Cournot allocation. In other words, if we allow for multiple public firms, a sufficiently high number of public firms makes collusion among private firms unprofitable. However, this effect is alleviated when public firms are relatively inefficient compared to private firms.

5 Concluding comments

We have developed a theoretical framework to study how the timing of policy measures affects the market competition and policy outcomes in a mixed-oligopoly with a relatively inefficient (possibly semi) public firm. Imperfect competition among private firms has been modeled using an interdependent payoff structure in the fashion of a delegation contract. Two different policy measures are considered in our welfare analysis. Regarding privatization, we obtain that, especially when its deterring effects on cooperation among private firms are considered, the existence of a (at least to some extent) public firm seems a more appropriate policy unless a public firm is very inefficient compared to private firms. Full privatization is only advisable if the public firm is markedly inefficient. On the other hand, we also showed that, through a larger production subsidy, the degree of competition is smaller when the regulator is not able to correctly anticipate that private firms' cooperation depends on the subsidy. Our results might have an important implication in mixed oligopolies. As mentioned earlier, the optimal degree of privatization and production subsidies are popular in the literature on mixed oligopolies. However, our results suggest that, when the degree of competition is considered, these policies can yield contrasting results according to the timing chosen for the policy. Therefore, it is necessary to accurately predict the competitive response from private firms when discussing the implications of a privatization policy or a subsidy in a mixed oligopoly.

The framework we have worked with is only a particular approach to a more general issue. To analyze real-world mixed oligopolies with collusive private firms, further research is required. Possible extensions include a repeated non-cooperative game where private firms tacitly collude. Additionally, incorporating price or supply function competition, spillovers in the case of a privatization policy affecting the production cost, foreign ownership or free entry of private firms would probably enrich our analysis. We believe that those are subjects for future research.

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Appendix

Proof of Proposition 1 We just have to check in the private firms' profit function $\Pi_i(\alpha,\beta)$ that

$$\frac{\partial \Pi_i(\alpha,\beta)}{\partial \alpha}=\frac{a^2(1-\beta+c)^2(1-N)(\beta-c+N(1-\beta)+cN-2+(\beta-c-2)(N-1)\alpha)}{(2(\beta-c)+N(\beta-1)-cN-4+(\beta-2-c)(N-1)\alpha)^3}>0$$

only if $\alpha<\bar{\alpha}\equiv[2+c+\beta(N-1)-(1+c)N]/[(2-\beta+c)(1-N)]$, whereas if $\alpha>\bar{\alpha}$ the contrary holds. It is also checked that,

$$\frac{\partial\bar{\alpha}}{\partial N}=\frac{1}{[(2-\beta+c)(N-1)^2]}>0,\quad \frac{\partial\bar{\alpha}}{\partial c}=\frac{N}{[(2-\beta+c)^2(N-1)]}>0$$
$$\frac{\partial\bar{\alpha}}{\partial \beta}=-\frac{N}{[(2-\beta+c)^2(N-1)]}<0.$$

□

Proof of Lemma 1 By adding firms' profits and consumer surplus we obtain the welfare expression that we denote by

$$W(\alpha,\beta)=\frac{a^2((2(1-\beta)+c)(2+\alpha)^2+(4+2(c-\beta)+\alpha)^2+2(1-\beta+c)^2(3+2\alpha))}{2(2(3+\alpha)-\beta(4+\alpha)+c(4+\alpha))^2}.$$

The partial derivative w.r.t β is

$$\partial W(\alpha,\beta)/\partial\beta=\frac{a^2(2+\alpha)(2c(1+\alpha)-6-\alpha(4+\alpha)+\beta(6+\alpha(4+\alpha)))}{\beta(4+\alpha)-2(3+\alpha)-c(4+\alpha))^3}$$

which is negative only if $c>3$. On the contrary, $W(\alpha,\beta)$ is maximized with respect to β whenever $\partial W(\alpha,\beta)/\partial\beta=0$. This is true if $\beta(\alpha)=1-[2c(1+\alpha)]/[6+\alpha(4+\alpha)]$. We also check that second order condition holds since

$$\frac{\partial^2 W(\alpha,\beta)}{\partial^2\beta}=-\frac{a^2(2+\alpha)(2\beta(4+\alpha)(6+\alpha(4+\alpha))-(6+\alpha)(6+\alpha(4+\alpha))+c(4+\alpha)(12+\alpha(10+\alpha)))}{(2(3+\alpha)-\beta(4+\alpha)+c(4+\alpha))^4}<0.$$

□

Proof of Proposition 2 The outputs expressed in (1) stands for the equilibrium at the third stage. On the other hand, the $\widehat{\alpha}$ provided in Proposition 1 (for the case $N=2$) and the $\beta(\alpha)$ provided in Lemma 1 are the solution of the second stage for the Models 2 and 1 respectively. Therefore, regarding the first stage, in the Model 1 we just have to maximize the private firms' profit function provided in Sect. 2, $\Pi_i(\alpha,\beta)$, when $\beta=\beta(\alpha)$ if $1\le c<3$, and when $\beta(\alpha)=0$ if $c\ge 3$. Then, if $1\le c<3$, α_1^* is the solution to

$$\frac{\partial \Pi_i(\alpha,\beta(\alpha))}{\partial \alpha}=-\frac{a^2c^2(4+\alpha)(6+c(\alpha-1)(4+\alpha)^2+\alpha(22+\alpha(11+\alpha)))}{(6+\alpha(4+\alpha)+c(4+\alpha)^2)^3}=0$$

for α and $\beta_1^*=\beta(\alpha_1^*)$. Unfortunately, the explicit expression for α_1^* cannot be simplified to be included in the paper. We used the program *Wolfram Mathematica 7.0* to simplify the expressions and tedious but straightforward details for this and subsequent proofs are available at https://goo.gl/c8tMyd or from the authors upon request. If $c\ge 3$, $\frac{\partial \Pi_i(\alpha,0)}{\partial\alpha}=-\frac{a^2(1+c)^2(c(\alpha-1)+2\alpha)}{(2(3+\alpha)+c(4+\alpha))^3}=0$ gives $\alpha_1^*=\frac{c}{2+c}$. Regarding the Model 2, the solution of the first stage of the game is given by the solution for β to the equation

$$\frac{\partial W(\widehat{\alpha},\beta)}{\partial\beta}=\frac{a^2(41\beta^4-\beta^3(201+119c)-((c-8)c-12)(8+c(11+4c))+3\beta^2(124+c(141+37c))-\beta(308+c(500+c(243+29c))))}{(5\beta-5c-6)^3(\beta-c-2)^3}=0.$$

It leads us to β_2^* where α_2^* corresponds to $\widehat{\alpha}$ evaluated at β_2^*. It can also be checked that $\frac{\partial W(\widehat{\alpha},\beta)}{\partial\beta}<0$ if $4+2\sqrt{7}\le c$. Finally, as long as a just appears as a single multiplicative parameter in β_1^*, β_2^*, α_1^*, and α_2^*, we just have to plot them in order to observe how they change with c. □

Proof of Lemma 2 First, in the Model 2, at the second stage private firms optimally choose α contingent to any value of s. The associated level of profits for private firms at the second stage of the game can be denoted by $\Pi_i(\alpha,\beta,s)$, where

$$\Pi_i(\alpha,\beta,s)=\frac{[(3+2\alpha)(a(1-\beta)+ac+s(1+c))^2]}{[(2(2(3+\alpha)+(c-\beta)(4+\alpha))^2)]}.$$

By taking the first order condition $\partial\Pi_i(\alpha,\beta,s)/\partial\alpha=0$, it is obtained $\alpha=(c-\beta)/(2+c-\beta)$ which does not depend on s. Second, in the Model 1, at the second stage, the public firm chooses s taken as given the level of α. By using (3) welfare at the second stage, that we denote by $W(\alpha,\beta,s)$, can be easily obtained. Hence, by taking the first order condition $\partial W(\alpha,\beta,s)/\partial s=0$, we obtain

$$s(\alpha)=\frac{a(6+(2c+c^2)2(1+\alpha)+\beta^2(2+\alpha)(4+\alpha)+\alpha(6+\alpha)-2\beta(7+c+(6+c)\alpha+\alpha^2))}{(1+c)(18+6c+(\beta-2)b(4+\alpha)^2+\alpha(8+\alpha))}.$$

□

Proof of Proposition 3 First, the optimal subsidy in the Model 2 is obtained at the first stage by solving $\frac{\partial W(\alpha,\beta,s)}{\partial s}=0$, when $\alpha=(c-\beta)/(2+c-\beta)$, (which is the α_2^*) and yields s_2^*. It is also immediate to check that $\frac{\partial\alpha_2^*}{\partial c}=\frac{2}{(2-\beta+c)^2}>0$. In the Model 1, at the second stage (Lemma 2) we have to obtained the solution for s: $s(\alpha)$. At the first stage, private firms decide the degree of competition by maximizing individual profits with respect to α when $s=s(\alpha)$. Then, we can check that $\frac{\partial\Pi_i(\alpha,\beta,s(\alpha))}{\partial\alpha}>0$

is always true which yields $\alpha_1^* = 1$ and the optimal subsidy is the $s(\alpha)$ provided in Lemma 2 evaluated at $\alpha = 1$. Then, $s_1^* = \frac{a(13+15\beta^2+4c(2+c)-4\beta(7+c))}{(1+c)(27+25(\beta-2)\beta+6c)} > s_2^*$. See the Proof of Proposition 2 for further details. □

Proof of Proposition 4 It can be easily checked that profits for private firms are given by $\Pi_i(K) = [a^2c^2(3+2(N-1)\alpha)]/[2(c(2+N)+2K+(N-1)(c+K)\alpha)^2]$. Then,

$$\frac{\partial \Pi_i(K)}{\partial \alpha} = -\frac{a^2c^2(N-1)(c-cN+K+(N-1)(c+K)\alpha)}{(c(2+N)+2K+(N-1)(c+K)\alpha)^3} < 0$$

$$\text{if } \alpha < \hat{\alpha} \equiv \frac{c(N-1)-K}{(K+c)(N-1)}.$$

Therefore, $\hat{\alpha} \leq 0$ if either $1 < c < 2$ and $N = 2$ or $K \geq c(N-1)$ .when $N > 2$. Finally, we just have to check that $\frac{\partial \bar{\alpha}}{\partial K} = -\frac{cN}{(K+c)^2(N-1)} < 0$. □

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