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A fuzzy quality cost estimation method

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Abstract

Quality cost control is one of the most important aspects in the development of a quality management system. This paper presents a method for the estimation of quality cost that aims to take into account the so-called hidden quality costs, which are typically unobserved or unknown. Although this is a subject that has already been approached in other studies, subjectivity and uncertainty are not included in their formal approach, which any attempt to address hidden quality costs should include. Our methodology begins by observing the position each business occupies in Crosby's Quality Management Maturity Grid. Obtaining the stage index on the basis of the experts' opinions permits the valuation of the company's membership for each of the stages of Crosby's Maturity Grid. The application of Crosby's corrector coefficient to an adequate weighting of the stage index makes it possible to obtain the fuzzy number quality cost. The measures obtained and their short-term predictions enable us to know the situation at all times and act accordingly, establishing precise corrective plans that will correct tendencies and make continuous improvement possible.

Keywords: Quality cost; Production research; Economics; Applications; Fault detection and diagnosis

1. Introduction

Measuring and reporting quality cost is an important step in a quality management program [1]. In this paper, we will develop a methodological proposal that mainly aims to facilitate the acquisition, and then the analysis and short-term prediction of reliable quality cost values.

Ever since Juran [2] introduced the concept of quality costs in the 50s, the subject has spurred an interest in many authors. Among the many existing definitions, in our opinion the one standing out because of its clarity is that by Campanella [3], who considers quality costs "the total of the costs incurred by investing in the prevention of non-conformances to requirements, appraising a product or service for conformance to requirements and failure to meet requirements." This means, as Juran and Godfrey [4] indicated, they are the costs that would disappear if there were no possibility of making mistakes.

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Feigenbaum [5] classified the quality cost into four broad categories, which are prevention, appraisal, internal and external failure costs. However, within the costs provoked by errors, failure costs, it is possible to introduce a new subdivision according to the components of cost. These are based on either objective criteria, and as a result, quantification should be relatively simple – *visible quality costs*, or, it may be necessary to resort to essentially subjective and totally unconventional criteria for estimation which complicates this task considerably – *invisible or hidden quality costs* [6].

In spite of the difficulty involved in measuring hidden quality costs, it is necessary to be aware not only of their existence but also of their importance [7]. It is not without reason that they have been the cause of the closure of many companies, because they are doubly dangerous. On the one hand, they represent significant amounts of money, and on the other, they remain hidden.

The hidden quality cost par excellence is the loss of income as a consequence of deterioration in the image of the company, resulting from clients' dissatisfaction because of faulty products or services [8]. In spite of their importance, costs brought about by the loss of image are not in the least the only hidden quality cost to be found in a company. In fact, as pointed out by Love and Irani [9], only some elements integrated in quality cost can be estimated with a certain degree of accuracy and objectiveness. Consequently, the real quality cost values are not only going to coincide with the calculated values by the business, but possibly may be far superior to them.

First, this paper develops a new method for the estimation of hidden quality costs based on fuzzy logic. Our method will allow any business to improve its estimations of quality costs, which is possible by observing the organization's position on Crosby's Quality Management Maturity Grid. Finally, short-term prediction of reliable quality cost values will then be made using possibilistic regression. To do so, we propose the regression method by Bisserier et al. [10]. It is clear that as the company progresses in quality management, there is a steady reduction in quality costs [11 and 12], which leads to a decrease in outspreads over time; consequently, this regression model adapts perfectly to quality cost behavior.

This paper is organized as follows: Section 2 gives a literature review; the main concepts of the fuzzy regression used are presented in Section 3; Section 4 outlines the methodological proposal for the estimation of quality costs; in Section 5, a case study is provided; and finally, Section 6 contains the concluding remarks.

2. Literature review

A great part of the literature written about the measurement of hidden quality costs stems from the study by Kotler [13], and in particular that of Albright and Roth [14], where different methods for calculating such costs are outlined. Since then, several authors have dealt with the quantification of hidden quality costs from different perspectives (see, for example [15–17] or more recently [18–20]). The papers by Kim and Liao [21] and Sedatole [22] stand out for their use of the "function of the loss of quality" by Taguchi.

However, subjectivity and uncertainty are not included in their formal approaches, which any attempt to approach hidden quality costs should include. To overcome this limitation, we propose using fuzzy logic.

The application of fuzzy set theory is a suitable approach in cases where uncertainty is due to the presence of limited and vague information. Applying fuzzy logic in management accounting is not new. Zebda [23] and Korvin, Strawser and Siegel [24] have applied fuzzy logic in cost–benefit analysis researching deviations; Kaufmann [25] did so in zero-based budgeting; Tanaka, Okuda and Asai [26] employ this instrument to resolve capital budgeting problems; Chan and Yuan [27] apply this methodology in their cost–volume–profit analysis to assist the accountant facing uncertainty and risk; Mansur [28] uses this to assess opportunity costs, and there are even application precedents of fuzzy logic toward quality costs [29–31], although dealing with work centered on the quantification of specific elements of the cost and not their posterior analysis.

Furthermore, the originality of the proposed model stand out because by simply observing the quality culture of the company, it is possible to approach the quality cost values it really has.

Subsequently, this study focuses on the analysis of the values obtained, and proposes the use of possibilistic regression to do so. To be exact, as already pointed out in the introduction, due to the behavior of quality costs, the regression method by Bisserier et al. [10] is proposed.

Fuzzy regression was introduced by Tanaka et al. [32,33]. In Tanaka and Ishibushi [34], quadratic membership functions are considered to propose an identification method of interactive fuzzy parameters in possibilistic linear systems. Fung et al. [35] propose an asymmetric fuzzy linear regression approach to estimate the functional relationships for product planning based on quality function deployment, integrating least-squares regression into fuzzy linear regression. Chen and Ko [36] propose fuzzy nonlinear programming models based on Kano's concept [37] to determine the fulfillment levels of partial characteristics so as to achieve determined levels of contribution to design requirements for customer satisfaction. Furthermore, the complete specification of regression problems strongly depends on the nature of input-output data [38]. In this way, Bisserier et al. [10] propose a tendency problem solution introducing the shift term. This model makes it possible to represent output spreads, which increase or decrease in relation to inputs. As far as the use of this type of regression is concerned, previous studies to be mentioned are Tozz et al. [39], which provides realistic predictions of the air temperature in the 21st century; and Brotons et al. [40], which describes the relationship between air temperature and the loss of greenness in lemon rind.

3. Preliminaries

Let us consider a set of N observed data samples defined on an interval $D = [x_{\min}, x_{\max}]$. Let the jth simple be represented by the couple (x_j, Y_j), $j = 1 \dots N$, where x_j are crisp and Y_j are the corresponding fuzzy output. The objective is to determine a predicted functional relationship

$$Y(x) = A_0 \oplus A_1.x \tag{1}$$

defined on the domain D. The parameters A_0 and A_1 are trapezoidal fuzzy coefficients, $\tilde{A}_i = ([K^-_{A_i}, K^+_{A_i}], [S^-_{A_i}, S^+_{A_i}])$, where support: $S_A = [S^-_A, S^+_A]$, kernel: $K_A = [K^-_A, K^+_A]$. As a result, the output is fuzzy as well. In order to identify the parameters A_0 and A_1, it must be imposed that all the observed data are included in the predicted ones for any α-cut. As the output of the model is a trapezoidal fuzzy number (TrFN), two constraints must be taken into consideration in order to guarantee the total inclusion of the data in the predicted one for each level α:

$$[Y_j]_{\alpha=0} \subseteq [\hat{Y}_j]_{\alpha=0}, \quad \text{and} \quad [Y_j]_{\alpha=1} \subseteq [\hat{Y}_j]_{\alpha=1}. \tag{2}$$

The output model tendencies are not taken into account in the conventional method. In order to solve this problem, Bisserier et al. [10] propose a modified model expression in which the model output can have any kind of spread variation for any sign of x by introducing a shift term in the original model input:

$$Y(x) = A_0 \oplus A_1(x - \mathit{shift}). \tag{3}$$

When the model has an increasing radius, $\mathit{shift} = x_{\min}$ will be taken, and $\mathit{shift} = x_{\max}$ will be taken on the contrary if the model has a decreasing radius. Denoting $w_j = x_j - \mathit{shift}$, the output of the fuzzy model is a trapezoidal interval given by:

$$\forall w \in D: \begin{cases} K^-_{\hat{Y}} = K^-_{A_0} + \big(M(K_{A_1}) - R(K_{A_1}).\Delta\big)w, \\ K^+_{\hat{Y}} = K^+_{A_0} + \big(M(K_{A_1}) + R(K_{A_1}).\Delta\big)w, \\ S^-_{\hat{Y}} = S^-_{A_0} + \big(M(S_{A_1}) - R(S_{A_1}).\Delta\big)w, \\ S^+_{\hat{Y}} = S^+_{A_0} + \big(M(S_{A_1}) + R(S_{A_1}).\Delta\big)w, \end{cases} \tag{4}$$

where $\Delta = \operatorname{sign}(w_{\min} + w_{\max})$, and $M()$ is the midpoint and $R()$ is the radius (for example, $M(K_{A_1}) = (K^-_{A_1} + K^+_{A_1})/2$ and $R(K_{A_1}) = (K^-_{A_1} - K^+_{A_1})/2$, and the output spread is given by $R([S_{\hat{Y}_j}]) = R([A_0]) + R([A_1])w_j$.

In conventional methods [33] the used criteria are only based on the available data, their minimization does not guarantee that the identified model has the least global imprecision that could be achieved on the whole domain [41]. In this case, the global imprecision of the model is covered by its output, and considering levels 0 and 1, it is necessary to consider the vertical dimension. The output area delimited by the TrFN is given by:

$$\operatorname{area}(w) = \frac{K^+_{\hat{Y}} + S^+_{\hat{Y}}}{2} - \frac{K^-_{\hat{Y}} + S^-_{\hat{Y}}}{2}$$

and the volume delimited by the model output is

$$\mathit{Volume} = (w_{\max} - w_{\min})\big(R(K_{A_0}) + R(S_{A_0})\big) + \frac{1}{2}\big(w^2_{\max} - w^2_{\min}\big)\big(R(K_{A_1}) + R(S_{A_1})\big)\Delta. \tag{5}$$

The constraints (2) must be respected

- For $\alpha = 1$

$$K_{Y_j} \in \left[K^-_{\hat{Y}_j}, K^+_{\hat{Y}_j}\right] \Leftrightarrow \left|M(K_{\hat{Y}_j}) - K_{Y_j}\right| \le R(K_{\hat{Y}_j}) \tag{6}$$

where

$$\begin{aligned} M(K_{\hat{Y}_j}) &= M(K_{A_0}) + M(K_{A_1})w_j, \\ R(K_{\hat{Y}_j}) &= R(K_{A_0}) + R(K_{A_1})w_j\Delta. \end{aligned} \tag{7}$$

- For $\alpha = 0$

$$[K_{Y_j} - R_{Y_j}, K_{Y_j} + R_{Y_j}] \subseteq \left[S^-_{Y_j}, S^+_{Y_j}\right] \Leftrightarrow \left|M(S_{\hat{Y}_j}) - K_{Y_j}\right| \le R(S_{\hat{Y}_j}) - R_{Y_j} \tag{8}$$

where

$$\begin{aligned} M(S_{\hat{Y}_j}) &= M(S_{A_0}) + M(S_{A_1})w_j, \\ R(S_{\hat{Y}_j}) &= R(S_{A_0}) + R(S_{A_1})w_j\Delta. \end{aligned} \tag{9}$$

The inclusion of the kernel into the support:

$$\left[K^-_{\hat{Y}_j}, K^+_{\hat{Y}_j}\right] \subseteq \left[S^-_{\hat{Y}_j}, S^+_{\hat{Y}_j}\right] \Leftrightarrow \left|M(S_{\hat{Y}_j}) - M(K_{\hat{Y}_j})\right| \le \left|R(S_{\hat{Y}_j}) - R(K_{\hat{Y}_j})\right|. \tag{10}$$

To sum up, the optimization program is performed by minimizing the criterion (5) under the constraints (6), (8) and (10).

4. Quality cost estimation method

The uncertainty and subjectivity inherent in the process of estimating various quality cost components advise treating them properly. In this regard, fuzzy logic is an especially appropriate tool, as it allows processing the information present, and not in specific terms, but instead by incorporating the existing ambiguity and uncertainty into the model. The problem facing us is, on the one hand, seeking a simple and easy method for businesses themselves to implement, whose cost for obtaining the information is not burdensome. On the other hand, we must be mindful that the degree of accuracy of the estimated values will depend upon data availability as well as the calculation processes utilized in each case, which logically will be different for each business.

The objectivity and developmental level of the quality cost quantification system will expectedly be determined by the firm's maturity and involvement in areas relating to quality management. Therefore, we immediately propose a method for using the position each business occupies on Crosby's celebrated Quality Management Maturity Grid [11] to allow transforming the costs calculated by each business into the costs that they can realistically achieve.

Studying quality cost behavior in organizations is an important reference in Crosby. This author, through his Quality Management Maturity Grid (Table 1), analyzes the evolution of such costs in relation to the development of quality management by simply observing the attitude of the organization's human component about quality management.

According to Crosby, businesses found in the first stage, Uncertainty, do not make any quality cost estimations. In the Awakening stage, they are only capable of quantifying one-sixth of the quality costs. As the business strengthens its quality functions and advances along the maturity grid stages, it perfects the quality cost quantification system, and so the values reported become ever nearer to those real.

Based on the definition of Crosby's Maturity Grid and the stage we can assign the company to, the proposed model (Fig. 1) consists of the following phases:

1. Obtaining the stage i index for internal and external consultants Two groups of experts, called internal (in) and external (out), according to whether they are members of company staff or not, will assess the company's membership for each of Crosby's proposed stages: Awakening (A_2), Enlightenment (A_3), Wisdom (A_4) and Certainty (A_5). The Grid's first stage (Uncertainty) is not considered because quality cost estimations cannot be made here. The experts will use a scale of six elements, 1 (totally disagree), 2 (strongly disagree), 3 (disagree), 4 (neutral), 5 (true) and 6 (very true). The respondent's position for each proposition, which is uncertain, is considered a fuzzy subset, and the six

Table 1
Quality cost in the Crosby's Quality Management Maturity Grid [11] introducing Crosby's corrector coefficient.

Stage: Uncertainty •Quality is the responsibility of the quality department. •Quality is hidden within manufacturing or engineering. No inspection. •Problems are fought as they occur. •There are no organized quality improvement activities. •The quality cost is unknown.	
Stage: Awakening •While quality management may be valuable, the organization is not willing to commit resources. •A quality leader is appointed, but the emphasis is on appraisal and moving the product. •Teams address major problems, but long-range solutions are not solicited. •Activities are limited to short-range, motivational efforts. •The quality cost is reported at 3%, but it could be about 18% of sales.	**Crosby's corrector coefficient** $18/3 = \mathbf{6.00}$
Stage: Enlightenment •Management adopts a supportive and helpful stance. •Quality is elevated to a functional level equivalent to engineering, marketing, etc. •Problems are resolved openly and in an orderly way. •The fourteen-step quality improvement program developed by Crosby [11] is implemented. •The quality cost is reported as 8%, but it could be about 12% of sales.	**Crosby's corrector coefficient** **1.50**
Stage: Wisdom •Top management participates in and understands quality. •The quality manager is an officer of the company. •Problems are identified in early development. •The Crosby's fourteen-step quality improvement program is continual and accompanied by follow-up training. • The quality cost is reported as 6.5% but it could be about 8% of sales.	**Crosby's corrector coefficient** **1.23**
Stage: Certainty •Quality is an essential part of the organization. •A quality manager serves on the board of directors. •Problems are prevented. •Quality improvement is normal and continual. • The quality cost is reported as 2.5%, which is what it really is.	**Crosby's corrector coefficient** **1.00**

possible values the respondent may take is what we will call referential. Thus, we can speak of a level of membership μ_j, $j = 1, \dots, 6$. The membership function assigned to each of the previous labels is shown in Table 2.

In short, this attempts to overcome the problems of measuring the different alternatives for each situation. The results made available by the experts for each group are summarized in Table 3, where a_{ij} indicates the number of experts that value stage i with the j grade on the previous scale of six elements. For its part, the stage i index is obtained as

$$I_i = \frac{\sum_{j=1}^{6} \mu_i a_{ij}}{m}. \tag{11}$$

In this way, four indices will be obtained every year for internal evaluators ($I_i(\text{in})$, $i = 2, \dots, 5$) and another four will be obtained for external evaluators ($I_i(\text{out})$, $i = 2, \dots, 5$).

2. Obtaining the weighting factor for each year for internal experts $WF_{\text{in}}(year_i)$ and external $WF_{\text{out}}(year_i)$ The weighting factor is the value that permits the estimation of the company's hidden quality costs every year, based on the costs initially quantified by the company. This, therefore, facilitates the estimation of the quality cost values that could be reached.

The weighting factor is obtained by multiplying Crosby's corrector coefficient (see Table 1) by the quotient between the corresponding stage index (I) and the sum of the whole stage index for that business and year (Table 4).

On the basis of the information provided by the group of experts, a weighting factor for the year s will be obtained for internal evaluators $WF_{\text{in}}(year_s)$ and another for external evaluators $WF_{\text{out}}(year_s)$.

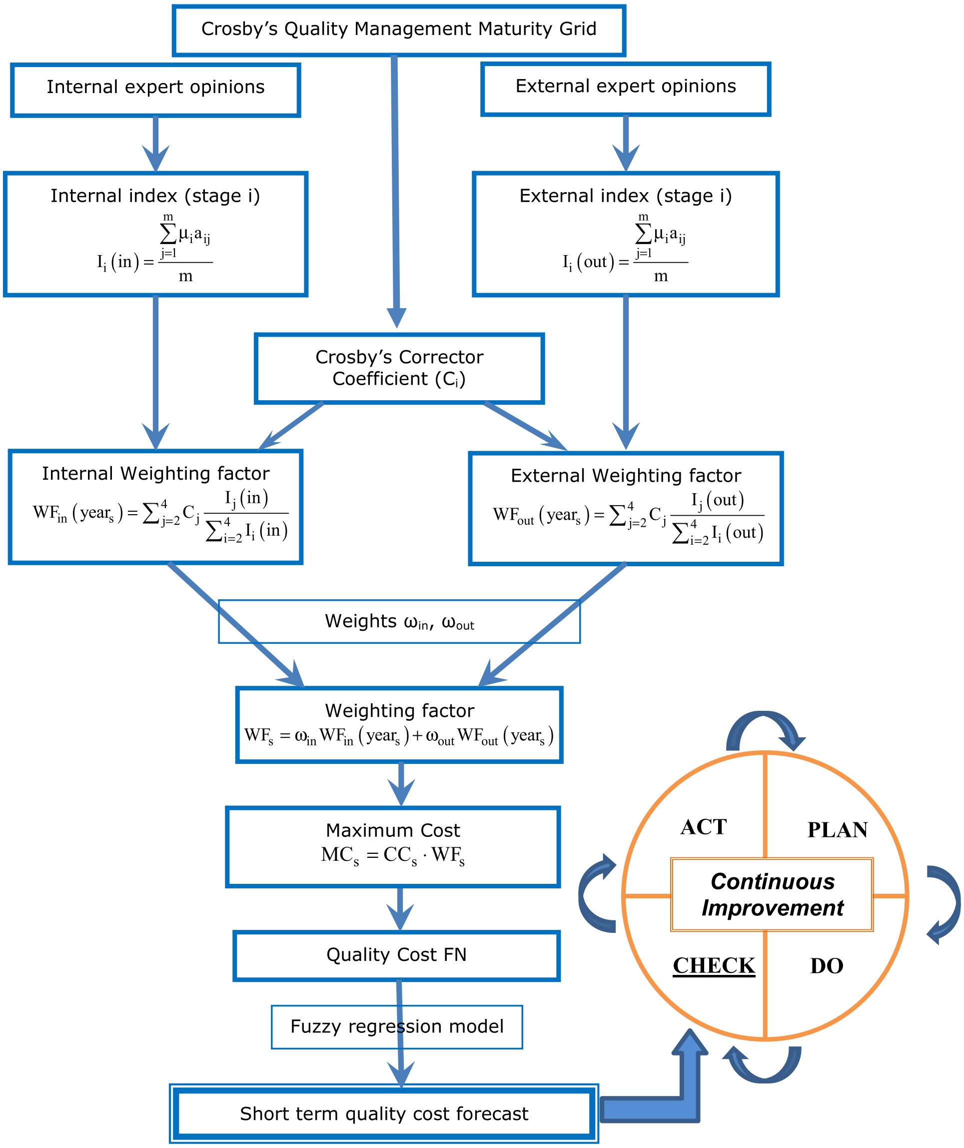

Fig. 1. Quality cost estimation method.

Table 2
Values assigned to the linguistic labels.

Linguistic label	μ_j
1: totally disagree	0.00
2: strongly disagree	0.20
3: disagree	0.40
4: neutral	0.60
5: true	0.80
6: very true	1.00

3. Weighting factor aggregate, WF_s Sometimes companies will not be able to count on the external experts' opinion, or even when they can count on both internal and external groups of evaluators, confidence in their valuations might not be the same. Consequently, it is appropriate to now evaluate what information is most relevant; whether the

Table 3
Experts results and stage i index.

	1	2	3	4	5	6	I_i
A_2	a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	I_2
A_3	a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	a_{36}	I_3
A_4	a_{41}	a_{42}	a_{43}	a_{44}	a_{45}	a_{46}	I_4
A_5	a_{51}	a_{52}	a_{53}	a_{54}	a_{55}	a_{56}	I_5

Table 4
Obtaining the weighting factor for the year s.

Stage	Crosby's corrector coefficient (C_i)	Stage index (I)	$E_i = C_i \frac{I_i}{\sum_{i=2}^{4} I_i}$
2. Awakening	C_2	I_2	E_2
3. Enlightenment	C_3	I_3	E_3
4. Wisdom	C_4	I_4	E_4
5. Certainty	C_5	I_5	E_5

Weighting factor year s $WF(year_s) = \sum_{i=2}^{4} E_i$.

information provided by the internal experts from the company or that provided by the external experts. To do so, some coefficients must be assigned, $\omega_{\text{in}} \geq 0$ and $\omega_{\text{out}} \geq 0$, with $\omega_{\text{in}} + \omega_{\text{out}} = 1$, which indicate the weight the opinions of each group of experts should have. The result permits obtaining the aggregate factor for the year s (WF_s),

$$WF_s = \omega_{\text{in}} WF_{\text{in}}(year_s) + \omega_{\text{out}} WF_{\text{out}}(year_s). \tag{12}$$

4. Calculated costs and maximum costs Calculated costs (CC) are defined as the quality costs initially quantified by businesses. For their part, maximum costs (MC) will be defined as the quality costs that may occur in the businesses because there is an intangible component that was not considered in the first estimations made. The maximum costs for the year s, MC_s, are obtained as the product of the CC_s by the aggregate factor for the year s, WF_s.

$$MC_s = CC_s \cdot WF_s \tag{13}$$

5. Obtaining the quality cost fuzzy number Every year, the business incurs the calculated cost (CC_s) as a minimum, but the existence of the so-called hidden quality costs will make the business's quality cost oscillate between the calculated cost (CC_s) and the maximum cost (MC_s). As the quality cost will be situated between both values, except in extreme cases, the greatest possibility of occurrence will be assigned to the average value of this interval average cost (AC_s). Consequently, quality cost can be represented by a triangular fuzzy number (TFN) $\tilde{Q}_t = (CC, AC, MC)$, with a maximum presumption level in the average cost.

6. Short-term prediction of quality costs Possibilistic regression with trapezoidal fuzzy coefficients:

1) Allows each year's estimation to not only be a concrete value, but moreover an interval and central point, values representative of the business's quality costs fuzzy number.
2) The use of trapezoidal fuzzy numbers (TrFN) ensures the inclusion of the observed costs (calculated cost, average cost and maximum cost) in the predicted costs by the regression model for any significance level [10].
3) The type of model utilized adequately incorporates the trend evolution of the difference between calculated and maximum costs. This difference annually decreases accordingly as the business advances along Crosby's Maturity Grid, and therefore, improves the quality measurement systems.

Because of this, the following possibilistic regression model is proposed, which is based on the use of trapezoidal fuzzy coefficients to estimate quality costs:

$$\tilde{Q}_t = \tilde{A}_0 + \tilde{A}_1 \cdot (Year_t - shift) \tag{14}$$

Table 5
Quality costs (% of sales) for companies A and B.

Year	Quality costs (% sales)	
	A	B
2004	4.57	
2005	4.89	
2006	6.43	3.29
2007	7.32	3.75
2008	7.02	4.35
2009	5.50	4.17

$\tilde{Q}_t$ is the TrFN representative of the quality costs, and shift is the maximum or minimum of years that minimizes Bisserier's objective function.

Finally, as can be seen in Fig. 1, within the PDCA (Plan, Do, Check, Act) circle or Deming circle [42], our methodology is situated in C as a checking instrument. The measures obtained and their short-term predictions enable us to know the situation at all times and act accordingly, establishing precise corrective plans that will correct tendencies and make continuous improvement possible.

5. Case study

This section aims to illustrate the model developed in the previous section through its application.

5.1. Application

We asked two Spanish footwear manufacturers to collaborate, both of which had implemented quality cost systems; one since 2004 (named A) and the other since 2006 (named B). The signing of research projects with the Miguel Hernández University encouraged the involved companies to quantify quality cost. They are situated in the southeast of Spain, where more than 65% of national production is focused, and almost two-thirds of the enterprises and workers from the footwear industry can be found. The companies analyzed are small and medium enterprises. They can be considered average-sized companies in the context of the characteristics of this sector in Spain.

Table 5 shows the results provided by the companies about the quality cost calculated from the moment that they began quantifying it until 2009. To compare figures from different periods, certain homogeneity in the values is required. Because the business volume of all companies will vary over time, if exclusively absolute values are used to measure possible improvements, the results could be wrongly interpreted. For this reason, it is recommended that quality costs should be analyzed through the comparison with other reference variables or bases of comparison. Due to its simplicity, net sales are the comparison basis used most among businesses.

In both cases (companies A and B), the most important elements related to vagueness and ambiguity of the quality cost data are the costs of lost image and lost sales among external failure costs, and the loss due to inefficient work time.

Table 6 shows the results obtained by the survey and the stage i index for internal experts for each of the four stages in question over the six-year study period.

Table 7 shows the weighting factor aggregates for each company for every year. Although the companies analyzed did not have access to external experts, the model permits the exclusive use of internal evaluators by applying the coefficients $\omega_{\text{in}} = 1$ and $\omega_{\text{out}} = 0$. Table 7 also reflects the calculated costs (CC_i), the average cost (AC_i) and the maximum costs (MC_i), which consequently reports the annual TFN quality cost.

For example, the quality costs for the year 2006 for company A can be represented by the fuzzy number $[6.43, 11.69, 16.94]$, where 6.43 is the calculated cost, 11.69 is the average cost, which is assigned the maximum presumption value, and 16.94 is the maximum cost, which would only be reached in extreme situations.

Now, we propose making the quality cost prediction for 2010, starting with the information made available by each company for the period considered.

Applying expression (14), where $Year_t$ is the year (2004 to 2009 for Company A and 2006 to 2009 for Company B) and shift is the maximum year for both companies, 2009, the coefficients for Company A are $\tilde{A}_0 =$

Table 6
Survey results and acquisition of the business's membership function for each year at each of Crosby's Maturity Grid stages in each company. A_2–A_5 represent the four stages considered, numbers 1–6 are the linguistic labels considered, and I_i () is the stage i index.

Year	Company A								Company B							
2004		1	2	3	4	5	6	I_i (2004)								
	A_2				2	2	1	0.76								
	A_3	1	3	1				0.20								
	A_4	5						0.00								
	A_5	5						0.00								
2005		1	2	3	4	5	6	I_i (2005)								
	A_2			1	3	1		0.60								
	A_3		1	3	1			0.40								
	A_4	5						0.00								
	A_5	5						0.00								
2006		1	2	3	4	5	6	I_i (2006)		1	2	3	4	5	6	I_i (2006)
	A_2		2	3				0.32	A_2				1	3	1	0.80
	A_3				1	3	1	0.80	A_3		2	2	1			0.36
	A_4	2	3					0.12	A_4	4	1					0.04
	A_5	5						0.00	A_5	5						0.00
2007		1	2	3	4	5	6	I_i (2007)		1	2	3	4	5	6	I_i (2007)
	A_2	3	2					0.08	A_2				4	1		0.64
	A_3				1	2	2	0.84	A_3		1	1	2	1		0.52
	A_4	1	4					0.16	A_4	1	4					0.16
	A_5	5						0.00	A_5	5						0.00
2008		1	2	3	4	5	6	I_i (2008)		1	2	3	4	5	6	I_i (2008)
	A_2	3	2					0.08	A_2		1	2	2			0.44
	A_3					2	3	0.92	A_3				1	3	1	0.80
	A_4	1	3	1				0.20	A_4	2	3					0.12
	A_5	5						0.00	A_5	5						0.00
2009		1	2	3	4	5	6	I_i (2009)		1	2	3	4	5	6	I_i (2009)
	A_2	3	2					0.08	A_2	1	2	2				0.24
	A_3					3	2	0.88	A_3				2	2	1	0.76
	A_4	1	3	1				0.20	A_4	2	3					0.12
	A_5	5						0.00	A_5	5						0.00

Table 7
Obtaining the weighting factor, calculated costs, average and maximum costs for each company.

Year	Company A				Company B			
	WF_s	TFN quality cost			WF_s	TFN quality cost		
		CC_i	AC_i	MC_i		CC_i	AC_i	MC_i
2004	5.06	4.57	13.85	23.14	–	–	–	–
2005	4.20	4.89	12.71	20.54	–	–	–	–
2006	2.64	6.43	11.69	16.94	4.49	3.29	9.03	14.78
2007	1.79	7.32	10.22	13.13	3.65	3.75	8.72	13.68
2008	1.76	7.02	9.67	12.32	2.93	4.35	8.55	12.75
2009	1.76	5.50	7.60	9.70	2.44	4.17	7.16	10.16

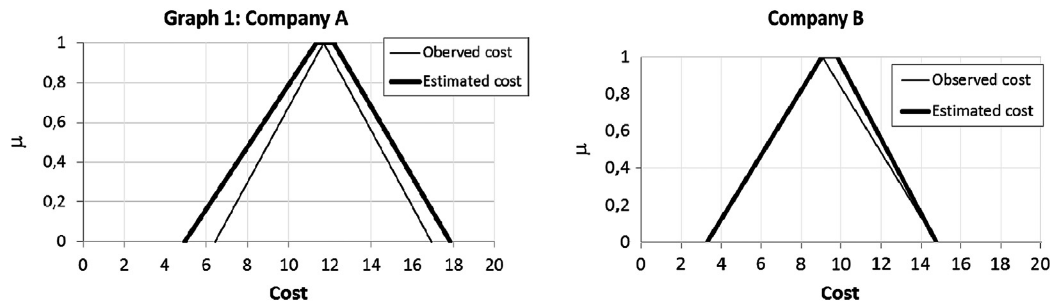

Fig. 2. Observed and estimated costs for 2006 for company A and B.

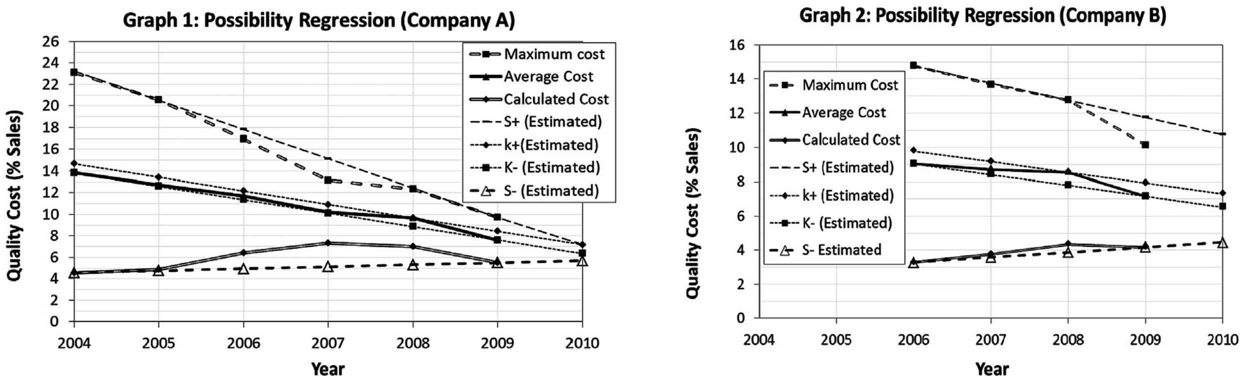

Fig. 3. Evolution of the costs estimated by the possibilistic model from 2004 until 2010 and the observed costs up to 2009 in companies A and B.

[[7.60, 8.42], [5.50, 9.70]] and $\tilde{A}_1 = [[-1.25, -1.25], [-2.71, 0.19]]$. Applying the previous coefficients permits the prediction for all the target years of this study. This is illustrated by the result obtained for 2006 (Fig. 2) for company A, which is $\tilde{Q}_A = [[11.35, 12.17], [4.94, 17.83]]$, i.e., the calculated cost (6.43) must be higher than the minimum value obtained by the regression (4.94), the business's average costs (11.69) must belong to the core of the estimation [11.35, 12.17], and the maximum costs (16.94) must be inferior to the maximum value obtained by the regression (superior extreme, 17.83).

For the same year in Company B, the coefficients are $\tilde{A}_0 = [[7.16, 7.93], [4.17, 11.74]]$ and $\tilde{A}_1 = [[-0.62, -0.62], [-1.01, 0.29]]$; for 2006, the business's calculated, average and maximum costs are 3.29, 9.03 and 14.78, respectively, and the result of the estimation is $\tilde{Q}_B = [[9.03, 9.80], [3.29, 14.78]]$.

Fig. 3 shows the evolution of the calculated, average and maximum costs. The differential between the calculated and maximum costs diminishes over time, and therefore, the shift term introduced in the prediction has been of great help. The prediction for 2010 for company A is [[6.35, 7.17], [5.69, 7.17]] and for company B it is [[6.54, 7.31], [4.46, 10.73]].

Finally, although it is not the main aim of this section, in order to demonstrate the superiority of the possibilistic regression model for predicting quality cost in contrast to probabilistic alternatives, we believe it is of interest to apply a linear regression model to the two companies analyzed.

The linear regression between the quality costs calculated by the company and the year permits the following results to be obtained: $Q_t = -677.975 + 0.341 \cdot year_t$, with $R^2 = 0.314$ for company A, and $Q_t = -477.952 + 0.240 \cdot year_t$ and $R^2 = 0.727$ for company B. In both companies, the results of the estimation are very weak, and because of this, the linear model is not adequate for explaining the evolution of quality costs.

The linear regression model does not explain quality cost evolution. Even if it is assumed that the results obtained were acceptable, this model only offers a point estimate, which is hardly applicable in a case like this where the information regarding quality costs is in the form of an interval, or better said, in the form of a fuzzy number.

Table 8
Results 2010.

	Company A							Company B						
	1	2	3	4	5	6	I_i	1	2	3	4	5	6	I_i
A_2	4	1					0.04	4	1					0.16
A_3					3	2	0.88					3	2	0.84
A_4	1	2	2				0.24	1	2	2				0.12
A_5	5						0.00	5						0.00

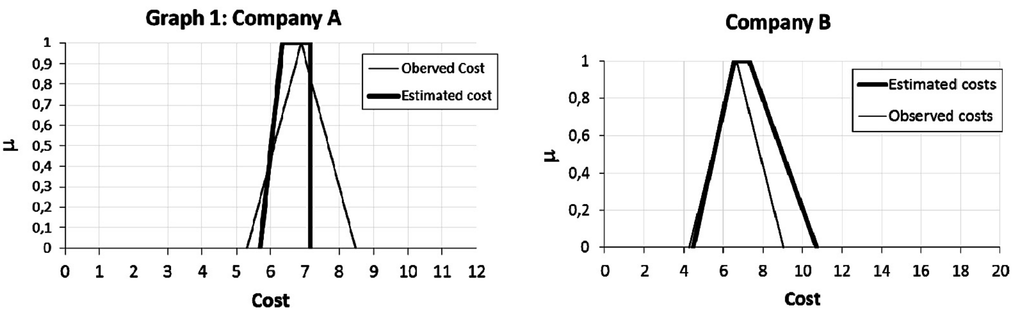

Fig. 4. Comparison of the costs observed by the business and the costs estimated by possibilistic regression for 2010.

5.2. Validation

Starting from the business's calculated costs, in this section we intend to acquire the maximum costs and average cost in order to validate the prediction made in the preceding section. To do this, the experts were asked to assess the stage of Crosby's Maturity Grid the business was found to be in for 2010 in the same way as they did for the previous years. As the companies only resorted to an internal evaluation for the period of this study, only internal experts were used for validation.

The results and the index for each stage are shown in Table 8 for both companies. For company A, the results permit a weighting factor of 1.60 to be obtained. Consequently, the maximum cost is 8.48. The weighting factor for company B is 2.11 and the maximum cost is 9.05.

Fig. 4 shows the comparison between the observed costs for 2010 and the estimated costs by possibilistic regression based on the information available. For company A, the costs observed by the business in 2010 are the following: the business quantifies costs of 5.30% (calculated cost), although these could reach 8.48% of sales (maximum cost), so the average cost is 6.89. A similar interpretation can be obtained for company B. This latter result was obtained through the proper treatment of the opinions provided by the aforementioned experts.

Possibilistic estimation by trapezoidal fuzzy numbers ensures the inclusion of all calculated costs by the business and the maximum cost in the estimated exterior intervals, as well as all the average costs in the estimated interior intervals for the calculated period. For its part, the 2010 quality cost prediction for company A [[6.35, 7.17], [5.69, 7.17]] offers an estimated central interval (6.35, 7.17) that includes the observed average cost (6.89) and an external interval (5.69, 7.17). This estimation includes a large part of the TFN quality cost obtained by the company (5.30, 6.89, 8.48).

On the other hand, for company B, the central estimation for year 2010 [6.54, 7.31] includes the central business value (6.66). For its part, the estimated external interval (4.46, 10.73) includes practically all the support from the TFN quality cost (4.28, 6.66, 9.05). That is to say, the cost calculated by the company (4.28) and the maximum cost (9.05).

6. Conclusions

One of the main problems with quality cost estimation in businesses is the existence of certain elements that are called hidden quality costs, whose quantification is at best uncertain and subjective. This is why a unique tool has been developed, based on the position each business occupies on Crosby's reputed quality Management Maturity Grid, which allows a company's calculated quality costs to be transformed into fuzzy numbers.

In particular, obtaining the stage index on the basis of the experts' opinions permits the valuation of the company's membership for each of the stages of Crosby's Maturity Grid. The application of Crosby's corrector coefficient to an adequate weighting of the stage index makes it possible to obtain the maximum costs the company could incur (maximum cost) based on initial estimations (calculated costs), thus permitting the construction of the TFN quality cost.

The next step is the short-term estimation of quality costs by means of regression. To do so, we propose the application of the model by Bisserier et al. [10]. These concepts come closer to natural reason in situations of uncertainty better than the classical statistical regression does, which makes rigid assumptions about the statistical properties of the model; e.g., the normality of error terms and predictions, in addition to offering a series of instruments that improve the transmission and interpretation of information, like for example fuzzy numbers. Possibilistic regression provides an interval within which the costs initially quantified by the business (calculated costs), as well as those that can actually be obtained (maximum costs), must both be found. Lastly, given that uncertainty (measured as the difference between the calculated and maximum costs) is reduced over time as the business advances in its quality management and measurement, the proposed model, with the inclusion of the shift term, can adapt to this circumstance, reducing the amplitude of the estimated interval every year. Furthermore, as possibilistic regression incorporates all the existing uncertainty, it is much more effective than the probabilistic regression in situations such as those presented in this paper, especially in company B, where there is very few data for a probabilistic regression.

In conclusion, it was thought suitable to proceed with the validation of the estimations made by the possibilistic model (the scarce significance of the simple linear regression model directly discourages its use). The comparison between the predictions made by the model for 2010 and the values observed that year, in accordance with the methodology introduced in the first part of this paper, validate the results obtained for the two companies we applied it to.

An axiom of quality management is "what cannot be measured, cannot be improved." In order to support continuous improvement towards the achievement of results, there is a need for tools that will facilitate indicators of the level of quality in any company, independently of its size or organizational structure. It is here that our model could be of particular interest for company directors.

The simplicity of the tools proposed facilitates their use in small and medium enterprises; nevertheless, the great virtue of the developed model is its ability to generalize its application to any organization from any sector, no matter how complex its cost structure may be. There is a direct relationship between the development of quality cost models and the culture of quality existing at enterprises [43], and so the most evolved quantification systems will occur at organizations that have advanced farther along Crosby's Maturity Grid. At these enterprises, failure costs, especially those external, are smaller [44], so the importance of the hidden costs will in turn be less, something that was kept in mind when developing the model, and from there it is perfectly suited for any type of organization. In this way, we provide a simple and efficacious tool for quality management in order to improve the estimation of quality costs, including hidden costs, making it possible to make short-term predictions.

Although the model designed is of interest, its application by means of a case study could be controversial.

The use of a case study as a research method is recommended when the phenomenon that we want to study cannot be understood independently from its context and its natural environment, and when we have to consider a large number of elements [45]. That is to say when we want to understand a real phenomenon, considering each and every one of the variables that are relevant to it [46]. The evaluation of the quality culture in an organization is, in this case, one of these situations.

Nevertheless, it is also true that some weaknesses inherent in its methodology, and which limit its scientific potential, can be observed. The most important refers to the problems with respect to the generalization of the results obtained, which are based on a reduced number of case studies. As happens in our work, case studies do not usually represent a significant sample. Gummesson [47] and Hamel et al. [48] stress this criticism based on three arguments: their lack of statistical validity; their usefulness for generating hypotheses, but not for testing them; and the lack of representativeness of the phenomenon the study is aimed at, which does not allow for generalizations based on case studies. In this sense, perhaps the most reasonable counterargument is the one given by Yin [49], who places emphasis on the purpose of the research, since according to him, the method can be considered as correctly fitting when, as in our case, it is fundamentally in pursuit of illustrating or presenting a theoretical model. Nevertheless, we consider that a future line of research would be to attempt to broaden the application of the proposed model to a larger sample, which would also give statistical consistency to its subsequent validation.

Finally, we would like to point out that although our model foresaw the possibility of resorting to a double evaluation of quality culture, neither of the companies in the cases studied had access to the opinion of independent experts. In the future, it would be interesting to be able to count on companies who can and want to undergo this double evaluation, which would undoubtedly improve the final results.

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