Bifurcation analysis for the Internet congestion

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***Poster Abstract*— The bifurcation and chaotic behavior of TCP may cause heavy oscillation of average queue length at routers and induce network instability. In this context, congestion control in the Internet is a challenging problem of key importance. This work presents a new discrete dynamical model of Random Early Detection (RED) using beta distribution for controlling bifurcations and chaos in the internet congestion control. The numerical analysis and the simulation experiments show that this new Active Queue Management (AQM) model can obtain the stable average queue length to the desired fix point. The model programmed with Python and Mathematica incorporates new parameters (α, β) that make it possible to stabilize oscillations of averaged router queue length and to be close to stationary state.**

Index Terms*— Congestion control, Active Queue Management (AQM), Internet, Discrete dynamical systems.

I. Introduction

The majority of Internet traffic is generated and controlled by the TCP protocol which has chaotic properties [1]. The poor controlling of congestion may lead to a network being partly or fully unreachable and degrade the performance. Congestion control is a challenging problem which requires of fine-tuning of when and how to govern buffering and queue management at routers [2]. How to stabilize the router queue length around an expected target independent of nonlinear traffic loads has been and is an open problem of congestion control.

II. Dynamical Model for Congestion Control

Discrete dynamical systems and bifurcation theory offer original solutions to the congestion control field, i.e., the problem can be bridged by modelling the behavior of aggregate TCP flows while reproducing the complexity of Internet. Our novel AQM solution based on RED modelled as a discrete-time dynamical system manages the congestion control from the perspective of bifurcation theory. Our model uses the beta distribution configured for tuning decisions of dropping or accepting packets so that the queue occupancy level is kept at a given target stationary level, thereby eliminating aggressive fluctuations of buffer underflow and overflow. We incorporated parameters (*α >0, β >0*) to stabilize high oscillations of Averaged Queue Length (AQL). These parameters are used to calculate the probability p of dropping or accepting packets:

$$p(x;\alpha,\beta) = \begin{cases} 0 & x < q_{min} \\ p_{max}\, I_z(\alpha,\beta) & q_{min} \le x \le q_{max} \\ 1 & x > q_{max} \end{cases}$$

$$I_z(\alpha,\beta) = \frac{B(z;\alpha,\beta)}{B(1;\alpha,\beta)}, \qquad z = \frac{x - q_{min}}{q_{max} - q_{min}}$$

where $B(z;\alpha,\beta) = \int_0^z t^{\alpha-1}(1-t)^{\beta-1}dt$ is the incomplete beta function and $I_z(\alpha,\beta)$ is the regularized incomplete beta function. Value of x represents AQL, q_{min} and q_{max} are minimum and maximum thresholds of queue size, p_{max} is a selected drop probability. For $\alpha = \beta = 1$ the model is typical RED. We analysed that most of the interesting dynamics of our model happened between borders b_1 and b_2 defined as in [3] by

$$b_1 = \begin{cases} I_{p_1}^{-1}(\alpha,\beta)(q_{max} - q_{min}) + q_{min} & if\ p_1 \le 1 \\ q_{\max} & otherwise \end{cases}$$

$$b_2 = I_{p_2}^{-1}(\alpha,\beta)(q_{max} - q_{min}) + q_{min}$$

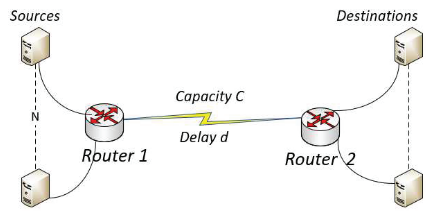
where $I_p^{-1}(\alpha,\beta)$ is the inverse of the regularized incomplete beta function and

$$p_1 = \frac{1}{p_{max}}\left(\frac{NMK}{Cd}\right)^2, \qquad p_2 = \frac{1}{p_{max}}\left(\frac{NMK}{BM+Cd}\right)^2.$$

M is the packet size, B is the buffer size and K is a constant. If AQL $\ge b_1$, the sending rates would be too small to keep the link highly utilized; if AQL is $\le b_2$, the sending rates would be too large to keep low delay. One advantage of our model is that the parameters α, β contribute to detect where the stabilization is.

III. Bifurcation Analysis for Congestion

This new model of AQM has been programmed successfully with Python and Mathematica. It was evaluated from the same perspective of former studies for network congestion control [3]. It was validated through numerical analysis of our parameters *α, β* on the AQL. As a result, our model can reach the stable AQL to the desired fix point. We used a simulation scenario (fig. 1) as a network where a bottleneck link is shared by many connections and a dominant bottleneck Internet link. Scenario ran with a set of parameters as given in [3].



$$C = 75\frac{Mb}{s}, K = \sqrt{\frac{3}{2}}, B = 3750\text{packets}, d = 0.1\text{sec},$$

$$M = 4000\ \text{b}, N = 250, \text{w} = 0.15, q_{\ max} = 750, \quad q_{\ min} = 250, p_{\ max} = 0.1$$

Fig 1. Network topology

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In this scenario, we interpreted the shared link as an intercontinental Internet link with capacity *C* and we assumed that the set of connections *N* are uniformly having the same round-trip propagation delay *d* (without any queueing delay). Rather than interpreting this assumption as a requirement that the connections must have the same propagation delay, we consider *d* as the effective delay that represents the overall propagation delay of the connections, or this could describe a case where the bottleneck link has a large propagation delay that dominates the round-trip delays of the connections. Secondly, it is known that any network manager may have control over the selection of AQM parameters such as q_{max}, q_{min}, w (exponential averaging weight) but other parameters like the number of connections *N* and the round-trip propagation delay *d* are out of the network manager's control. We demonstrated how these different parameters affect the stability of our model by varying the spectrum field of parameters α, β. Numerical simulations demonstrated one of the main advantages of our new model: the parameters α, β can be adapted to reach a better first bifurcation point (BP), i.e., the value that stabilizes oscillations of AQL. We ran simulations between borders b_1 and b_2 and α, β between 0 and 1, and from them we selected the area for values α, β between 0.4 and 1 as the best and most robust. After that, we ran simulations in this region by getting 44.100 BP for values α, β between 0.4 and 1. Fig 2(a) shows the impact of the number of connections on the AQL and Fig 2(b) shows how the AQL stability can be reached improving the classical RED *(α=β=1)*. Our model stabilizes better with lower number of connections, reduces AQL in benefit of lower delays and hence the end to end delay for the network. We suggest that instability and chaos could appear for area represented in white. Besides we varied the control parameter *w* to study how affects to the AQL while other parameters are fixed. Results showed how our model manages an effective way of controlling and delaying the instability. Fig 3(a) shows the impact of the value *w* and Fig 3(b) shows the impact on AQL stability. The model is better with lower value of *w* and AQL, that improved the classical RED as before with value of N. In the figures we can see the optimal regions of the values *α, β* for each parameter. See [4] for results of specific values α, β in different scenarios.

IV. Conclusions

In this work a new AQM model has been presented and validated into a network scenario from same perspective of former studies for network congestion control. We explored numerically the parameter space to demonstrate the successful dynamical features of our model.

Acknowledgments

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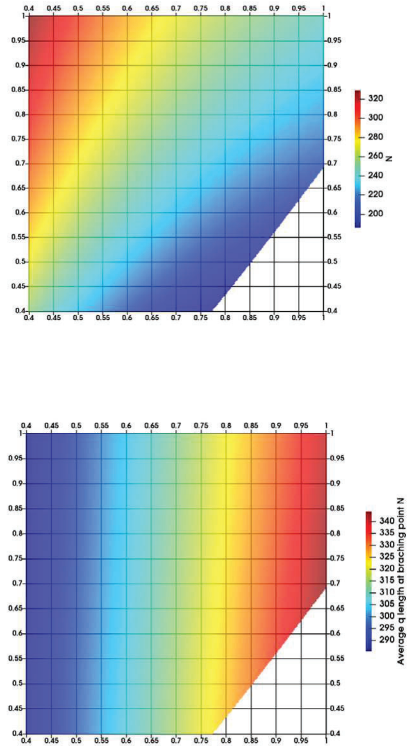

Fig. 2. Number of connections N (a) and AQL (b) varying α and β

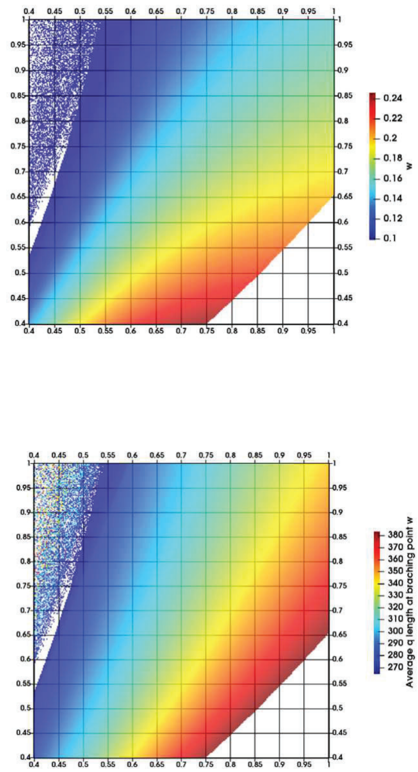

Fig. 3. Value of *w* (a) and AQL (b) varying α and β