International Journal of Uncertainty,
Fuzziness and Knowledge-Based Systems
Vol. 26, Supp. 1 (December 2018) 1–19

DOI: 10.1142/S0218488518400019

Fuzzy Systems in Business Valuation

José M. Brotons* and Manuel E. Sansalvador†
Miguel Hernández University, Economic and Financial Department,
Edificio La Galia, Avda de la Universidad, s/n Elche, 03202, Spain
**jm.brotons@umh.es*
†manri@umh.es

Received 6 May 2018
Revised 16 November 2018

This research aims to develop a model that is able to integrate and objectify information provided by the different business valuation methods, incorporating quality management in its formal approach, which to date has not been considered in the literature about business valuation or quality management. Firstly, the company is valued using the methods which best adapt to its specific characteristics. Because of the subjectivity inherent in any valuation process, the results will be expressed through Triangular Fuzzy Numbers (TFN). These Fuzzy Numbers will be aggregated and summarized by applying Basic Defuzzification Distribution Uncertain Probabilistic Ordered Weighted Averaging operator (BADD-UPOWA). The weighting factors will be: the degree of confidence in each of the business valuation methods applied, and the innovative use of the company's position on Crosby's Quality Administration Grid. The results from application of the model in a case study show a significant reduction in uncertainty in contrast to the initial valuations. Moreover, the proposed methodology is seen to increase the final value of the company as its advances in quality management.

Keywords: Business valuation; discounted cash flow; quality management; fuzzy logic; ordered weighted average; case study.

1. Introduction

Due to an increase in business transfer and an interest in setting goals in terms of value, business valuation has become increasingly significant for companies. Among the reasons why a company may be valued are: a business acquisition, a company merger, the sale of a business, business liquidation, stock analyses, or even because management want to identify key areas within their company, so they can improve or enhance their operational efficiency and shareholder value. In this regard, Blanco-Mesa and Gil-Lafuente[1] examine stakeholder dynamics through a causality relationship process, and Blanco-Mesa, Gil-Lafuente and Merigó[2] analyse the dynamic interactions of stakeholders to explain how a set of agents can act by considering positions of power or influence. Ionita, Stoica and Grigore[3] demonstrate the possibility of correctly determining goodwill and the causal factors behind it through the use of econometric models based on the subtle sets theory.

There are several valuation methods,[4,5] and each of them has operational and strategic perspectives associated with managing a business and valuing business activities. It is therefore impossible to determine a single way to correctly assess companies. There is a wide range of alternatives, all of which have advantages and disadvantages.[6]

According to Castaño,[7] to undertake a business valuation, it is necessary to carry out a strict examination of accounting assessment over a period of several years, and if it is a listed company, its value on the secondary market should be verified. In any case, multiple methods should be used to obtain a company's value by comparing its figures with those of comparable companies, and to make a final valuation though the discount cash flow method (DCF).

According to Trigueros[8] business valuation is currently performed using three methods or a combination of three methods: the asset-based valuation method involves estimating the fair value of a company's assets and liabilities; the comparable company method[9] involves finding publicly traded companies that closely resemble the target company, which is then assigned a similar price-to-earnings ratio derived from the comparable companies. Finally, the DCF method involves estimating future earnings and calculating the present value of a future earnings stream.

However, there are some drawbacks. First, the asset-based valuation method is based on historical data and depreciation that do not represent the current fair market value of assets and liabilities if they are to be bought or sold on the open market.[10] The disadvantage in the comparable company method is that stock values may not accurately represent the actual value of the company.[11] Finally, although the DCF is widely used, it has a high level of subjectivity in both the estimation of future cash flow and the discount rate to be applied, so in this case the use of fuzzy math is very effective. For example, Yao, Chen, and Lin[12] extend the classic DCF model by developing a fuzzy logic system that takes vague cash flow and imprecise discount rate into account. On the other hand, Tsao[13] presents a series of pragmatic algorithms for calculating the net present values (NPVs) of capital investments in an environment that is subject to uncertainty from randomness of outcomes and vagueness of estimation. Discounted cash flow analysis in a fuzzy environment and real options were used by Sansalvador and Brotons[14] to obtain the ISO 9001 certification value. Real options is another methodology that is widely used to minimize the effects of uncertainty, rather than the traditional methods of valuation[15] or probabilistic approaches.[16]

The fair market value of a company is the price at which property would change hands between a willing buyer and a willing seller, where neither of them are under any compulsion to buy or sell and they both have a reasonable knowledge of the relevant facts. Company valuation consists of estimating the company's price. However, value and price are not the same because certainty only exists in price, which is a reality, while value is only a possibility.[17] The valuation process not only involves the uncertain nature of the variables that appear in the assessment processes, but it also contains an evident measure of subjectivity. All these inaccuracies denote that most of the estimations carried out will be controversial, regardless of the method chosen to assess the business. It is only possible to contemplate the estimation process for valuating a company by accepting that the conditions of uncertainty and subjectivity in the valuations unequivocally determine the reliability of the information obtained. Fuzzy logic provides a natural conceptual framework for knowledge representation and inference from knowledge bases that are imprecise, incomplete, or not totally reliable.[18] For this reason, it can be seen as an adequate reference framework that sustains the design of models which permit business valuation.

Applying fuzzy logic in accounting is not new, and the following authors use it in various contexts. Zebda[19] and Korvin, Strawser and Siegel[20] for cost-benefit analysis researching deviations; Kaufmann[21] in zero-based budgeting; Tanaka, Okuda and Asai[22] for solving capital budgeting problems; Chan and Yuan[23] for cost-volume-profit analysis to assist the accountant facing uncertainty and risk. Magni, Malagoli and Mastroleo[24] believe that fuzzy logic is a good tool for describing the value of a firm. These authors construct a fuzzy expert system replicating the reasoning of a human expert as an alternative to the decision models and evaluation models existing in the literature. More recently, Gil-Lafuente, Castillo-López and Blanco-Mesa[25] propose an overview of the business valuation process associated to uncertainty modelisation.

This paper aims primarily to develop a fuzzy model that is able to objectify the information derived from the valuation processes of companies, and in addition includes quality management as a variable to be considered.

As there is no single method that gives an objective business valuation, several methods will be used, and all the information obtained will be summarized. Several studies have approached the problem of aggregation, some of which are outlined below. Peng and Wag[26] develop a method for addressing multicriteria group decision-making problems with Z-numbers under the condition that the weight information is completely unknown. Torra[27] proposes hesitant fuzzy sets, and Wei[28] proposes hesitant fuzzy multiple attribute decision making (MADM) problems in which the attributes are on different priority levels. Wang, Peng and Wang[29] use probability hesitant interval neutrosophic sets in order to address the practical middle-level manager selection rather than fuzzy sets. In Yu *et al.*[30] a mathematical model was designed to select appropriate hotels on websites. Zhang, Wang and Hu[31] focus on multi-criteria decision-making based on picture 2-tuple linguistic information.

Another interesting instrument is Ordered Weighted Averaging operators (OWAs), which permits the aggregation of different values and gives consistency to the final results. Since Yager[32] introduced this aggregation technique, extensive literature has been published about it,[33–36] which has also been extended to different contexts like probabilistic aggregation,[37–39] or the use of intervals and fuzzy numbers.[40,41] In fact, Weighted Averaging operator (WA) proposed by Harsanyi[42] and Ordered Weighted Averaging operator (OWA) are the two most commonly used operators.[43] Recently, different extensions to OWAs have appeared, as outlined below. Blanco-Mesa, Gil-Lafuente and Merigó[44] propose a new method using a family of selection indices with OWA operator that allows information to be aggregated according to the level of importance and level of objectivity and subjectivity in the same formulation within the decision-making process. Blanco-Mesa, Merigó and Kacprzyk[45] develop new aggregation operators using Boferroni means, OWA operators and some distance measures: Induced Heavy Ordered Weighted Moving Average (IHOWMA) operator,[46] Heavy Ordered

Weighted Moving Average (HOWMA) operator[47] or the Prioritized Induced Probabilistic Ordered Weighted Average (PIPOWA) operator.[48]

However, this paper introduces a completely innovative application of this tool with respect to the aggregation of results obtained from the different business valuation methods applied. These results are weighted according to the level of confidence in each valuation method and the company's position on Crosby's Quality Management Maturity Grid.[49] The latter should be noted as a novel and original aspect of this approach, which shows that if a company is at an advanced stage on the Grid, the methods with greater valuations will have a higher weighting. Although to date nobody has questioned the importance of quality as a fundamental business management strategy,[50–52] it is the first time that a company's attitude towards quality management is formally included in business valuation models. Furthermore, this is an important contribution to the development of business valuation methodologies, since different authors have demonstrated the implementation of quality management strategies increases the value of companies. For example, in the studies by Nicolau and Sellers,[53] Sharma,[54] and Sansalvador and Brotons[14,55] in the area of quality certification systems ISO 9000, and by Chung *et al.*[56] in relation to Total Quality Management.

This paper is organized as follows: Section 2 outlines the methodological proposal for the estimation of business value; in Section 3, a case study is presented; and finally, Section 4 contains the concluding remarks.

2. Theoretical Model

This section gives a detailed outline of the model proposed for improving the business valuation process. The stages are as follows:

2.1. *Selection of the H methods to be used for company valuation*

As there is no single infallible business valuation method, in this section several methods will be selected. While there are several acceptable options to choose from, we highlight the discounted cash flow (DCF) method. As indicated in the introduction, different authors have used this method in a fuzzy environment.

In addition to the DCF method, other methods selected will be those that adapt better the particular characteristics of an organization. Combining methods with different operational approaches is also recommended, so the following methods are suggested: methods based on business equity and accounting information, methods based on company income generation capacity, compound methods, comparative methods, and if it is a listed company, its value on the secondary market should be verified.[4,5,7,8]

2.2. *Application of the selected methods*

The methods mentioned in the previous section will be applied, and different H valuations of the company will be obtained. To reflect the value of a company in an objective way, triangular fuzzy numbers (TFNs) can be used to express the attribute value. This not only considers the interval, but it also highlights the possibility of various values within this interval.

$$\tilde{Q}_i = \left(q_i^1, q_i^2, q_i^3\right), \forall i = 1,...,H \tag{1}$$

A fuzzy number is a fuzzy subset defined over real numbers. It is the main instrument for quantifying uncertain quantities in Fuzzy Set Theory. Two properties are required for a fuzzy number: it must be a normal fuzzy set and it must be convex. For practical purposes, the most used fuzzy numbers are TFNs.[57] This class of fuzzy numbers has been used extensively in different applications because of their computational simplicity, and they are useful for representing data and information processing in a fuzzy environment.

2.3. *Applying BADD-UPOWA for the quantification of business value*

The results given by the different business valuation methods are aggregated and summarized by applying basic defuzzification distribution (BADD) probabilistic uncertain OWA operator (BADD-UPOWA). To do so, the results obtained are weighted according to the level of confidence the company has in each of them and its position on Crosby's Quality Management Maturity Grid.[49] OWA methodology can be followed, for instance, in Yager,[32,58] Yager and Filev,[59] Leao and Costa[60] and Merigó and Wei.[61]

For aggregated elements $X = \left(x_1, x_2,..., x_n\right), x_i \in [a,b]$ and $f\left(x_i\right) \geq 0, f\left(x_i\right) \neq 0$ for at least one *i*, an additive neat OWA (ANOWA) operator determined by weighting function $f\left(x\right)$ is a neat OWA operator with weights $W_f = \left(\omega_1, \omega_2,..., \omega_n\right)$ defined as[62]:

$$\omega_i = \frac{f\left(x_i\right)}{\sum_{j=1}^{n} f\left(x_i\right)} \tag{2}$$

It includes the BADD aggregation operator as a special case with $f\left(x_i\right) = x^{\alpha}, \alpha \in \left(-\infty, +\infty\right)$. With different $f\left(x_i\right)$, we can obtain different forms of neat OWA operators.

A BADD probabilistic uncertain OWA operator (BADD-UPOWA) is defined as a mapping of dimension *n*, $F : \Omega^n \rightarrow \Omega$ that has an associated weighting vector W of dimension *n*, $\mathrm{W} = \left[\mathrm{w}_1, \mathrm{w}_2, \ldots \mathrm{w}_\mathrm{n}\right]$ such that $\mathrm{w}_\mathrm{j} = \mathrm{b}_\mathrm{j}^{\alpha} / \sum_{\mathrm{j}=1}^{\mathrm{n}} \mathrm{b}_\mathrm{j}^{\alpha}$, and a vector of probabilities such that $v_j \in [0,1]$ and $\sum_{j=1}^{n} v_j = 1$ where

$$BADD-UPOWA\left(\tilde{a}_1, \tilde{a}_2,..., \tilde{a}_n\right) = \beta \cdot \sum_{j=1}^{n} w_j \cdot b_j^* + \left(1-\beta\right) \sum_{j=1}^{n} v_j \cdot \tilde{a}_i\ , \tag{3}$$

where b_j^* is the *j*th largest of the $\tilde{a}_i$, $\tilde{a}_i$ are TFN, and $\beta \in [0,1]$. Ordering the NBs is at times complex and it is necessary to resort to subjective criteria. This can be consulted in Buckley,[63] Chen,[64] Dubois and Prade[65] and Kim or Park.[66] As can be observed, if $\beta = 0$, only probability is obtained, and if $\beta = 1$, the BADD operator is obtained.

2.3.1. *Business valuation: using Crosby's Quality Management Maturity Grid as weighting factor*

The grid created by Crosby[49] identifies five stages of maturity which describe the different phases a company goes through. They progress from ignorance and total mistrust towards quality until the ideal situation is reached, where administering quality is considered an essential part of the organization (Table 1).

Table 1. Crosby's Quality Management Maturity Grid[49].

Stage	Quality Management
Uncertainty	• Quality is the responsibility of the quality department • Quality is hidden within manufacturing or engineering. No inspection. • Problems are fought as they occur. • There are no organized quality improvement activities.
Awakening	• While quality management may be valuable, the organization is not willing to commit resources. • A quality leader is appointed, but the emphasis is on appraisal and moving the product. • Teams address major problems, but long-range solutions are not solicited. • Activities are limited to short-range, motivational efforts.
Enlightenment	• Management adopts a supportive and helpful stance. • Quality is elevated to a functional level equivalent to engineering, marketing, etc. • Problems are resolved openly and in an orderly way. • The fourteen-step quality improvement program developed by Crosby (1979) is implemented.
Wisdom	• Top management participates in and understands quality • The quality manager is an officer of the company. • Problems are identified in early development. • The Crosby´s fourteen-step quality improvement program is continual and accompanied by follow-up training.
Certainty	• Quality is an essential part of the organization. • A quality manager serves on the board of directors. • Problems are prevented. • Quality improvement is normal and continual.

Quality Management may help reduce internal costs, optimize the use of labour and production equipment, increase production, and ultimately ensure greater efficiency. As a company advances on Crosby's Maturity Grid by suitably reinforcing quality management, business value will increase. However, although a company's favourable attitude to quality is a factor that should be considered throughout the business valuation process, this does not occur. On the basis of this premise, the company's position on the Grid will be introduced as a weighting factor. In order to position the company on the Grid, expert opinion will be necessary.

2.3.1.1. *Determining the company's membership for each stage of Crosby's Maturity Grid*

A group of experts were asked to assess at which stage the business was situated on Crosby's Maturity Grid: Uncertainty (A_1), Awakening (A_2), Enlightenment (A_3), Wisdom (A_4) and Certainty (A_5). They were then asked to what extent the business belonged to each stage, based on the following agreement or disagreement scale: 1 (totally disagree), 2 (strongly disagree), 3 (disagree), 4 (neutral), 5 (true) and 6 (very true). The experts' responses for each stage are considered a fuzzy subset, and the six possible values the expert may give is what we will call referential. Thus, we can speak of a level of membership μ_k, k = 1, 2, 3, 4, 5, 6. The membership function assigned to each of the previous labels is 0.0, 0.2, 0.4, 0.6, 0.8,1.0.

Table 2 shows the results obtained and the membership function value for each of the five stages considered.

Table 2. Experts results and stage *I* Index.

	1	**2**	**3**	**4**	**5**	**6**	I_j
A_1	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	I_1
A_2	a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	I_2
A_3	a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	a_{36}	I_3
A_4	a_{41}	a_{42}	a_{43}	a_{44}	a_{45}	a_{46}	I_4
A_5	a_{51}	a_{52}	a_{53}	a_{54}	a_{55}	a_{56}	I_5

The table's elements are denoted as a_{jk}, which indicates the number of experts that value stage *j* with the *k* grade on the previous scale of six elements.

For each stage *j*, an index is obtained as

$$I_j = \frac{\sum_{k=1}^{6} \mu_i a_{ik}}{L}, \; j = 1,2,\ldots 5 \tag{4}$$

where *L* is the number of experts that value a company's membership function for each stage of Crosby's Maturity Grid.

The total stage index I_T is obtained as:

$$I_T = \sum_{j=1}^{5} j \cdot I_j \Big/ \sum_{j=1}^{5} I_j \tag{5}$$

The stage a company is situated on Crosby's Maturity Grid is determined through the total Stage index I_T. However, it is now necessary to define exactly what degree of membership each one belongs to.

Degree of membership to Uncertainty stage (A_1),

$$\mu_1(I_T) = \begin{cases} 2 - I_T & 1 < I_T < 2 \\ 0 & otherwise \end{cases} \tag{6}$$

Degree of membership from Awakening stage (A_2) to Wisdom stage (A_4),

$$\mu_i(I_T) = \begin{cases} I_t - i + 1 & i-1 < I_T < i \\ i + 1 - I_t & i < I_T < i+1 \\ 0 & \textit{otherwise} \end{cases} \tag{7}$$

Degree of membership to Certainty stage (A_5).

$$\mu_5(I_T) = \begin{cases} I_t - 4 & 4 < I_T < 5 \\ 0 & \textit{otherwise} \end{cases} \tag{8}$$

The result will indicate a company's simultaneous membership to two of the stages outlined previously with their corresponding membership functions. In any case, the total of all the membership functions will be equal to 1.

2.3.1.2. *Determination of business value at each stage*

The different business values obtained through each of the selected business valuation methods are aggregated by applying BADD-OWA, with different weightings for each of the TFN extremes, and by applying the following coefficients: {2 for A_5, 1 for A_4, 0 for A_3, −1 for A_2, −2 for A_1} for each stage of Crosby's Maturity Grid. In this way, the business valuation methods that give higher business values are weighted much more if a company is at stage A_5, a little more if it is at stage A_4, all equally at stage A_3, a little more for those that provide lower values at phase A_2 and a lot more for those that provide lower values at phase A_1. As can be observed, the weighting of the methods that give a higher value will increase as the company advances in quality management.

Different weightings are used for each of the extremes of the company's TFN value provided by the H methodologies (1). That is to say, the weightings of the central values and those of the lower and higher extremes differ from each other.

$$\omega_{S_j}^{r} = q_l^{r,j-3} \Big/ \sum_{l=1}^{m} q_l^{r,j-3}, r = 1,2,3 \textit{ and } j = 1,\ldots 5 , \tag{9}$$

where $\omega_{S_j}^{r}$ is the weighting of extreme *r* (1, lower; 2, central and 3, higher) of the stage of Crosby's Maturity Grid (*j*), and l indicates *l*-th method with the greatest value communicated. Based on these weightings, the TFN value of the company is obtained for stage j, $\tilde{Q}_{sj} = \left(Q_{S_j}^{1}, Q_{S_j}^{2}, Q_{S_j}^{3}\right), j = 1,2,\ldots,5$, where,

$$Q_{S_j}^{r} = \sum_{l=1}^{m} q_l^{r,j-2} \Big/ \sum_{l=1}^{m} q_l^{r,j-3}, r = 1,2,3 \textit{ and } j = 1,\ldots 5 , \tag{10}$$

$j = 1,\ldots,5$ corresponding to the phases ... In this way, a different business value is obtained for each of the stages of Crosby's Maturity Grid the company belongs to.

2.3.1.3. *Business valuation taking the company's position on Crosby's Maturity Grid as the weighting factor*

Multiplying the value obtained in each phase by the company's membership function will be enough to determine its business value. The value of each stage is determined by expression (10) and the membership function to each phase is determined by expressions (6) to (8). The result is the TFN business value weighted according to the company's position on Crosby's Maturity Grid $\tilde{Q}_{Sj} = \left(Q_{S\,j}^{\,1}, Q_{S\,j}^{\,2}, Q_{S\,j}^{\,3}\right)$, $j = 1, 2, \ldots, 5$, being

$$Q_C^r = \sum_{j=1}^{5} \mu_j \left(I_T\right) \cdot Q_{S\,j}^{\,r}, \; r = 1,2,3 \tag{11}$$

2.3.2. *Business value: incorporating the importance attributed to each valuation methodology as weighting factor*

Although the business value obtained according to expression (11) weights the initial valuations of the H methods (1) in relation to the company's position on Crosby's Maturity Grid (expressions (6) to (8)), it does not take into account the level of confidence the company believes the H valuation methods merit.

Through BADD-UPOWA, it is possible to obtain the business value by considering the company's position on Crosby's Maturity Grid and incorporating confidence into each of the selected H methods. The aggregation of the two weighting factors considered is reflected in expression (12), whose result is the final business value $\tilde{Q}_F = \left(Q_F^1, Q_F^2, Q_F^3\right)$

$$\tilde{Q}_F = \beta \cdot \tilde{Q}_C + (1-\beta) \sum_{i=1}^{H} \upsilon_i \cdot \tilde{Q}_i \tag{12}$$

where

- β: takes values between 0 and 1 and indicates the importance assigned to quality management as weighting factor.
- $1-\beta$: indicates the importance assigned to the degree of confidence in the different business valuation methods as weighting factor, regardless of the values they expressed.
- υ_i: probability assigned to the methods i according to the degree of confidence the company has in the method.

As can be seen, through β each organization will decide which of the two weighting factors (an existing culture of quality or confidence in the methods) will have a greater specific weight in the final valuation.

$\tilde{Q}_F = \left(Q_F^1, Q_F^2, Q_F^3\right)$ summarizes the final business value through a TFN.

3. Case Study

This section aims to illustrate the model developed in the previous section through its application to a company. To do so, we collaborated with a Spanish company (NACE code 1621) dedicated to the manufacture of veneer sheets and wood-based panels. A non-listed company has been selected rather than a listed company since it is not possible to use the quoted price as a reference, and so the valuation process is more complex.

Table 3 shows the key figures in the company's financial statement available on the date the valuation was made.

Table 3. Company data in 2016.

Variable	Amount
Net sales	592.093.194
Material	313.492.980
Personnel costs	87.524.991
Depreciation of fixed assets	17.371.244
Other expenses	114.028.352
Financial performance	-6.232.829
Corporate tax	13.490.739
Net profit	39.952.059
Cash flow	57.323.303
Fixed assets	295.132.564
Net working capital	375.093.896
Equity	421.598.976
Non-current liabilities	116.340.700
Current liabilities	132.286.784

3.1. *Selection of the methods to be used for company valuation*

Among the most used methods in companies with continuity expectations are those based on the discount cash flow method. These methods consider the organization as any other individual investment project, and consequently its value depends on its future income-generating capacity.

As indicated in Section 2.1 of the theoretical model, business valuation based on the value of expected future earnings is not the only method available, other methods that are commonly used are based on the value reflected in financial statements, the value obtained after comparison with other companies, and the value reflected in the financial markets for listed companies.

Since the target company is not listed, in addition to the discount cash flow (DCF) method, three other methods, reflecting the valuation aspects highlighted above, have been selected:

- Asset-based Valuation, based on historical information.
- Comparable Company Valuation, where the value of a company is obtained and compared with listed companies belonging to same sector.
- The simplified abbreviated goodwill income method is a mixed method where business valuation is obtained as the sum of Asset-based Valuation plus the current value of annual income generated.

Table 4 presents a brief outline of the mathematical expressions used for business valuation in each method.

Table 4. Expressions based on the most well-known methods, from which company value can be obtained.

Method	Notes
Discount cash flow	T: time horizon.
$V=\sum_{t=1}^{T}\frac{CF_t}{(1+i)^t}+\frac{CF_T(1+g)}{(1+i)^T(i-g)}$	CF: cash flow. g: growth rate in the residual period.
Asset-based Valuation	
V=*Assets-Current liabilities*	
Comparable Company Valuation	*Variable*: PER, net profit, total assets
$V=\frac{Variable_s}{Variable_{ref}}\cdot quotation_{ref}$	s: company under evaluation ref: reference company
Abbreviated income	A: assets
$V=A+a_{nli}\cdot(B-A\cdot i)$	a_{nli} : present value of a unitary ordinary annuity for n years with interest rate i B: company's net profit

Table 5. Discount rate used in percentages.

	Optimistic	**Base**	**Pessimistic**
Risk premium	4.20	6.35	8.50
Risk free interest	1.36	1.36	1.36
Discount rate	5.56	7.71	9.86

3.2. *Implementation of the methods chosen to obtain several business valuation values*

It is important to highlight the following aspects of the discounted cash flow (DCF) method:

- The discount rate has been considered as a TFN (Table 5) in order to represent all the uncertainty inherent in the risk premium that it incorporates. For this purpose, risk-free interest is estimated using the interest rate on Spanish 10-year bonds [67] plus a risk premium which is situated between 4.2 and 8.5%. As a result, the discount rate ranges between 5.56 and 9.86, the most possible value being 7.71.
- Growth in the time horizon: cash flows in the time horizon have been obtained from the income statement projections for the next five years. To do so, the growth rate of net sales for the last years has been used, which in this case has a negative value of 1.67%.
- Growth during the residual period: There is a pseudo axiom that must be considered in the long-term evolution of a company[31]: the growth rate of a company's cash flow in the long-term cannot be higher than the growth rate of a country's nominal gross domestic product (GDP). For this reason and considering that the growth of Spain's GDP for the period 1961–2016 was 3.45%, the considered growth rate for the residual period is 1.72%, which is 50% of the growth of this variable in recent years. Sales and cash flow estimated for the 2017–2020 period are given in Table 6.

  With respect to the rest of the business valuation methods applied, it should be pointed out that:

Table 6. Sales and cash flow estimated (2017-2022)

	2017	2018	2019	2020	2021	2022
Estimated sales	592.093.194	582.176.138	572.425.185	562.837.552	553.410.503	562.970.288
Estimated cash flow	59.687.209	55.787.764	51.953.630	48.183.716	44.476.944	48.235.908

- Asset-based Valuation, Multiples and the simplified abbreviated goodwill income method are based on information from the last financial year available (2016).
- PER (price earnings ratio), net profit, and the company's total assets have been used to apply Comparable Company Valuation. The result obtained through these methods has been considered as a TFN: lower and higher bounds are the minimum and maximum values obtained, and the central point is the average value obtained. The principal magnitudes for the calculation of Comparable Company Valuation are given in Table 7.
- Three, five and seven years have been considered to apply the simplified abbreviated goodwill income method. As in the previous method, the results have been considered as a TFN.

Table 7. Main figures for the Comparable Company Valuation Method.

	Listed company	Analysed company
Sales	337.189.000	592.093.194
Net profit	37.957.000	39.952.059
Total Assets	972.679.000	670.226.460
Share price	2,465	

Table 8. Value of the company provided by the different methodologies, with indication of level of confidence assigned to each methodology.

Method (i)	Value of the company			Confidence in Method i (υ_i)
	q_i^1	q_i^2	q_i^3	
Discount cash flow	571,241,225	767,721,209	1,183,767,054	0,350
Net book value	421,598,976	421,598,976	421,598,976	0,250
Multiples	376,467,406	575,072,551	959,383,009	0,200
Abbreviated income	499,637,641	549,933,885	598,889,482	0,200

Table 8 shows the valuations obtained by the four selected methods through triplets of confidence, and the degree of confidence that the company has in each of them, which is one of the two weighting factors considered. As can be observed, the discount cash flow provides a TFN (571,241,225, 767,721,209, 1,183,767,054). According to this method, the business value ranges from 571,241,225 to 1,183,767,054, the most possible value being 767,721,209. Despite this, the results from each valuation method are different. If all the results are checked, it could be inferred at first that the business value will range between € 376,467,406 for the lower extreme of the TFN of the multiple method and € 1,183,767 for the higher extreme of the TFN of the discount cash flow method.

3.3. *Applying BADD-UPOWA for the quantification of business value*

The proposed method, using BADD-UPOWA, enables the valuations from different methods to be combined. This is possible because the weighting factors introduced are the degree of confidence for each of the methods used (see Table 8) and, what is especially innovative, the current quality philosophy of the company.

In order to assess the quality philosophy of the company, five experts were asked to assess at which stage the analysed business was situated on Crosby's Maturity Grid. According to Robbins,[69] the number of participants required for decision making problems varies between 5 and 7, so five experts were chosen. Table 9 summarizes the experts' opinions, the value of the membership function for each of the stages as defined by Crosby, and finally the Total stage index (I_T) obtained according to expression (5) and whose membership function is represented graphically in Figure 1.

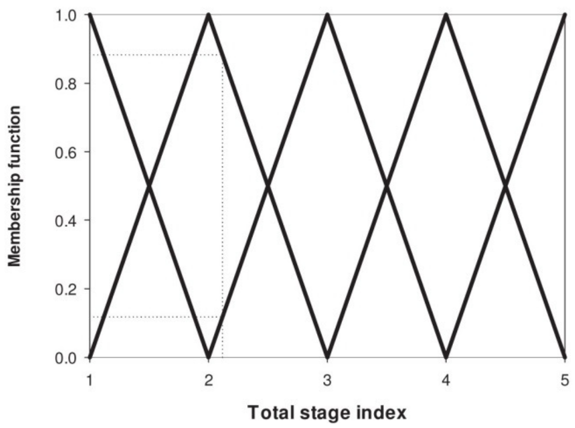

Fig. 1. Membership function of the total stage index.

By taking $\beta = 0,5$ (last line of Table 10), in other words, by giving the same specific weight in the final valuation to each of the two weighting factors considered, the result of the application of the proposed methodology is the TFN [468,748,622; 577,606,567; 759,861,374]. Thus the business value ranges between 468,748,622 and 759,861,374 €, the maximum possible value being 577,606,567 €.

The advantage of the model is clear: the initial business valuation ranges from 376,467,406 (minimum of the lower limits) to 1,183,767,054 (maximum of the higher limits), that is to say, an uncertainty of 807,299,648 €, and the final estimation ranges from 468,748,622 to 759,861,374 €, that is to say, an uncertainty of 291.112.752 €. As a result, uncertainty has been reduced to 516,186,896 €, 63.9% of the first value.

Finally, if point estimation is required, the previous TFN can be defuzzified using any of the known methods. For example, if it is used through the median, the business value rises to 595,955,782 € (last line of Table 10). Also, this value takes into account the company's quality management measured according to its position on Crosby's Maturity Grid, which is an important contribution to the literature on business valuation.

Table 9. Experts' opinion about company's membership to each of the phases of Crosby's Maturity Grid.

Stage	Totally disagree	Disagree a lot	Slightly disagree	Slightly agree	Quite agree	Totally agree	I_i
A_1	1	2	1	1			0,28
A_2			1	1	3		0,68
A_3		2	2	1			0,36
A_4	4	1					0,04
A_5	5						0,00
						I_T	2,12

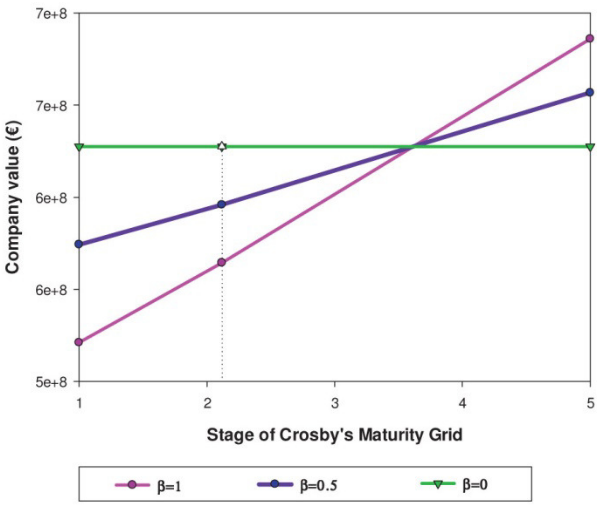

Fig. 2. Business value in relation to position on Cosby's Maturity Grid, according to the weighting factor considered: quality management (β = 1), confidence each method deserves (β = 0) or both (β = 0,5).

This contribution stands out because it carries out two simulations that modify solely the information initially provided by the experts about quality management initially reflected in (Table 9). The first simulation assumes that all the experts consulted fully agree that the company is at the first stage (A_1) of Crosby's Maturity Grid, unlike their opinions presented in Table 9. The second simulation proposes the opposite: it understands that all the experts now agree that the company is situated at the most advanced stage of Crosby's Maturity Grid (A_5). On applying the proposed methodology, the first simulation, for $\beta = 0.5$, obtains a final business value of TFN [462,537,855; 564,393,095; 705,908,902], whose defuzzified value would reach 574,308,237 €, lower than in the original case study. In contrast, the second simulation, again for β = 0,5, obtains a final business value of TFN [485,650,359;615,176,881; 911,042,597], whose defuzzified value is approximately 656,761,679 €, which is clearly higher than the initial 595,955,782 €. Figure 2 relates the business value to the company's position on Crosby's maturity Grid according to the defuzzified business values taking into account the experts' real opinions (Table 9) as well as the two simulations. As can be seen, as the company advances in quality, the value given by the model increases. Logically, this is reinforced if its position on Crosby's maturity grid ($\beta = 1$) is considered as the only weighting factor.

Table 10. Obtaining business value according to the weighting factor β.

	Lower extreme	Central value	Higher extreme	Defuzzified value
$\beta = 1$ (Only weighting factor quality management)	456.942.062	556.109.679	688.350.036	564.377.864
$\beta \neq 0$ (Only weighting factor degree of confidence)	480.555.182	599.103.454	831.372.711	627.533.701
$\beta = 0{,}5$ (Used in the case study)	468.748.622	577.606.567	759.861.374	595.955.782

4. Conclusions

In the context of the qualitative significance and operational frequency required for business valuation, this subject is clearly relevant to the business community. The subjectivity involved in any business valuation process requires the use of a fuzzy-logic-based computational framework for sustaining model design. Thus, this study aims to develop a fuzzy model that is able to integrate and objectify the information provided by different business valuation methods, incorporating quality management in its formal approaches and methods.

A description is given of the different stages of the proposed theoretical model. First, the most suitable business valuation methods are selected according to the characteristics of the business. They are then applied, bearing in mind that several operative approaches must be provided. The information is provided by way of TFNs in order to properly include the conditions of uncertainty and subjectivity that exist in the evaluations carried out. BADD-UPOWA is used to aggregate and summarize the subjective results obtained, establishing two weighting factors, which are: the level of confidence that the company has in each business valuation method used and the company's position on Crosby's Quality Administration Grid, which should be especially noted for its originality. As the company advances further on Crosby's Quality Administration Grid, the methods with greater valuations will have a higher weighting. In this way, the company's commitment to quality management is positively evaluated.

This model is doubly innovative on two levels. Although there are several studies that approach the problem of aggregation in general, and especially the use of Ordered Weighted Averaging Operators, the application of this tool as a means of improving business valuation processes is innovative. Moreover, for the first time a business valuation model incorporates quality management in its formal approach and method. This contribution is highly significant, since different authors have indicated the positive relation existing between quality and business value, as discussed in the introduction.

Finally, a case study is used to illustrate the proposed methodology, which is an effective research tool for demonstrating or presenting a theoretical model.[70] In addition, its use is especially recommended when the phenomenon that we want to study cannot be understood independently from its context and its natural environment, and when a large number of elements have to be considered[71]; the evaluation of quality culture in an organization is one of these situations. After the application of the proposed model, there is a 63.9% reduction in uncertainty compared to the initial business valuations. Also, the comparison of the results obtained in the case study with the two simulations carried out clearly show how this method increases the final business value as it advances in quality management.

Although the study carried out is of interest, future work in this area will involve increasing the number of companies analysed and experimenting with other methods such as a method called Hesitant Fuzzy Sets and triangular numbers that combines aggregation operators and takes into account membership function and non-membership function.

Acknowledgements

We would like to thank the anonymous reviewers for valuable comments that have improved the quality of the paper.

References

1. F. Blanco-Mesa and A.M. Gil-Lafuente, *A methodological approach for analysing stakeholder dynamics in decision-making Process: An application in family compensation Funds*, in Gil-Lafuente, A., Merigó, J., Dass, B., and Verma, R. (eds) Applied Mathematics and Computational Intelligence. Advances in Intelligent Systems and Computing (Springer, Cham, 2018).
2. F. Blanco-Mesa, A. M. Gil-Lafuente and J. M. Merigo, Dynamics of stakeholder relations with multi-person aggregation, *Kybernetes* **47**(9) (2018) 1801−1820.
3. I. Ionita, M. Stoica and C. Grigore, The determination of goodwill factors contribution using econometric models based on the subtle sets theory, *Annals of Faculty of Economics, University of Oradea, Faculty of Economics* **2**(1) (2009) 666–670.
4. AECA, *Métodos Prácticos de Valoración de Empresas* (Spanish Accounting and Business Administration Association, Documentos AECA, Madrid, 1989).
5. AECA, *Valoración de Pymes* (Spanish Accounting and Business Administration Association, Documentos AECA, Madrid, 2005).
6. A. Rojo and D. García, Firm valuation in Spain: an empirical research, *Spanish Journal of Finance and Accounting* **35**(132) (2006) 913–934.
7. C. Castaño, "Valoración de pequeñas empresas: una aplicación a la marca denominación de origen dehesa de Extremadura", PhD. Thesis (National Distance Education University, 2008).
8. J. A. Trigueros, *Concurso de acreedores. Aspectos prácticos de contabilidad y de valoración* (Francis Lefebvre, Madrid, 2018).
9. S. P. Pratt, R. F. Riley and R. P. Schweihs, *Valuing a Business: The Analysis and Appraisal of Closely Held Companies* (McGraw-Hill, New York, 2000).
10. S. Miller and R.F. Rielly, The asset-based approach − the adjusted net asset value method, *Insights*, Winter (2018) 28–40.
11. S. C. Hall, Comparable-companies business valuation in the current financial markets environment, *Journal of Financial Service Professionals* **57**(1) (2003) 9–10.

12. J. S. Yao, M. S. Chen and H. W. Lin, Valuation by using a fuzzy discounted cash flow model, *Expert Systems with Applications* **28**(2) (2005) 209–222.
13. C. T Tsao, Fuzzy net present values for capital investments in an uncertain environment, *Computers & Operations Research* **39**(8) (2012) 1885–1892.
14. M. E. Sansalvador and J. M. Brotons, Valuation of the option of abandoning ISO 9001 certification: an empirical study in Spain, *Total Quality Management & Business Excellence* **26**(11-12) (2015) 1255–1268.
15. I. Ucal and C. Kahraman, Fuzzy real option valuation for oil investments, *Ukio Technologinis ir Ekonominis Vystymas,* **15**(4) (2009) 646–669.
16. C. Carlsson and R. Fuller, A fuzzy approach to real option valuation, *Fuzzy Sets and Systems* **139**(2) (2003) 297–312.
17. M. C. Valls, Métodos clásicos de valoración de empresas, *European Research on Management and Business Economics* **7**(3) (2001) 49–66.
18. L. A. Zadeh, The role of fuzzy logic in the management of uncertainty in expert systems, *Fuzzy Sets and Systems* **11**(1-3) (1983) 199–227.
19. A. Zebda, The investigation of cost variances: a fuzzy sets theory approach, *Decision Sciences* **15**(3) (1984) 359–388.
20. A. Korvin, J. Strawer, and P. Siegel, An application of control system to cost variance analysis, *Managerial Finance* **21**(3) (1995) 17–38.
21. A. Kaufmann, Fuzzy set zero-based budgeting, *TIMS/Studies in Management Sciences* **20** (1984) 489–494.
22. H. Tanaka, T. Okuda, and K. Asai, A foundation of fuzzy decision problems and its application to an investment problem, *Kybernetes* **5** (1976) 25–30.
23. Y. Chan and Y. Yuan, Dealing with fuzziness in cost–volume–profit analysis, *Accounting and Business Research* **20**(78) (1990) 83–95.
24. C. Magni, S. Malagoli and G. Mastroleo, An alternative approach to firms' evaluation: Expert systems and fuzzy logic, *International Journal of Information Technology & Decision Making* **5**(1) (2006) 195–225.
25. A. M. Gil-Lafuente, C. Castillo-López and F. Blanco-Mesa, *A Paradigm Shift in Business Valuation Process Using Fuzzy Logic*, in A. Gil-Lafuente, J. Gil-Lafuente and J. Merigó-Lindahl (eds.), *Soft Computing in Management and Business Economics. Studies in Fuzziness and Soft Computing*, Vol. 287 (Springer, Berlin, Heidelberg, 2012).
26. H.-G. Peng and J.-Q. Wang, A multicriteria group decision-making method based on the normal cloud model with Zadeh's Z-numbers, *IEEE Transactions on Fuzzy Systems*, DOI:10.1109/TFUZZ.2018. 2816909 (in press) (2018).
27. V. Torra, Hesitant fuzzy sets, *International Journal of Intelligent Systems* **25**(6) (2010) 529–539.
28. G. W. Wei, Hesitant fuzzy prioritized operators and their application to multiple attribute decision making, *Knowledge-Based Systems* **31** (2012) 176–182.
29. X. K. Wang, H.-G. Peng and J.-Q. Wang, Hesitant linguistic intuitionistic fuzzy sets and their application in multi-criteria decision-making problems, *International Journal for Uncertainty Quantification* **8**(4) (2018) 321–341.
30. S. M. Yu, J. Wang, J. Q. Wang and L. Li, A multi-criteria decision-making model for hotel selection with linguistic distribution assessments, *Applied Soft Computing* **67** (2018) 739–753.
31. X. Y. Zhang, J. Q Wang and J. H. Hu, On novel operational laws and aggregation operators of picture 2-tuple linguistic information for MCDM problems, *International Journal of Fuzzy Systems* **20**(3) (2018) 958–969.
32. R. R. Yager, On ordered weighted averaging aggregation operators in multicriteria decision-making, *IEEE Transactions on Systems, Man and Cybernetics* **18**(1) (1988) 183–190.

33. J. M. Merigó, M. Casanovas and J. B.Yang, Group decision making with expertons and uncertain generalized probabilistic weighted aggregation operators, *European Journal of Operational Research* **235**(1) (2014) 15−224.
34. Y. Dong, Y. Xu, H. Li and B. Feng, The OWA-based consensus operator under linguistic representation models using position indexes, *European Journal of Operational Research* **203**(2) (2010) 455–463.
35. M. Zarghami and F. Szidarovszky, Revising the OWA operator for multi criteria decision making problems under uncertainty, *European Journal of Operational Research* **198**(1) (2009) 259–265.
36. R. Sadiq and S. Tesfamariam, Probability density functions based weights for ordered weighted averaging (OWA) operators: An example of water quality indices, *European Journal of Operational Research* **182**(3) (2007) 1350–1368.
37. J. M. Merigó, Fuzzy decision making with immediate probabilities, *Computers & Industrial Engineering* **58**(4) (2010) 651–657.
38. R. R. Yager, K. J. Engemann and D. P. Filev, On the concept of immediate probabilities. *International Journal of Intelligent Systems* **10**(4) (1995) 373–397.
39. G. L. Yang, J. B. Yang, W. B. Liu and X. X. Li, Cross-efficiency aggregation in DEA models using the evidential-reasoning approach, *European Journal of Operational Research* **231**(2) (2013) 393–404.
40. J. M. Merigó and M. Casanovas, The fuzzy generalized OWA operator and its application in strategic decision making, *Cybernetics & Systems* **41**(5) (2010) 359–370.
41. J. M. Merigó and M. Casanovas, The uncertain induced quasi-arithmetic OWA operator, *International Journal of Intelligent Systems* **26**(1) (2011) 1–24.
42. J. C. Harsanyi and C. Welfare, *Individualistic Ethics, and Interpersonal Comparisons of Utility* (Springer, Berlin, 1976).
43. H. Mo and Y. Deng, A new aggregating operator for linguistic information based on D numbers, *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems* **24**(6) (2016) 831–846.
44. F. Blanco-Mesa, A. M. Gil-Lafuente and J. M. Merigo, New aggregation operators for decision-making under uncertainty: an applications in selection of entrepreneurial opportunities, *Technological and Economic Development of Economy* **24**(2) (2018) 335−357.
45. F. Blanco-Mesa, J. M. Merigó and J. Kacprzyk, Bonferroni means with distance measures and the adequacy coefficient in entrepreneurial group theory, *Knowledge-Based Systems* **111**(1) (2016) 217−227.
46. E. León-Castro, E. Avilés-Ochoa, J. M. Merigó and A. M. Gil-Lafuente, Heavy moving averages and their application in econometric forecasting, *Cybernetics and Systems* **49**(1) (2018) 26–43.
47. E. León-Castro, E. Avilés-Ochoa and J.M. Merigó, Induced heavy moving averages, *International Journal of Intelligent Systems* **33** (9) (2018) 1823−1839.
48. L. A. Pérez-Arellano, E. León-Castro, E. Avilés-Ochoa and J. M. Merigó, Prioritized induced probabilistic operator and its application in group decision making, *International Journal of Machine Learning and Cybernetics*, doi: 10.1007/s13042-017-0724-2 (in press) (2017).
49. P. Crosby, *Quality is free: The Art of Making Quality Certain* (Penguin Books, New York, 1979).
50. J. A. Trigueros and M. E. Sansalvador, Management and measurement of quality in ISO 9000 organizations: An empirical study in Spain, *Total Quality Management & Business Excellence* **19**(5) (2008) 481−492.
51. A. Honarpour, A. Jusoh and K. Nor, Total quality management, knowledge management, and innovation: an empirical study in R&D units, *Total Quality Management & Business Excellence* **29**(7-8) (2018) 798−816.

52. H. Banna, R. Ahmad and E. H. Y. Koh, How does total quality management influence the loan quality of the bank? *Total Quality Management & Business Excellence* **29**(3-4) (2018) 287−300.
53. J. L. Nicolau and R. Sellers, The stock market's reaction to quality certification: Empirical evidence from Spain, *European Journal of Operational Research* **142**(2) (2002) 631−641.
54. D. S. Sharma, The association between ISO 9000 certification and financial performance, *International Journal of Accounting* **40**(2) (2005) 151−172.
55. J. M. Brotons and M. E. Sansalvador, El valor de la certificación ISO 9001 en las Pymes españolas dedicadas a la producción agrícola: incidencia de la crisis económica, *ITEA-Información Técnica Económica Agraria* **112**(1) (2016) 72−87.
56. Y.-C. Chung , S.-W. Tien, C.-H. Hsieh and C.-H. Tsai, A study of the business value of Total Quality Management, *Total Quality Management & Business Excellence* **19**(4) (2008) 367−379.
57. A. Terceño, J. Andres, M. G. Barbera and T. Lorenzana, Using fuzzy set theory to analyze investments and select portfolios of tangible investments in uncertain environments, *The International Journal of Uncertainty, Fuzziness and Knowledge-based Systems* **11**(3) (2003) 263−281.
58. R. R. Yager, Families of OWA operators, *Fuzzy Sets and Systems* **59**(2) (1993) 125−148.
59. R. R. Yager and D. P. Filev, *Essentials of Fuzzy Modeling and Control* (John & Wiley Sons, New York, 1994).
60. J. Leao and D. Costa, Group decision model based on ordered weighted distance to aid decisions on logistics, *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems* **26**(2) (2018) 233−254.
61. J. M. Merigó and G. Wei, Probabilistic aggregation operators and their application in uncertain multi-person decision-making, *Technological and Economic Development of Economy* **17**(2) (2011) 335−351.
62. X. Liu and H. Lou, Parameterized additive neat OWA operators with different orness levels, *International Journal of Intelligent Systems* **21**(10) (2006) 1045–1072.
63. J. Buckley, Ranking alternatives using fuzzy numbers, *Fuzzy Sets and Systems* **15**(1) (1985) 21−31.
64. S. H. Chen, Ranking fuzzy numbers with maximizing set and minimizing set, *Fuzzy Sets and Systems* **17**(2) (1985) 113−129.
65. D. Dubois and H. Prade, Ranking of fuzzy numbers in the setting of possibility theory, *Information Sciences* **30**(3) (1983) 183−224.
66. K. Kim and K. Park, Ranking fuzzy numbers with index of optimism, *Fuzzy Sets and Systems* **35**(2) (1990) 143−150.
67. Bank of Spain, Boletín del Mercado de Deuda Pública, available on the website https://www.bde.es/webbde/es/secciones/informes/banota/b161222.pdf (2017).
68. M. Casanovas, *Valoración de empresas: bases conceptuales y aplicaciones prácticas* (Profit Editorial, Barcelona, 2009).
69. S. P. Robbins, *Organizational behavior* (Prentice-Hall, New Jersey, 1994).
70. R. K. Yin, *Case Study Research: Design and Methods* (*Applied Social Research Methods Series*) (Sage Publications, London, 1989).
71. W. Johnston, M. Leach and A. Liu, Theory testing using case studies in business-to-business research, *Industrial Marketing Management* **28**(3) (1999) 201−213.